Introduction to Neutrino Oscillation Physics

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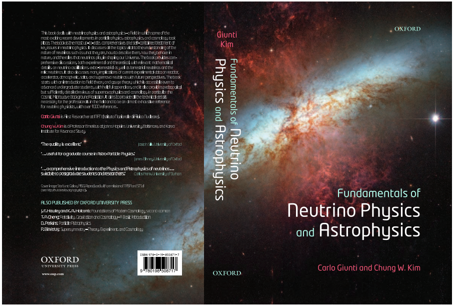

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Fundamentals of Neutrino Physics and Astrophysics

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Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses
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Part I

Theory of Neutrino Masses and Mixing

Dirac Neutrino Masses

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Dirac Mass

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$

- Dirac Lagrangian: $\mathscr{L}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$

- Chiral decomposition: $\nu_L \equiv \dfrac{1-\gamma^5}{2}\,\nu\,, \qquad \nu_R \equiv \dfrac{1+\gamma^5}{2}\,\nu$

$$\nu = \nu_L + \nu_R$$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

- In SM only $\nu_L \Longrightarrow$ no Dirac mass

- Oscillation experiments have shown that neutrinos are massive

- Simplest extension of the SM: add ν_R

Higgs Mechanism in SM

- SM: fermion masses are generated through the Higgs mechanism
- Higgs Doublet: $\Phi(x) = \begin{pmatrix} \phi^{+}(x) \\ \phi^{0}(x) \end{pmatrix}$
- Higgs Lagrangian: $\mathscr{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$
- Higgs Potential: $V(\Phi) = \mu^2 \, \Phi^\dagger \Phi + \lambda \, (\Phi^\dagger \Phi)^2$
- $\mu^2 < 0$, $\lambda > 0 \Longrightarrow V(\Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$, with $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$
- Vacuum: V_{min} for $\Phi^\dagger \Phi = \frac{v^2}{2} \Longrightarrow \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- Spontaneous Symmetry Breaking: $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_Q$
- Unitary Gauge: $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \ell_R \qquad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = -y^\ell \, \overline{L_L} \, \Phi \, \ell_R - y^\nu \, \overline{L_L} \, \widetilde{\Phi} \, \nu_R + \mathrm{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\tau_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathscr{L}_{H,\mathrm{L}} = & - \frac{y^\ell}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & - \frac{y^\nu}{\sqrt{2}} \begin{pmatrix} \overline{\nu_L} & \overline{\ell_L} \end{pmatrix} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \mathrm{H.c.} \end{aligned}$$

$$\mathscr{L}_{\mathrm{H,L}} = -\, y^{\ell}\, \frac{v}{\sqrt{2}}\, \overline{\ell_L}\, \ell_R - y^{\nu}\, \frac{v}{\sqrt{2}}\, \overline{\nu_L}\, \nu_R - \frac{y^{\ell}}{\sqrt{2}}\, \overline{\ell_L}\, \ell_R\, H - \frac{y^{\nu}}{\sqrt{2}}\, \overline{\nu_L}\, \nu_R\, H + \mathrm{H.c.}$$

$$m_{\ell} = y^{\ell}\, \frac{v}{\sqrt{2}} \qquad\qquad m_{\nu} = y^{\nu}\, \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^{\ell}}{\sqrt{2}} = \frac{m_{\ell}}{v} \qquad\qquad g_{\nu H} = \frac{y^{\nu}}{\sqrt{2}} = \frac{m_{\nu}}{v}$$

Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
ν'_{eR}	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \Phi \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{L'_{\alpha L}} \, \widetilde{\Phi} \, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\tau_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \sum_{\alpha,\beta=e,\mu,\tau} \left[Y'^{\ell}_{\alpha\beta} \, \overline{\ell'_{\alpha L}} \, \ell'_{\beta R} + Y'^{\nu}_{\alpha\beta} \, \overline{\nu'_{\alpha L}} \, \nu'_{\beta R} \right] + \mathrm{H.c.}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right) \left[\overline{\boldsymbol{\ell}'_L} \, Y'^{\ell} \, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L} \, Y'^{\nu} \, \boldsymbol{\nu}'_R \right] + \mathrm{H.c.}$$

$$\boldsymbol{\ell}'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \qquad \boldsymbol{\ell}'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y'^{\ell} \equiv \begin{pmatrix} Y'^{\ell}_{ee} & Y'^{\ell}_{e\mu} & Y'^{\ell}_{e\tau} \\ Y'^{\ell}_{\mu e} & Y'^{\ell}_{\mu\mu} & Y'^{\ell}_{\mu\tau} \\ Y'^{\ell}_{\tau e} & Y'^{\ell}_{\tau\mu} & Y'^{\ell}_{\tau\tau} \end{pmatrix} \qquad Y'^{\nu} \equiv \begin{pmatrix} Y'^{\nu}_{ee} & Y'^{\nu}_{e\mu} & Y'^{\nu}_{e\tau} \\ Y'^{\nu}_{\mu e} & Y'^{\nu}_{\mu\mu} & Y'^{\nu}_{\mu\tau} \\ Y'^{\nu}_{\tau e} & Y'^{\nu}_{\tau\mu} & Y'^{\nu}_{\tau\tau} \end{pmatrix}$$

$$M'^{\ell} = \frac{v}{\sqrt{2}} \, Y'^{\ell} \qquad M'^{\nu} = \frac{v}{\sqrt{2}} \, Y'^{\nu}$$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}'_L}\, Y'^{\ell}\, \boldsymbol{\ell}'_R + \overline{\boldsymbol{\nu}'_L}\, Y'^{\nu}\, \boldsymbol{\nu}'_R\right] + \mathrm{H.c.}$$

Diagonalization of Y'^{ℓ} and Y'^{ν} with unitary V^{ℓ}_L, V^{ℓ}_R, V^{ν}_L, V^{ν}_R

$$\boldsymbol{\ell}'_L = V^{\ell}_L\, \boldsymbol{\ell}_L \qquad \boldsymbol{\ell}'_R = V^{\ell}_R\, \boldsymbol{\ell}_R \qquad \boldsymbol{\nu}'_L = V^{\nu}_L\, \mathbf{n}_L \qquad \boldsymbol{\nu}'_R = V^{\nu}_R\, \mathbf{n}_R$$

Kinetic terms are invariant under unitary transformations of the fields

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\boldsymbol{\ell}_L} V^{\ell\dagger}_L Y'^{\ell} V^{\ell}_R \boldsymbol{\ell}_R + \overline{\boldsymbol{\nu}_L} V^{\nu\dagger}_L Y'^{\nu} V^{\nu}_R \boldsymbol{\nu}_R\right] + \mathrm{H.c.}$$

$$V^{\ell\dagger}_L \; Y'^{\ell} \; V^{\ell}_R = Y^{\ell} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\, \delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

$$V^{\nu\dagger}_L \; Y'^{\nu} \; V^{\nu}_R = Y^{\nu} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\, \delta_{kj} \qquad (k, j = 1, 2, 3)$$

Real and Positive y^{ℓ}_{α}, y^{ν}_{k}

$$V_L^\dagger \, Y' \, V_R = Y \qquad \Longleftrightarrow \qquad \begin{array}{ccccc} Y' & = & V_R^\dagger & Y & V_L \\ 2N^2 & & N^2 & N & N^2 \\ 18 & & 9 & 3 & 9 \end{array}$$

Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger}\,\ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger}\,\ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger}\,\boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger}\,\boldsymbol{\nu}'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell_L}\,Y^\ell\,\ell_R + \overline{\mathbf{n}_L}\,Y^\nu\,n_R\right] + \text{H.c.}$$

$$= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^\ell_\alpha\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y^\nu_k\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \text{H.c.}$$

Massive Dirac Lepton Fields

$$\ell_\alpha \equiv \ell_{\alpha L} + \ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \qquad (k = 1, 2, 3)$$

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell \, v}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha - \sum_{k=1}^{3} \frac{y_k^\nu \, v}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \qquad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^\ell}{\sqrt{2}} \, \overline{\ell_\alpha} \, \ell_\alpha \, H - \sum_{k=1}^{3} \frac{y_k^\nu}{\sqrt{2}} \, \overline{\nu_k} \, \nu_k \, H \qquad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^\ell \, v}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \qquad\qquad m_k = \frac{y_k^\nu \, v}{\sqrt{2}} \qquad (k = 1, 2, 3)$$

Lepton-Higgs coupling $\propto$ Lepton Mass

Quantization

$$\nu_k(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E_k} \sum_{h=\pm1} \left[a_k^{(h)}(p)\, u_k^{(h)}(p)\, e^{-ip\cdot x} + b_k^{(h)\dagger}(p)\, v_k^{(h)}(p)\, e^{ip\cdot x} \right]$$

$$p^0 = E_k = \sqrt{\vec{p}^2 + m_k^2}$$

$$(\not{p} - m_k)\, u_k^{(h)}(p) = 0$$
$$(\not{p} + m_k)\, v_k^{(h)}(p) = 0$$

$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, u_k^{(h)}(p) = h\, u_k^{(h)}(p)$$
$$\frac{\vec{p}\cdot\vec{\Sigma}}{|\vec{p}|}\, v_k^{(h)}(p) = -h\, v_k^{(h)}(p)$$

$$\{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} = \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = (2\pi)^3\, 2E_k\, \delta^3(\vec{p} - \vec{p}\,')\, \delta_{hh'}$$
$$\{a_k^{(h)}(p), a_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), a_k^{(h')\dagger}(p')\} = 0$$
$$\{b_k^{(h)}(p), b_k^{(h')}(p')\} = \{b_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$
$$\{a_k^{(h)}(p), b_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$
$$\{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')}(p')\} = 0$$

Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}}\, j_W^\rho\, W_\rho + \mathrm{H.c.}$$

Weak Charged Current: $\qquad j_W^\rho = j_{W,\mathrm{L}}^\rho + j_{W,\mathrm{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathrm{L}}^\rho = \sum_{\alpha=e,\mu,\tau} \overline{\nu'_\alpha}\, \gamma^\rho \left(1-\gamma^5\right)\, \ell'_\alpha = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{\alpha L}}\, \gamma^\rho\, \ell'_{\alpha L} = 2\, \overline{\boldsymbol{\nu}'_L}\, \gamma^\rho\, \boldsymbol{\ell}'_L$$

$$\boldsymbol{\ell}'_L = V_L^\ell\, \boldsymbol{\ell}_L \qquad\qquad \boldsymbol{\nu}'_L = V_L^\nu\, \mathbf{n}_L$$

$$j_{W,\mathrm{L}}^\rho = 2\, \overline{\mathbf{n}_L}\, V_L^{\nu\dagger}\, \gamma^\rho\, V_L^\ell\, \boldsymbol{\ell}_L = 2\, \overline{\mathbf{n}_L}\, V_L^{\nu\dagger}\, V_L^\ell\, \gamma^\rho\, \boldsymbol{\ell}_L = 2\, \overline{\mathbf{n}_L}\, U^\dagger\, \gamma^\rho\, \boldsymbol{\ell}_L$$

Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger}\, V_L^\ell \qquad\qquad U = V_L^{\ell\dagger}\, V_L^\nu$$

- Definition: Left-Handed Flavor Neutrino Fields

$$\nu_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- They allow us to write the Leptonic Weak Charged Current as in the SM:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\nu_L}\,\gamma^{\rho}\,\ell_L = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Each left-handed flavor neutrino field is associated with the corresponding charged lepton field which describes a massive charged lepton (e, μ, τ).

- In practice left-handed flavor neutrino fields are useful for calculations in the SM approximation of massless neutrinos (interactions).

- If neutrino masses must be taken into account, it is necessary to use

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L = 2\sum_{k=1}^{3}\sum_{\alpha=e,\mu,\tau}U^{*}_{\alpha k}\,\overline{\nu_{kL}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining
Flavor Lepton Numbers
as in the SM

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Standard Model: Lepton numbers are conserved

- Leptonic Weak Charged Current is invariant under the global U(1) gauge transformations

$$\ell_{\alpha L} \to e^{i\varphi_\alpha}\,\ell_{\alpha L} \qquad \nu_{\alpha L} \to e^{i\varphi_\alpha}\,\nu_{\alpha L} \qquad (\alpha = e, \mu, \tau)$$

- If neutrinos are massless (SM), Noether's theorem implies that there is, for each flavor, a conserved current:

$$j_\alpha^\rho = \overline{\nu_{\alpha L}}\,\gamma^\rho\,\nu_{\alpha L} + \overline{\ell_\alpha}\,\gamma^\rho\,\ell_\alpha \qquad \partial_\rho j_\alpha^\rho = 0$$

and a conserved charge:

$$\mathrm{L}_\alpha = \int \mathrm{d}^3x\, j_\alpha^0(x) \qquad \partial_0 \mathrm{L}_\alpha = 0$$

$$:\mathrm{L}_\alpha: \; = \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E} \left[a_{\nu_\alpha}^{(-)\dagger}(p)\, a_{\nu_\alpha}^{(-)}(p) - b_{\nu_\alpha}^{(+)\dagger}(p)\, b_{\nu_\alpha}^{(+)}(p) \right]$$

$$+ \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E} \sum_{h=\pm1} \left[a_{\ell_\alpha}^{(h)\dagger}(p)\, a_{\ell_\alpha}^{(h)}(p) - b_{\ell_\alpha}^{(h)\dagger}(p)\, b_{\ell_\alpha}^{(h)}(p) \right]$$

- Lepton-Higgs Yukawa Lagrangian:

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y^{\nu}_{k}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + \mathrm{H.c.}$$

- Mixing: $\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\,\nu_{kL} \qquad \Longleftrightarrow \qquad \nu_{kL} = \sum_{\alpha=e,\mu,\tau} U^{*}_{\alpha k}\,\nu_{\alpha L}$

$$\mathscr{L}_{H,\mathrm{L}} = -\left(\frac{v+H}{\sqrt{2}}\right)\sum_{\alpha=e,\mu,\tau}\left[y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \overline{\nu_{\alpha L}}\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}\right] + \mathrm{H.c.}$$

- Invariant for

$$\ell_{\alpha L} \to e^{i\varphi_\alpha}\,\ell_{\alpha L}\,, \quad \nu_{\alpha L} \to e^{i\varphi_\alpha}\,\nu_{\alpha L}$$

$$\ell_{\alpha R} \to e^{i\varphi_\alpha}\,\ell_{\alpha R}\,, \quad \sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR} \to e^{i\varphi_\alpha}\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}$$

- But kinetic part of neutrino Lagrangian is not invariant

$$\mathscr{L}^{(\nu)}_{\text{kinetic}} = \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} i\partial\!\!\!/ \nu_{\alpha L} + \sum_{k=1}^{3} \overline{\nu_{kR}} i\partial\!\!\!/ \nu_{kR}$$

because $\sum_{k=1}^{3} U_{\alpha k}\,y^{\nu}_{k}\,\nu_{kR}$ is not a unitary combination of the ν_{kR}'s

$$\mathscr{L}^{\mathrm{D}}_{\mathrm{mass}} = -\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \mathrm{H.c.}$$

L_e, L_μ, L_τ are not conserved

L is conserved: $\quad L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Total Lepton Number

- Dirac neutrino masses violate conservation of Flavor Lepton Numbers
- Total Lepton Number is conserved, because Lagrangian is invariant under the global U(1) gauge transformations

$$\nu_{kL} \to e^{i\varphi}\,\nu_{kL}\,, \qquad \nu_{kR} \to e^{i\varphi}\,\nu_{kR} \qquad (k = 1, 2, 3)$$
$$\ell_{\alpha L} \to e^{i\varphi}\,\ell_{\alpha L}\,, \qquad \ell_{\alpha R} \to e^{i\varphi}\,\ell_{\alpha R} \qquad (\alpha = e, \mu, \tau)$$

- From Noether's theorem:

$$j^\rho = \sum_{k=1}^{3} \overline{\nu_k}\,\gamma^\rho\,\nu_k + \sum_{\alpha=e,\mu,\tau} \overline{\ell_\alpha}\,\gamma^\rho\,\ell_\alpha \qquad \partial_\rho j^\rho = 0$$

Conserved charge: $\mathrm{L}_\alpha = \int \mathrm{d}^3x\, j^0_\alpha(x) \qquad \partial_0 \mathrm{L}_\alpha = 0$

$$:\mathrm{L}: = \sum_{k=1}^{3} \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)\dagger}_{\nu_k}(p)\, a^{(h)}_{\nu_k}(p) - b^{(h)\dagger}_{\nu_k}(p)\, b^{(h)}_{\nu_k}(p) \right]$$
$$+ \sum_{\alpha=e,\mu,\tau} \int \frac{\mathrm{d}^3p}{(2\pi)^3\,2E} \sum_{h=\pm1} \left[a^{(h)\dagger}_{\ell_\alpha}(p)\, a^{(h)}_{\ell_\alpha}(p) - b^{(h)\dagger}_{\ell_\alpha}(p)\, b^{(h)}_{\ell_\alpha}(p) \right]$$

Mixing Matrix

- Leptonic Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\boldsymbol{\ell}_L$

- $U = V_L^{\ell\dagger}\,V_L^{\nu} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \quad \Longrightarrow \quad \begin{array}{ll} \dfrac{N\,(N-1)}{2} = 3 & \text{Mixing Angles} \\ \dfrac{N\,(N+1)}{2} = 6 & \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, U_{\alpha k}^{*} \, \gamma^{\rho} \, \ell_{\alpha L}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations

$$\nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1, 2, 3) \, , \qquad \ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

- Performing this transformation, the Charged Current becomes

$$j^{\rho}_{W,\mathrm{L}} = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, e^{-i\varphi_k} \, U_{\alpha k}^{*} \, e^{i\varphi_\alpha} \, \gamma^{\rho} \, \ell_{\alpha L}$$

$$j^{\rho}_{W,\mathrm{L}} = 2 \, \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_{1} \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, \underbrace{e^{-i(\varphi_k - \varphi_1)}}_{N-1=2} \, U_{\alpha k}^{*} \, \underbrace{e^{i(\varphi_\alpha - \varphi_e)}}_{N-1=2} \, \gamma^{\rho} \, \ell_{\alpha L}$$

- There are $1 + (N-1) + (N-1) = 2\,N - 1 = 5$ arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- $2N - 1$ and not $2N$ phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant $\Longleftrightarrow$ conservation of Total Lepton Number.

