Since 25/02/2001 TOPAZ0 includes Q_W among the set of PO. An attempt has benn made to evaluate the theoretical uncertainty at the level of electroweak and of QCD corrections. TOPAZ0 will return, among PO, the two quantities, C_{1u} and C_{1d}. They are defined as follows:

$$\begin{aligned} C_{1u} &= -\frac{1}{2}\,\rho'_{\rm PV}\,\Big[1-\frac{8}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat\theta(M_W^2)\Big], \\ C_{1d} &= \frac{1}{2}\,\rho'_{\rm PV}\,\Big[1-\frac{4}{3}\,\kappa'_{\rm PV}(0)\sin^2\hat\theta(M_W^2)\Big], \end{aligned} \tag{1}$$

where $\sin^2\hat\theta(M_W^2)$ is the $\overline{MS}$ weak-mixing angle at the scale $\mu = M_W$ and where

$$\begin{aligned} \rho'_{\rm PV} &= \rho - \frac{\alpha}{2\,\pi}\Big[1+\frac{1}{\hat s^2}+4\,\hat v_e\,B_{\rm p(np)}+\frac{9}{16\,\hat s^2\hat c^2}\left(1-\frac{16}{9}\hat s^2\right)(1+\hat v_e^2)\Big], \\ \kappa'_{\rm PV}(0) &= \kappa_{\rm PV}(0) - \frac{\alpha}{2\,\pi\hat s^2}\Big[\frac{9-8\hat s^2}{8\,\hat s^2}-\frac{\hat v_e}{6}\left(\ln\frac{M_Z^2}{m_e^2}+\frac{1}{6}\right)+\left(\frac{9}{4}-4\hat s^2\right)\hat v_e\,B_{\rm p(np)} \\ &+ \frac{9}{16\,\hat s^2\hat c^2}\left(\frac{1}{2}\hat v_e+\frac{16}{9}\hat s^4\right)(1+\hat v_e^2)\Big], \end{aligned} \tag{2}$$

where $\hat v_e = 1-4\,\hat s^2$, $\hat s^2 = \sin^2\hat\theta(M_W^2)$ and where we have two different treatment of [2] $Z-\gamma$ boxes, perturbative

$$B_{\rm p} = \ln\frac{M_Z^2}{m^2}+\frac{3}{2}, \quad m = m_u = m_d = 75\ \text{MeV}, \tag{3}$$

and non-perturbative,

$$\begin{aligned} B_{\rm np} &= K + \frac{4}{5}\,(\xi_1)_B^p\,, \\ K &= M_Z^2\int_{M^2}^{\infty}\frac{du}{u(u+M_Z^2)}\Big[1-\frac{\alpha_{_S}(u)}{\pi}\Big], \quad (\xi_1)_B^p = 2.55, \end{aligned} \tag{4}$$

where M is a mass scale representing the onset of the asymptotic behavior, i.e. the regime where $\alpha_{_S}$ becomes small. We observe a plateau of stability in K for M centered around 0.5 GeV and this is the numerical value used. Furthermore [1]

$$\begin{aligned} \rho &= 1+\frac{\alpha}{4\,\pi\hat s^2}\Big[\frac{3}{4\,\hat s^2}\,\ln\hat c^2-\frac{7}{4}+\frac{3}{4}\,\frac{m_t^2}{M_W^2}\,(1+\delta_{\rm EW}+\delta_{\rm QCD}) \\ &+ \frac{3}{4}\,h\left(\frac{\ln(\hat c^2/h)}{\hat c^2-h}+\frac{1}{\hat c^2}\,\frac{\ln h}{1-h}\right)\Big], \\ \kappa_{\rm PV}(0) &= 1-\frac{\alpha}{2\,\pi\hat s^2}\Big[\frac{7}{9}-\frac{\hat s^2}{3}+\frac{Q_f}{3}\sum_f\left(I_e^{(3)}-2\,Q_f\,\hat s^2\right)\ln\frac{m_f^2}{M_W^2}\Big]. \end{aligned} \tag{5}$$

The strange quark mass is effectively choosen to be $m_s = 250$ MeV so that (with effective $m_u = m_d = 75$ MeV) we recover the dispersive analysis for the $Z-\gamma$ transition. Here $\delta_{\rm EW(QCD)}$ are the LO+NLO electroweak ($\mathcal{O}\left(\alpha_{_S}^2+\alpha_{_S}^3\right)$ QCD)

correction to ρ [3]. The evaluation of $\hat{s}^2$ includes the best available LO+NLO terms [4]

`TOPAZ0` default is the perturbative formulation of the above *factorized* result. There is the option of using some *additive* formulation where

$$\begin{aligned} C_{1u} &= -\frac{1}{2}\,\rho\left[1-\frac{8}{3}\,\kappa_{\rm PV}(0)\sin^2\hat{\theta}(M_{_W}^2)\right]+\Delta_u\,, \\ C_{1d} &= \frac{1}{2}\,\rho\left[1-\frac{4}{3}\,\kappa_{\rm PV}(0)\sin^2\hat{\theta}(M_{_W}^2)\right]+\Delta_d\,, \end{aligned} \tag{6}$$

where $\Delta_{u,d}$ are obtained from Eq.(1) by expanding and neglecting terms of $\mathcal{O}\left(\alpha^2\right)$. Results for $Q_{_W}$(Cs) are shown in the following table, corresponding to $M_{_Z}=91.1875\,$GeV, $M_{_H}=150\,$GeV and $\alpha_{_S}(M_{_Z}^2)=0.119$. The associated the-

m_t [GeV]	170	175	180
Fact/Pert	-73.0073	-72.9996	-72.9918
Fact/Non Pert	-73.2117	-73.2056	-73.1995
Add/Pet	-73.0095	-73.0023	-72.9951
Add/Non Pert	-73.2150	-73.2094	-73.2038

oretical uncertainty is approximately 2.8 per-mill, largely dominated by QCD. The two *new* options are controlled by the following flags:

- `OU9 = F(A)`. Factorized is default.
- `OU10 = P(N)`. Perturbative is default.

References

[1] W. J. MArciano and A. Sirlin, Phys. Rev. D27(1983)552.

[2] W. J. MArciano and A. Sirlin, Phys. Rev. D29(1984)75.

[3] D. Bardin, G. Passarino, Oxford, UK: Clarendon (1999) 685 p. (International series of monographs on physics. 104), chapter 8.

[4] D. Bardin, G. Passarino, Oxford, UK: Clarendon (1999) 685 p. (International series of monographs on physics. 104), chapter 6.