- The mixing matrix contains

$$\frac{N\,(N+1)}{2} - (2\,N-1) = \frac{(N-1)\,(N-2)}{2} = 1 \qquad \text{Physical Phase}$$

- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$\begin{aligned} U &= R_{23} \, W_{13} \, R_{12} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \end{aligned}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $\quad$ $s_{ab} \equiv \sin\vartheta_{ab}$ $\quad$ $0 \leq \vartheta_{ab} \leq \dfrac{\pi}{2}$ $\quad$ $0 \leq \delta_{13} \leq 2\pi$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

CP Violation

- $U = U^* \quad \Longleftrightarrow \quad$ CP symmetry
- General conditions for CP violation (14 conditions):
  1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or $\pi/2$ (6 conditions)
  3. The physical phase is different from 0 or π (2 conditions)
- These 14 conditions are combined into the single condition $\det C \neq 0$

$$C = -i\,[M'^{\nu}\,M'^{\nu\dagger}\,,\,M'^{\ell}\,M'^{\ell\dagger}]$$

$$\begin{aligned}\det C = -2\,J \left(m_{\nu_2}^2 - m_{\nu_1}^2\right)\left(m_{\nu_3}^2 - m_{\nu_1}^2\right)\left(m_{\nu_3}^2 - m_{\nu_2}^2\right) \\ \left(m_{\mu}^2 - m_{e}^2\right)\left(m_{\tau}^2 - m_{e}^2\right)\left(m_{\tau}^2 - m_{\mu}^2\right)\end{aligned}$$

- Jarlskog invariant: $J = \mathfrak{Im}\Big[U_{\mu 3}\,U_{e2}\,U_{\mu 2}^*\,U_{e3}^*\Big]$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunietz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

Example: $\vartheta_{12} = 0$

$$U = R_{23} R_{13} W_{12}$$

$$W_{12} = \begin{pmatrix} \cos\vartheta_{12} & \sin\vartheta_{12} e^{-i\delta_{12}} & 0 \\ -\sin\vartheta_{12} e^{-i\delta_{12}} & \cos\vartheta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vartheta_{12} = 0 \qquad \Longrightarrow \qquad W_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{1}$$

real mixing matrix $\qquad U = R_{23} R_{13}$

Example: $\vartheta_{13} = \pi/2$

$$U = R_{23} W_{13} R_{12}$$

$$W_{13} = \begin{pmatrix} \cos\vartheta_{13} & 0 & \sin\vartheta_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin\vartheta_{13} e^{i\delta_{13}} & 0 & \cos\vartheta_{13} \end{pmatrix}$$

$$\vartheta_{13} = \pi/2 \qquad \Longrightarrow \qquad W_{13} = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -e^{i\delta_{13}} & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} e^{i\delta_{13}} & 0 \\ s_{12} s_{23} - c_{12} c_{23} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} e^{i\delta_{13}} & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix}$$

$$\lambda_{\mu 1} - \lambda_{\mu 2} = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi \qquad \lambda_{\tau 1} - \lambda_{\mu 1} = \lambda_{\tau 2} - \lambda_{\mu 2} \pm \pi$$

$$\nu_k \to e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \qquad \ell_\alpha \to e^{i\varphi_\alpha} \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$U \to \begin{pmatrix} e^{-i\varphi_e} & 0 & 0 \\ 0 & e^{-i\varphi_\mu} & 0 \\ 0 & 0 & e^{-i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{i(-\delta_{13} - \varphi_e + \varphi_3)} \\ |U_{\mu 1}| e^{i(\lambda_{\mu 1} - \varphi_\mu + \varphi_1)} & |U_{\mu 2}| e^{i(\lambda_{\mu 2} - \varphi_\mu + \varphi_2)} & 0 \\ |U_{\tau 1}| e^{i(\lambda_{\tau 1} - \varphi_\tau + \varphi_1)} & |U_{\tau 2}| e^{i(\lambda_{\tau 2} - \varphi_\tau + \varphi_2)} & 0 \end{pmatrix}$$

$$\varphi_1 = 0 \qquad \varphi_\mu = \lambda_{\mu 1} \qquad \varphi_\tau = \lambda_{\tau 1} \qquad \varphi_2 = \varphi_\mu - \lambda_{\mu 2} = \lambda_{\mu 1} - \lambda_{\mu 2}$$

$$\varphi_2 = \varphi_\tau - \lambda_{\tau 2} \pm \pi = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi = \lambda_{\mu 1} - \lambda_{\mu 2}$$ OK!

$$U = \begin{pmatrix} 0 & 0 & \pm 1 \\ |U_{\mu 1}| & |U_{\mu 2}| & 0 \\ |U_{\tau 1}| & -|U_{\tau 2}| & 0 \end{pmatrix}$$

Example: $m_{\nu_2} = m_{\nu_3}$

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L$$

$$U = R_{12}\,R_{13}\,W_{23} \qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,W_{23}^{\dagger}\,R_{13}^{\dagger}\,R_{12}^{\dagger}\,\gamma^{\rho}\,\ell_L$$

$$W_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_{23} & \sin\vartheta_{23}\,e^{-i\delta_{23}} \\ 0 & -\sin\vartheta_{23}\,e^{-i\delta_{23}} & \cos\vartheta_{23} \end{pmatrix}$$

$$W_{23}\,\mathbf{n}_L = \mathbf{n}'_L \qquad R_{12}\,R_{13} = U' \qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}'_L}\,U'^{\dagger}\,\gamma^{\rho}\,\ell_L$$

ν_2 and ν_3 are indistinguishable

drop the prime $\qquad \Longrightarrow \qquad j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L$

real mixing matrix $\qquad U = R_{12}\,R_{13}$

Jarlskog Invariant

$$\boxed{J = \Im\mathfrak{m}\left[U_{\mu 3}\, U_{e2}\, U_{\mu 2}^{*}\, U_{e3}^{*}\right]}$$

- All the imaginary parts of the rephasing-invariant quartic products $U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}$ are equal up to a sign:

$$\Im\mathfrak{m}\left[U_{\alpha k}^{*}\, U_{\beta k}\, U_{\alpha j}\, U_{\beta j}^{*}\right] = \pm J$$

- In the standard parameterization

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^{2} s_{13} \sin\delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12}\, \sin 2\vartheta_{23}\, \cos\vartheta_{13}\, \sin 2\vartheta_{13}\, \sin\delta_{13} \end{aligned}$$

- The Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way

Maximal CP Violation

- Maximal CP violation is defined as the case in which $|J|$ has its maximum possible value

$$|J|_{\text{max}} = \frac{1}{6\sqrt{3}}$$

- In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4 \,, \quad s_{13} = 1/\sqrt{3} \,, \quad \sin\delta_{13} = \pm 1$$

- This case is called Trimaximal Mixing. All the absolute values of the elements of the mixing matrix are equal to $1/\sqrt{3}$:

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp\frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- The unitarity of V_L^ℓ, V_R^ℓ and V_L^ν implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$
\begin{aligned}
j^\rho_{Z,\mathrm{L}} =& 2\, g_L^\nu\, \overline{\boldsymbol{\nu}'_L}\, \gamma^\rho\, \boldsymbol{\nu}'_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}'_L}\, \gamma^\rho \boldsymbol{\ell}'_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}'_R}\, \gamma^\rho \boldsymbol{\ell}'_R \\
=& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, V_L^{\nu\dagger}\, \gamma^\rho\, V_L^\nu\, \mathbf{n}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, V_L^{\ell\dagger}\, \gamma^\rho\, V_L^\ell\, \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, V_R^{\ell\dagger}\, \gamma^\rho\, V_R^\ell\, \boldsymbol{\ell}_R \\
=& 2\, g_L^\nu\, \overline{\mathbf{n}_L}\, \gamma^\rho\, \mathbf{n}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R
\end{aligned}
$$

- The unitarity of U implies the same expression for the neutral weak current in terms of the flavor neutrino fields $\boldsymbol{\nu}_L = U\, \mathbf{n}_L$:

$$
\begin{aligned}
j^\rho_{Z,\mathrm{L}} &= 2\, g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, U\, \gamma^\rho\, U^\dagger\, \boldsymbol{\nu}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R \\
&= 2\, g_L^\nu\, \overline{\boldsymbol{\nu}_L}\, \gamma^\rho\, \boldsymbol{\nu}_L + 2\, g_L^l\, \overline{\boldsymbol{\ell}_L}\, \gamma^\rho \boldsymbol{\ell}_L + 2\, g_R^l\, \overline{\boldsymbol{\ell}_R}\, \gamma^\rho \boldsymbol{\ell}_R
\end{aligned}
$$

Lepton Numbers Violating Processes

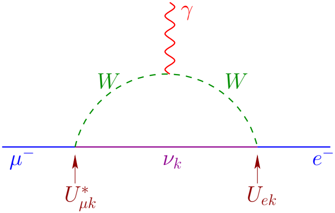
Dirac mass term allows L_e, L_μ, L_τ violating processes

Example: $\mu^\pm \to e^\pm + \gamma\,, \qquad \mu^\pm \to e^\pm + e^+ + e^-$

$$\boxed{\mu^- \to e^- + \gamma}$$

$$\sum_k U_{\mu k}^* U_{ek} = 0 \implies \text{only part of } \nu_k \text{ propagator} \propto m_k \text{ contributes}$$

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \underbrace{\frac{3\alpha}{32\pi} \left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor: $\displaystyle \frac{m_k}{m_W} \lesssim 10^{-11}$ for $m_k \lesssim 1\,\text{eV}$

$$(\text{BR})_{\text{the}} \lesssim 10^{-47} \qquad\qquad (\text{BR})_{\text{exp}} \lesssim 10^{-11}$$

Majorana Neutrino Masses

- Dirac Neutrino Masses
- Majorana Neutrino Masses
  - Two-Component Theory of a Massless Neutrino
  - Majorana Equation
  - Majorana Lagrangian
  - Majorana Antineutrino Jargon
  - Lepton Number
  - CP Symmetry
  - No Majorana Neutrino Mass in the SM
  - Effective Majorana Mass
  - Mixing of Three Majorana Neutrinos
  - Mixing Matrix
  - Neutrinoless Double-Beta Decay
  - Effective Majorana Neutrino Mass
  - Majorana Neutrino Mass $\Leftrightarrow$ $\beta\beta_{0\nu}$ Decay

Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- Dirac Equation: $(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0$
- Chiral components of a Fermion Field: $\psi = \psi_L + \psi_R$
- The equations for the Chiral components are coupled by the mass:

$$i\gamma^{\mu}\partial_{\mu}\psi_L = m\,\psi_R$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = m\,\psi_L$$

- They are decoupled for a massless fermion: Weyl Equations (1929)

$$i\gamma^{\mu}\partial_{\mu}\psi_L = 0$$
$$i\gamma^{\mu}\partial_{\mu}\psi_R = 0$$

- A massless fermion can be described by a single chiral field ψ_L or ψ_R (Weyl Spinor).

- ψ_L and ψ_R have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \qquad\qquad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix}$$

- The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to the violation of parity

- The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields $\Longrightarrow$ Two-component Theory of a Massless Neutrino (1957)

- $V - A$ Charged-Current Weak Interactions $\Longrightarrow$ ν_L

- In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of ν_R

Majorana Equation

- Can a two-component spinor describe a massive fermion? Yes! (E. Majorana, 1937)
- Trick: ψ_R and ψ_L are not independent.
- The relation connecting ψ_R and ψ_L must be compatible with the Dirac equation:

$$i\gamma^\mu \partial_\mu \psi_L = m \psi_R \qquad\qquad i\gamma^\mu \partial_\mu \psi_R = m \psi_L$$

- The two equations must be two ways of writing the same equation for one independent field, say ψ_L.
- Consider $i\gamma^\mu \partial_\mu \psi_R = m \psi_L$
- Take the Hermitian conjugate and multiply on the right with γ^0: $-i\partial_\mu \psi_R^\dagger \gamma^{\mu\dagger} \gamma^0 = m \overline{\psi_L}$
- $\gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu \implies -i\partial_\mu \overline{\psi_R} \gamma^\mu = m \overline{\psi_L}$
- Transpose and multiply on the left with $\mathcal{C}$ ($\mathcal{C} \gamma_\mu^T \mathcal{C}^{-1} = -\gamma_\mu$) $\implies$ $i\gamma^\mu \partial_\mu \mathcal{C} \overline{\psi_R}^T = m \mathcal{C} \overline{\psi_L}^T$

- $\mathcal{C}\,\overline{\psi_L}^T$ is right-handed and $\mathcal{C}\,\overline{\psi_R}^T$ is left-handed
- $i\gamma^\mu \partial_\mu \mathcal{C}\,\overline{\psi_R}^T = m\,\mathcal{C}\,\overline{\psi_L}^T$ has the same structure as $i\gamma^\mu \partial_\mu \psi_L = m\,\psi_R$
- We can consider them as identical by setting

$$\psi_R = \xi\,\mathcal{C}\,\overline{\psi_L}^T \qquad \text{with} \qquad |\xi|^2 = 1$$

- ξ is unphysical phase factor which can be eliminated by rephasing

$$\psi_L \to \xi^{1/2}\,\psi_L \qquad \Longrightarrow \qquad \boxed{\psi_R = \mathcal{C}\,\overline{\psi_L}^T}$$

- Majorana Equation: $\boxed{i\gamma^\mu \partial_\mu \psi_L = m\,\mathcal{C}\,\overline{\psi_L}^T}$
- The field $\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\,\overline{\psi_L}^T$ is called Majorana Field
- Majorana Condition: $\boxed{\psi = \mathcal{C}\,\overline{\psi}^T}$
- A Majorana Field has only two independent components
- Chiral representation: $\psi = \begin{pmatrix} i\sigma^2 \chi_L^* \\ \chi_L \end{pmatrix}$

- Charge Conjugation: $\psi_L^C = \mathcal{C}\,\overline{\psi_L}^T$
- Majorana Field: $\psi = \psi_L + \psi_L^C$ Majorana Condition: $\psi = \psi^C$
- The Majorana condition implies the equality of particle and antiparticle
- Only neutral fermions can be Majorana particles
- Dirac equation for fermion with charge q coupled to electromagnetic field A_μ:

$$(i\gamma^\mu\partial_\mu - q\,\gamma^\mu A_\mu - m)\,\psi = 0 \qquad \text{(particle)}$$

$$(i\gamma^\mu\partial_\mu + q\,\gamma^\mu A_\mu - m)\,\psi^C = 0 \qquad \text{(antiparticle)}$$

  If $q \neq 0$, ψ and ψ^C obey different equations and the Majorana equality cannot be imposed

- For a Majorana field, the electromagnetic current vanishes identically: $\overline{\psi}\gamma^\mu\psi = \overline{\psi^C}\gamma^\mu\psi^C = -\psi^T\mathcal{C}^\dagger\gamma^\mu\mathcal{C}\overline{\psi}^T = \overline{\psi}\mathcal{C}\gamma^{\mu\,T}\mathcal{C}^\dagger\psi = -\overline{\psi}\gamma^\mu\psi = 0$

Majorana Lagrangian

- Let us consider first the Dirac Lagrangian
$$\mathscr{L}^{\mathrm{D}} = \overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R)$$

- In order to write a Majorana Mass Term using ν_L alone, we make the substitution $\nu_R \to \nu_L^C = \mathcal{C}\,\overline{\nu_L}^T$

- Majorana Lagrangian:
$$\mathscr{L}^{\mathrm{M}} = \frac{1}{2}\left[\overline{\nu_L}\,i\partial\!\!\!/\,\nu_L + \overline{\nu_L^C}\,i\partial\!\!\!/\,\nu_L^C - m\left(\overline{\nu_L^C}\,\nu_L + \overline{\nu_L}\,\nu_L^C\right)\right]$$

- The overall factor $1/2$ avoids double counting in the derivation of the due to the fact that ν_L^C and $\overline{\nu_L}$ are not independent $(\nu_L^C = \mathcal{C}\overline{\nu_L}^T)$
$$\mathscr{L}^{\mathrm{M}} = \overline{\nu_L}\,i\partial\!\!\!/\,\nu_L - \frac{m}{2}\left(-\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \overline{\nu_L}\,\mathcal{C}\,\overline{\nu_L}^T\right)$$

- Majorana Field: $\nu = \nu_L + \nu_L^C$

- Majorana Condition: $\nu^C = \nu$

- Majorana Lagrangian: $\mathscr{L}^{\text{M}} = \frac{1}{2}\,\overline{\nu}\,(i\partial\!\!\!/ - m)\,\nu$

- The factor $1/2$ distinguishes the Majorana Lagrangian from the Dirac Lagrangian

- Quantized Dirac Neutrino Field:

$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2E} \sum_{h=\pm 1} \left[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + b^{(h)\,\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x} \right]$$

- Quantized Majorana Neutrino Field $[b^{(h)}(p) = a^{(h)}(p)]$

$$\nu(x) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2E} \sum_{h=\pm 1} \left[a^{(h)}(p)\, u^{(h)}(p)\, e^{-ip\cdot x} + a^{(h)\dagger}(p)\, v^{(h)}(p)\, e^{ip\cdot x} \right]$$

- A Majorana field has half the degrees of freedom of a Dirac field

Majorana Antineutrino Jargon

- A Majorana neutrino is the same as a Majorana antineutrino
- Neutrino interactions are described by the CC and NC Lagrangians

$$
\mathscr{L}_{\mathrm{I,L}}^{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_L} \, \gamma^\mu \, \ell_L \, W_\mu + \overline{\ell_L} \, \gamma^\mu \, \nu_L \, W_\mu^\dagger \right)
$$

$$
\mathscr{L}_{\mathrm{I},\nu}^{\mathrm{NC}} = -\frac{g}{2 \cos \vartheta_{\mathrm{W}}} \, \overline{\nu_L} \, \gamma^\mu \, \nu_L \, Z_\mu
$$

- In practice, since detectable neutrinos are always ultrarelativistic, the neutrino mass can be neglected in interactions

- In interaction amplitudes we neglect corrections of order m/E

- Dirac: $\begin{cases} \nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{cases} \\ \overline{\nu_L} \begin{cases} \text{destroys right-handed antineutrinos} \\ \text{creates left-handed neutrinos} \end{cases} \end{cases}$

- Majorana: $\begin{cases} \nu_L \begin{cases} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{cases} \\ \overline{\nu_L} \begin{cases} \text{destroys right-handed neutrinos} \\ \text{creates left-handed neutrinos} \end{cases} \end{cases}$

- Common definitions:

  Majorana neutrino with negative helicity $\equiv$ neutrino

  Majorana neutrino with positive helicity $\equiv$ antineutrino

Lepton Number

- The Majorana Mass Term
$$\mathscr{L}^{\text{M}}_{\text{mass}} = \frac{1}{2}\, m \left(\nu_L^T\, \mathcal{C}^\dagger\, \nu_L + \nu_L^\dagger\, \mathcal{C}\, \nu_L^* \right)$$
is not invariant under the global U(1) gauge transformation
$$\nu_L \to e^{i\varphi}\, \nu_L$$
$$\cancel{L = -1} \quad \longleftarrow \quad \boxed{\nu^c = \nu} \quad \longrightarrow \quad \cancel{L = +1}$$

- The Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

- However, the Total Lepton Number is conserved in interactions in the ultrarelativistic approximation of massless neutrinos

- Best process to find the violation of the Total Lepton Number: Neutrinoless Double-β Decay
$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + 2\,e^- \qquad (\beta\beta^-_{0\nu})$$
$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z-2) + 2\,e^+ \qquad (\beta\beta^+_{0\nu})$$

CP Symmetry

- Under a CP transformation

$$
\begin{aligned}
\mathrm{U}_{\mathrm{CP}} \nu_L(x) \mathrm{U}_{\mathrm{CP}}^{-1} &= \xi_\nu^{\mathrm{CP}} \, \gamma^0 \, \nu_L^C(x_{\mathrm{P}}) \\
\mathrm{U}_{\mathrm{CP}} \nu_L^C(x) \mathrm{U}_{\mathrm{CP}}^{-1} &= -\xi_\nu^{\mathrm{CP}*} \, \gamma^0 \, \nu_L(x_{\mathrm{P}}) \\
\mathrm{U}_{\mathrm{CP}} \overline{\nu_L}(x) \mathrm{U}_{\mathrm{CP}}^{-1} &= \xi_\nu^{\mathrm{CP}*} \, \overline{\nu_L^C}(x_{\mathrm{P}}) \, \gamma^0 \\
\mathrm{U}_{\mathrm{CP}} \overline{\nu_L^C}(x) \mathrm{U}_{\mathrm{CP}}^{-1} &= -\xi_\nu^{\mathrm{CP}} \, \overline{\nu_L}(x_{\mathrm{P}}) \, \gamma^0
\end{aligned}
$$

with $|\xi_\nu^{\mathrm{CP}}|^2 = 1$, $x^\mu = (x^0, \vec{x})$, and $x_{\mathrm{P}}^\mu = (x^0, -\vec{x})$

- The theory is CP-symmetric if there are values of the phase ξ_ν^{CP} such that the Lagrangian transforms as

$$\mathrm{U}_{\mathrm{CP}} \mathscr{L}(x) \mathrm{U}_{\mathrm{CP}}^{-1} = \mathscr{L}(x_{\mathrm{P}})$$

in order to keep invariant the action $I = \int \mathrm{d}^4 x \, \mathscr{L}(x)$

- The Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{\text{M}}(x) = -\frac{1}{2}\, m \left[\overline{\nu_L^C}(x)\, \nu_L(x) + \overline{\nu_L}(x)\, \nu_L^C(x) \right]$$

transforms as

$$\begin{aligned} \text{U}_{\text{CP}} \mathscr{L}_{\text{mass}}^{\text{M}}(x) \text{U}_{\text{CP}}^{-1} = & -\frac{1}{2}\, m \left[-(\xi_\nu^{\text{CP}})^2\, \overline{\nu_L}(x_{\text{P}})\, \nu_L^C(x_{\text{P}}) \right. \\ & \left. -(\xi_\nu^{\text{CP}*})^2\, \overline{\nu_L^C}(x_{\text{P}})\, \nu_L(x_{\text{P}}) \right] \end{aligned}$$

- $\text{U}_{\text{CP}} \mathscr{L}_{\text{mass}}^{\text{M}}(x) \text{U}_{\text{CP}}^{-1} = \mathscr{L}_{\text{mass}}^{\text{M}}(x_{\text{P}})$ for $\boxed{\xi_\nu^{\text{CP}} = \pm i}$

- The one-generation Majorana theory is CP-symmetric

- The Majorana case is different from the Dirac case, in which the CP phase ξ_ν^{CP} is arbitrary

No Majorana Neutrino Mass in the SM

- A Majorana Mass Term $\propto \left[\nu_L^T \, \mathcal{C}^\dagger \, \nu_L - \overline{\nu_L} \, \mathcal{C} \, \overline{\nu_L}^T\right]$ involves only the neutrino left-handed chiral field ν_L, which is present in the SM (one for each lepton generation)
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

		I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet	$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$\begin{matrix} 1/2 \\ -1/2 \end{matrix}$	-1	$\begin{matrix} 0 \\ -1 \end{matrix}$
lepton singlet	ℓ_R	0	0	-2	-1
Higgs doublet	$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$	$1/2$	$\begin{matrix} 1/2 \\ -1/2 \end{matrix}$	$+1$	$\begin{matrix} 1 \\ 0 \end{matrix}$

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2 \Longrightarrow$ needed Higgs triplet with $Y = 2$

Effective Majorana Mass

- Dimensional analysis: Fermion Field $\sim [E]^{3/2}$ Boson Field $\sim [E]$
- Dimensionless action: $I = \int d^4x \, \mathscr{L}(x) \Longrightarrow \mathscr{L}(x) \sim [E]^4$
- Kinetic terms: $\overline{\psi} i \partial\!\!\!/ \psi \sim [E]^4$, $(\partial_\mu \phi)^\dagger \partial^\mu \phi \sim [E]^4$
- Mass terms: $m \overline{\psi} \psi \sim [E]^4$, $m^2 \phi^\dagger \phi \sim [E]^4$
- CC weak interaction: $\overline{\nu_L} \gamma^\rho \ell_L W_\rho \sim [E]^4$
- Yukawa couplings: $\overline{L_{\alpha L}} \Phi \ell'_{\beta R} \sim [E]^4$
- Product of fields $\mathscr{O}_d$ with energy dimension $d \equiv$ dim-d operator
- Coupling constant of $\mathscr{O}_d$ has dimension $[E]^{-(d-4)}$
- $\mathscr{O}_{d>4}$ are not renormalizable

- SM Lagrangian includes all $\mathscr{O}_{d \leq 4}$ invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- SM cannot be considered as the final theory of everything
- SM is an effective low-energy theory
- It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- It is plausible that at low-energy there are effective non-renormalizable $\mathscr{O}_{d>4}$ [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- All $\mathscr{O}_d$ must respect $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies
- Approach analogous to effective non-renormalizable four-fermion Fermi theory of weak interactions, which is a low-energy manifestation of the SM

- $\mathscr{O}_{d>4}$ is suppressed by a coefficient $\mathcal{M}^{-(d-4)}$, where $\mathcal{M}$ is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}} \mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2} \mathscr{O}_6 + \ldots$$

- Analogy with $\mathscr{L}_{\text{eff}}^{(\text{CC})} = -\frac{G_{\text{F}}}{\sqrt{2}} j_{W\mu}^{\dagger} j_W^{\mu}$:

$$\mathscr{O}_6 \to j_{W\mu}^{\dagger} j_W^{\mu} \qquad\qquad \frac{g_6}{\mathcal{M}^2} \to \frac{G_{\text{F}}}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

- $\mathcal{M}^{-(d-4)}$ is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM

- The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality

- $\mathscr{O}_5 \quad \Longrightarrow \quad$ Majorana neutrino masses (Lepton number violation)

- $\mathscr{O}_6 \quad \Longrightarrow \quad$ Baryon number violation (proton decay)

- Only one dim-5 operator:

$$\begin{aligned}\mathscr{O}_5 &= (L_L^T\,\tau_2\,\Phi)\,\mathcal{C}^\dagger\,(\Phi^T\,\tau_2\,L_L) + \text{H.c.}\\ &= \frac{1}{2}\,(L_L^T\,\mathcal{C}^\dagger\,\tau_2\,\vec{\tau}\,L_L)\cdot(\Phi^T\,\tau_2\,\vec{\tau}\,\Phi) + \text{H.c.}\end{aligned}$$

$$\mathscr{L}_5 = \frac{g_5}{2\mathcal{M}}\,(L_L^T\,\mathcal{C}^\dagger\,\tau_2\,\vec{\tau}\,L_L)\cdot(\Phi^T\,\tau_2\,\vec{\tau}\,\Phi) + \text{H.c.}$$

- Electroweak Symmetry Breaking: $\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathscr{L}_{\text{mass}}^{\text{M}} = \dfrac{1}{2}\,\dfrac{g_5\,v^2}{\mathcal{M}}\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.} \quad \Longrightarrow \quad \boxed{m = \dfrac{g_5\,v^2}{\mathcal{M}}}$

- The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM
- $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_{\mathrm{D}}^2}{\mathcal{M}}$ natural explanation of smallness of neutrino masses (special case: See-Saw Mechanism)
- Example: $m_{\mathrm{D}} \sim v \sim 10^2\,\mathrm{GeV}$ and $\mathcal{M} \sim 10^{15}\,\mathrm{GeV} \implies m \sim 10^{-2}\,\mathrm{eV}$

Mixing of Three Majorana Neutrinos

- $\nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$
$$\begin{aligned} \mathscr{L}^{\text{M}}_{\text{mass}} &= \frac{1}{2} \left(\nu'^{T}_{L} \, \mathcal{C}^{\dagger} \, M^{L} \, \nu'_{L} - \overline{\nu'_{L}} \, M^{L\dagger} \, \mathcal{C} \, \nu'^{T}_{L} \right) \\ &= \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \left(\nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} - \overline{\nu'_{\alpha L}} \, M^{L*}_{\beta\alpha} \, \mathcal{C} \, \nu'^{T}_{\beta L} \right) \end{aligned}$$

- In general, the matrix M^L is a complex symmetric matrix

$$\begin{aligned} \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\beta L} &= - \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, M^{L}_{\alpha\beta} \, (\mathcal{C}^{\dagger})^{T} \, \nu'_{\alpha L} \\ &= \sum_{\alpha,\beta} \nu'^{T}_{\beta L} \, \mathcal{C}^{\dagger} \, M^{L}_{\alpha\beta} \, \nu'_{\alpha L} = \sum_{\alpha,\beta} \nu'^{T}_{\alpha L} \, \mathcal{C}^{\dagger} \, M^{L}_{\beta\alpha} \, \nu'_{\beta L} \end{aligned}$$

$$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha} \quad \Longleftrightarrow \quad M^{L} = M^{L\,T}$$

- $\mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} = \frac{1}{2}\left(\boldsymbol{\nu}'^{T}_{L}\,\mathcal{C}^{\dagger}\,M^{L}\,\boldsymbol{\nu}'_{L} - \overline{\boldsymbol{\nu}'_{L}}\,M^{L\dagger}\,\mathcal{C}\,\boldsymbol{\nu}'^{T}_{L}\right)$
- Diagonalization: $\boldsymbol{\nu}'_{L} = \mathbf{n}_{L}\,V^{\nu\,\dagger}_{L}$ with unitary V^{ν}_{L}
- $(V^{\nu}_{L})^{T}\,M^{L}\,V^{\nu}_{L} = M\,, \quad M_{kj} = m_{k}\,\delta_{kj} \quad (k,j=1,2,3)$
- Real and Positive m_k
- Left-handed chiral fields with definite mass: $\mathbf{n}_{L} = V^{\nu\,\dagger}_{L}\,\boldsymbol{\nu}'_{L} = \begin{pmatrix}\nu_{1L}\\ \nu_{2L}\\ \nu_{3L}\end{pmatrix}$

$$\begin{aligned}\mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} &= \frac{1}{2}\left(\mathbf{n}^{T}_{L}\,\mathcal{C}^{\dagger}\,M\,\mathbf{n}_{L} - \overline{\mathbf{n}_{L}}\,M\,\mathcal{C}\,\mathbf{n}^{T}_{L}\right)\\ &= \frac{1}{2}\sum_{k=1}^{3} m_{k}\left(\nu^{T}_{kL}\,\mathcal{C}^{\dagger}\,\nu_{kL} - \overline{\nu_{kL}}\,\mathcal{C}\,\nu^{T}_{kL}\right)\end{aligned}$$

- Majorana fields of massive neutrinos: $\nu_{k} = \nu_{kL} + \nu^{C}_{kL}$ $\quad \boxed{\nu^{C}_{k} = \nu_{k}}$
- $\mathbf{n} = \begin{pmatrix}\nu_{1}\\ \nu_{2}\\ \nu_{3}\end{pmatrix} \Longrightarrow \mathscr{L}^{\mathrm{M}} = \frac{1}{2}\sum_{k=1}^{3}\overline{\nu_{k}}\,(i\partial\!\!\!/ - m_{k})\,\nu_{k} = \frac{1}{2}\,\overline{\mathbf{n}}\,(i\partial\!\!\!/ - M)\,\mathbf{n}$

Mixing Matrix

- Leptonic Weak Charged Current:

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\mathbf{n}_L}\,U^{\dagger}\,\gamma^{\rho}\,\ell_L \qquad \text{with} \qquad U = V_L^{\ell\dagger}\,V_L^{\nu}$$

- Definition of the left-handed flavor neutrino fields:

$$\boldsymbol{\nu}_L = U\,\mathbf{n}_L = V_L^{\ell\dagger}\,\boldsymbol{\nu}'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- Leptonic Weak Charged Current has the SM form

$$j^{\rho}_{W,\mathrm{L}} = 2\,\overline{\boldsymbol{\nu}_L}\,\gamma^{\rho}\,\boldsymbol{\ell}_L = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\,\gamma^{\rho}\,\ell_{\alpha L}$$

- Important difference with respect to Dirac case:
  Two additional CP-violating phases: Majorana phases

- The Majorana Mass Term $\mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} = \frac{1}{2} \sum_{k=1}^{3} m_k \left(\nu_{kL}^T \, \mathcal{C}^\dagger \, \nu_{kL} - \overline{\nu_{kL}} \, \mathcal{C} \, \nu_{kL}^T \right)$ is not invariant under the global U(1) gauge transformations
$$\nu_{kL} \to e^{i\varphi_k} \, \nu_{kL} \qquad (k = 1, 2, 3)$$
- The left-handed massive neutrino fields cannot be rephased in order to eliminate the two phases that can be factorized on the right of the mixing matrix
$$\boxed{U = U^{\mathrm{D}} \, D^{\mathrm{M}}} \qquad D^{\mathrm{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$
- U^{D} is analogous to a Dirac mixing matrix, with one Dirac phase
- Standard parameterization:
$$U^{\mathrm{D}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$
- Jarlskog invariant: $J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$ as in the Dirac case

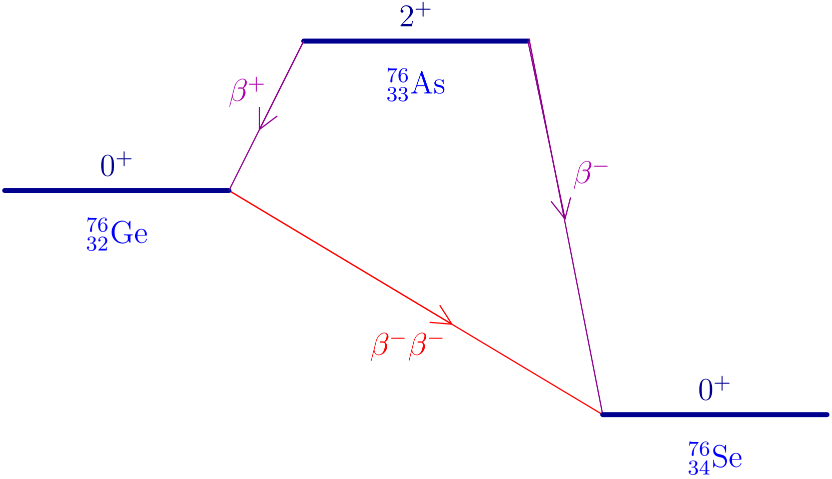
- $D^{\mathrm{M}} = \mathrm{diag}\left(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3}\right)$, but only two Majorana phases are physical

- All measurable quantities depend only on the differences of the Majorana phases
$$\ell_\alpha \to e^{i\varphi}\ell_\alpha \quad \Longrightarrow \quad e^{i\lambda_k} \to e^{i(\lambda_k - \varphi)}$$
$e^{i(\lambda_k - \lambda_j)}$ remains constant

- Our convention: $\lambda_1 = 0 \Longrightarrow D^{\mathrm{M}} = \mathrm{diag}\left(1, e^{i\lambda_2}, e^{i\lambda_3}\right)$

- CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:
$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

Neutrinoless Double-Beta Decay

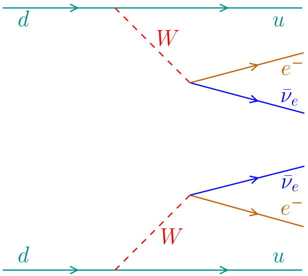


Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model

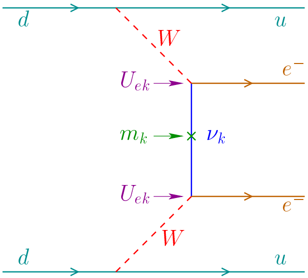


Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass
$$m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$$

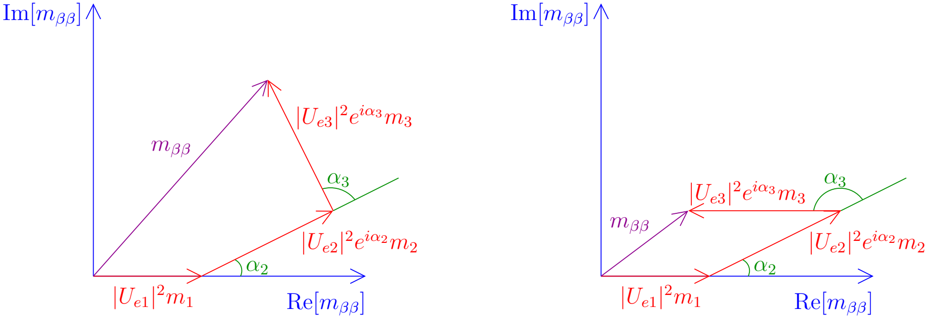


Effective Majorana Neutrino Mass

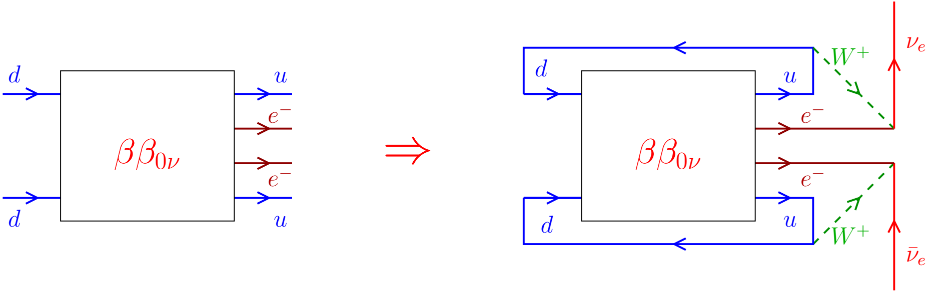
$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



Majorana Neutrino Mass $\Leftrightarrow$ $\beta\beta_{0\nu}$ Decay



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\boxed{\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2}\, m \left(\overline{\nu_L^c}\, \nu_L + \overline{\nu_L}\, \nu_L^c \right) = \frac{1}{2}\, m \left(\nu_L^T\, \mathcal{C}^\dagger\, \nu_L + \nu_L^\dagger\, \mathcal{C}\, \nu_L^* \right)}$$

two conditions: $\begin{cases} u,\ d,\ e \text{ are massive} \\ \text{standard left-handed weak interaction exists} \end{cases}$

cancellation with other diagrams is very unlikely

(no symmetry, unstable under perturbative expansion)

Dirac-Majorana Mass Term

One Generation

If ν_R exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \mathscr{L}_{\text{mass}}^{\text{D}} + \mathscr{L}_{\text{mass}}^{L} + \mathscr{L}_{\text{mass}}^{R}$$

$$\mathscr{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}}\,\overline{\nu_R}\,\nu_L + \text{H.c.}$$ Dirac Mass Term

$$\mathscr{L}_{\text{mass}}^{L} = \frac{1}{2}\,m_L\,\nu_L^T\,\mathcal{C}^\dagger\,\nu_L + \text{H.c.}$$ Majorana Mass Term

$$\mathscr{L}_{\text{mass}}^{R} = \frac{1}{2}\,m_R\,\nu_R^T\,\mathcal{C}^\dagger\,\nu_R + \text{H.c.}$$ New Majorana Mass Term!

- Column matrix of left-handed chiral fields: $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C}\,\overline{\nu_R}^T \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2}\, N_L^T\, \mathcal{C}^\dagger\, M\, N_L + \text{H.c.} \qquad M = \begin{pmatrix} m_L & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$$

- The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- Diagonalization: $n_L = U^\dagger\, N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T\, M\, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \text{Real } m_k \geq 0$$

- $\mathscr{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k\, \nu_{kL}^T\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k\, \overline{\nu_k}\, \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^C$$

- Massive neutrinos are Majorana! $\qquad \nu_k = \nu_k^C$

Real Mass Matrix

- CP is conserved if the mass matrix is real: $M = M^*$
- $M = \begin{pmatrix} m_L & m_\mathrm{D} \\ m_\mathrm{D} & m_R \end{pmatrix}$ we consider real and positive m_R and m_D and real m_L
- A real symmetric mass matrix can be diagonalized with $U = \mathcal{O}\,\rho$

$$\mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \qquad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \qquad \rho_k^2 = \pm 1$$

- $\mathcal{O}^T\,M\,\mathcal{O} = \begin{pmatrix} m_1' & 0 \\ 0 & m_2' \end{pmatrix} \qquad \tan 2\vartheta = \dfrac{2m_\mathrm{D}}{m_R - m_L}$

$$m_{2,1}' = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4\,m_\mathrm{D}^2}\right]$$

- m_1' is negative if $m_L m_R < m_\mathrm{D}^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m_1' & 0 \\ 0 & \rho_2^2 m_2' \end{pmatrix} \quad \Longrightarrow \quad \boxed{m_k = \rho_k^2\, m_k'}$$

- m'_2 is always positive:

$$m_2 = m'_2 = \frac{1}{2}\left[m_L + m_R + \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

- If $m_L m_R \geq m_{\text{D}}^2$, then $m'_1 \geq 0$ and $\rho_1^2 = 1$

$$m_1 = \frac{1}{2}\left[m_L + m_R - \sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2}\right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

- If $m_L m_R < m_{\text{D}}^2$, then $m'_1 < 0$ and $\rho_1^2 = -1$

$$m_1 = \frac{1}{2}\left[\sqrt{(m_L - m_R)^2 + 4\,m_{\text{D}}^2} - (m_L + m_R)\right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \quad \Longrightarrow \quad U = \begin{pmatrix} i\cos\vartheta & \sin\vartheta \\ -i\sin\vartheta & \cos\vartheta \end{pmatrix}$$

- If Δm^2 is small, there are oscillations between active ν_a generated by ν_L and sterile ν_s generated by ν_R^C:

$$P_{\nu_a \to \nu_s}(L, E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 \, L}{4 \, E}\right)$$

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R)\sqrt{(m_L - m_R)^2 + 4\, m_\mathrm{D}^2}$$

- It can be shown that the CP parity of ν_k is $\xi_k^{\mathrm{CP}} = i\,\rho_k^2$:

$$\mathrm{U}_{\mathrm{CP}} \nu_k(x) \mathrm{U}_{\mathrm{CP}}^{-1} = i\,\rho_k^2\,\gamma^0\,\nu_k(x_\mathrm{P})$$

- Special cases:
  - $m_L = m_R \implies$ Maximal Mixing
  - $m_L = m_R = 0 \implies$ Dirac Limit
  - $|m_L|, m_R \ll m_\mathrm{D} \implies$ Pseudo-Dirac Neutrinos
  - $m_L = 0 \quad m_\mathrm{D} \ll m_R \implies$ See-Saw Mechanism

Maximal Mixing

$$\boxed{m_L = m_R}$$

$$\vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_{\text{D}}$$

$$\begin{cases} \rho_1^2 = +1 \,, & m_1 = m_L - m_{\text{D}} \quad \text{if} \quad m_L \geq m_{\text{D}} \\ \rho_1^2 = -1 \,, & m_1 = m_{\text{D}} - m_L \quad \text{if} \quad m_L < m_{\text{D}} \end{cases}$$

$$m_2 = m_L + m_{\text{D}}$$

$$\underline{m_L < m_{\text{D}}}$$

$$\begin{cases} \nu_{1L} = \dfrac{-i}{\sqrt{2}} \left(\nu_L - \nu_R^C \right) \\ \nu_{2L} = \dfrac{1}{\sqrt{2}} \left(\nu_L + \nu_R^C \right) \end{cases}$$

$$\begin{cases} \nu_1 = \nu_{1L} + \nu_{1L}^C = \dfrac{-i}{\sqrt{2}} \left[\left(\nu_L + \nu_R \right) - \left(\nu_L^C + \nu_R^C \right) \right] \\ \nu_2 = \nu_{2L} + \nu_{2L}^C = \dfrac{1}{\sqrt{2}} \left[\left(\nu_L + \nu_R \right) + \left(\nu_L^C + \nu_R^C \right) \right] \end{cases}$$

Dirac Limit

$$\boxed{m_L = m_R = 0}$$

- $m'_{2,1} = \pm m_{\rm D} \quad \Longrightarrow \quad \begin{cases} \rho_1^2 = -1\,, & m_1 = m_{\rm D} \\ \rho_2^2 = +1\,, & m_2 = m_{\rm D} \end{cases}$
- The two Majorana fields ν_1 and ν_2 can be combined to give one Dirac field:
$$\nu = \frac{1}{\sqrt{2}}\left(i\nu_1 + \nu_2\right) = \nu_L + \nu_R$$
- A Dirac field ν can always be split in two Majorana fields:
$$\begin{aligned} \nu &= \frac{1}{2}\left[\left(\nu - \nu^C\right) + \left(\nu + \nu^C\right)\right] \\ &= \frac{i}{\sqrt{2}}\left(-i\,\frac{\nu - \nu^C}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\nu + \nu^C}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\left(i\nu_1 + \nu_2\right) \end{aligned}$$
- A Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities

Pseudo-Dirac Neutrinos

$$\boxed{|m_L|\,,\; m_R \ll m_{\mathrm{D}}}$$

- $m'_{2,1} \simeq \dfrac{m_L + m_R}{2} \pm m_{\mathrm{D}}$
- $m'_1 < 0 \quad \Longrightarrow \quad \rho_1^2 = -1 \quad \Longrightarrow \quad m_{2,1} \simeq m_{\mathrm{D}} \pm \dfrac{m_L + m_R}{2}$
- The two massive Majorana neutrinos have opposite CP parities and are almost degenerate in mass
- The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_{\mathrm{D}}\,(m_L + m_R)$$

- The oscillations occur with practically maximal mixing:

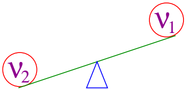
$$\tan 2\vartheta = \frac{2m_{\mathrm{D}}}{m_R - m_L} \gg 1 \quad \Longrightarrow \quad \vartheta \simeq \pi/4$$

See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

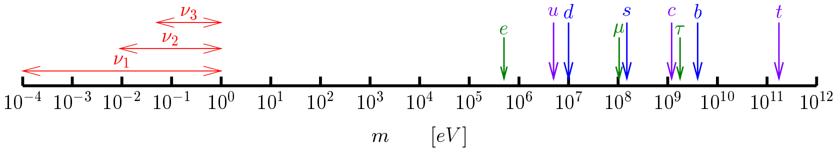
$$\boxed{m_L = 0 \quad m_{\mathrm{D}} \ll m_R}$$

- $\mathscr{L}^L_{\mathrm{mass}}$ is forbidden by SM symmetries $\Longrightarrow m_L = 0$
- $m_{\mathrm{D}} \lesssim v \sim 100\,\mathrm{GeV}$ is generated by SM Higgs Mechanism (protected by SM symmetries)
- m_R is not protected by SM symmetries $\Longrightarrow m_R \sim \mathcal{M}_{\mathrm{GUT}} \gg v$
- $$\left.\begin{array}{l} m'_1 \simeq -\dfrac{m_{\mathrm{D}}^2}{m_R} \\ m'_2 \simeq m_R \end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} \rho_1^2 = -1\,, & m_1 \simeq \dfrac{m_{\mathrm{D}}^2}{m_R} \\ \rho_2^2 = +1\,, & m_2 \simeq m_R \end{array}\right.$$



- Natural explanation of smallness of neutrino masses
- Mixing angle is very small: $\tan 2\vartheta = 2\,\dfrac{m_{\mathrm{D}}}{m_R} \ll 1$
- ν_1 is composed mainly of active ν_L: $\nu_{1L} \simeq -i\,\nu_L$
- ν_2 is composed mainly of sterile ν_R: $\nu_{2L} \simeq \nu_R^C$

Majorana Neutrino Mass?



known natural explanation of smallness of ν masses

New High Energy Scale $\mathcal{M}$ $\Rightarrow$ $\begin{cases} \text{See-Saw Mechanism (if } \nu_R\text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{cases}$

both imply $\begin{cases} \text{Majorana } \nu \text{ masses} \iff |\Delta L| = 2 \iff \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \dfrac{\mathcal{M}_{\text{EW}}^2}{\mathcal{M}} \end{cases}$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

Right-Handed Neutrino Mass Term

Majorana mass term for ν_R respects the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Standard Model Symmetry!

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2}\, m \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

Majorana mass term for ν_R breaks Lepton number conservation!

Three possibilities:

- Lepton number can be explicitly broken
- Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\mathcal{L}_\Phi = -y_d \left(\overline{L_L}\, \Phi\, \nu_R + \overline{\nu_R}\, \Phi^\dagger\, L_L \right) \xrightarrow[\langle\Phi\rangle\neq 0]{} -m_{\text{D}} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right)$$

$$\mathcal{L}_\eta = -y_s \left(\eta\, \overline{\nu_R^c}\, \nu_R + \eta^\dagger\, \overline{\nu_R}\, \nu_R^c \right) \xrightarrow[\langle\eta\rangle\neq 0]{} -\tfrac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$\eta = 2^{-1/2} \left(\langle\eta\rangle + \rho + i\,\chi \right) \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2} \left(\overline{\nu_L^c}\ \overline{\nu_R} \right) \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\underset{\text{scale of } L \text{ violation}}{m_R} \gg \underset{\text{EW scale}}{m_{\text{D}}} \implies \text{See-Saw: } \boxed{m_1 \simeq \frac{m_{\text{D}}^2}{m_R}}$$

ρ = massive scalar, χ = Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{i y_s}{\sqrt{2}}\,\chi \left[\overline{\nu_2}\gamma^5\nu_2 - \frac{m_{\text{D}}}{m_R} \left[\overline{\nu_2}\gamma^5\nu_1 + \overline{\nu_1}\gamma^5\nu_2 \right) + \left(\frac{m_{\text{D}}}{m_R} \right)^2 \overline{\nu_1}\gamma^5\nu_1 \right]$$

Three-Generation Mixing

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{L}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$\mathscr{L}^{\text{D}}_{\text{mass}} = -\sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}}\, M^{\text{D}}_{s\alpha}\, \nu'_{\alpha L} + \text{H.c.}$$

$$\mathscr{L}^{L}_{\text{mass}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'^{T}_{\alpha L}\, \mathcal{C}^{\dagger}\, M^{L}_{\alpha\beta}\, \nu'_{\beta L} + \text{H.c.}$$

$$\mathscr{L}^{R}_{\text{mass}} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'^{T}_{sR}\, \mathcal{C}^{\dagger}\, M^{R}_{ss'}\, \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \boldsymbol{\nu}'_L \\ \boldsymbol{\nu}'^{C}_R \end{pmatrix} \qquad \boldsymbol{\nu}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \qquad \boldsymbol{\nu}'^{C}_R \equiv \begin{pmatrix} \nu'^{C}_{1R} \\ \vdots \\ \nu'^{C}_{N_S R} \end{pmatrix}$$

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \frac{1}{2}\, \mathbf{N}'^{T}_L\, \mathcal{C}^{\dagger}\, M^{\text{D+M}}\, \mathbf{N}'_L + \text{H.c.} \qquad M^{\text{D+M}} = \begin{pmatrix} M^L & M^{\text{D}\,T} \\ M^{\text{D}} & M^R \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term $\Longrightarrow$ massive Majorana neutrinos

- See-Saw Mechanism $\Longrightarrow$ sterile right-handed neutrinos have large masses and are decoupled from the low-energy phenomenology

- At low energy we have an effective mixing of three Majorana neutrinos

Number of Massive Neutrinos?

$$Z \to \nu\bar{\nu} \quad \Rightarrow \quad \nu_e \ \nu_\mu \ \nu_\tau$$ active flavor neutrinos

mixing $\Rightarrow$ $$\nu_{\alpha L} = \sum_{k=1}^{N} U_{\alpha k} \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$N \geq 3$
no upper limit!

Mass Basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor Basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
	ACTIVE			STERILE		

STERILE NEUTRINOS

singlets of SM $\Longrightarrow$ no interactions!

active $\to$ sterile transitions are possible if $\nu_4, \ldots$ are light (no see-saw)

$\Downarrow$

disappearance of active neutrinos

Part II

Neutrino Oscillations in Vacuum and in Matter

Neutrino Oscillations in Vacuum

- Neutrino Oscillations in Vacuum
  - Ultrarelativistic Approximation
  - Neutrino Oscillations in Vacuum
  - Neutrinos and Antineutrinos
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Question: Do Charged Leptons Oscillate?
- Neutrino Oscillations in Matter

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered} \nu + A \to B + C \\ \Downarrow \\ s = 2 E m_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2 m_A} - \frac{m_A}{2} \end{gathered}}$$

$$\begin{array}{ll} \nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\ \nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\ \bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\ \nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\ \nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2 m_e} = 10.9\,\text{GeV} \end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO), $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

[Bilenky, Pontecorvo, Nuovo Cim. Lett. 17 (1976) 569] [Bilenky, Pontecorvo, Phys. Rep. 41 (1978) 225]

Flavor Neutrino Production: $j^{\rho}_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \, \gamma^{\rho} \, \ell_{\alpha L}$

$$\boxed{\nu_{\alpha L} = \sum_k U_{\alpha k} \, \nu_{kL}}$$

Fields $\overline{\nu_{\alpha L}} = \sum_k U^*_{\alpha k} \, \overline{\nu_{kL}} \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k} \, |\nu_k\rangle$ States

$$|\nu_k(t,x)\rangle = e^{-iE_k t + i p_k x} \, |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U^*_{\alpha k} \, e^{-iE_k t + i p_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} \, |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha \to \nu_\beta}(t,x) = \left| \langle \nu_\beta | \nu_\alpha(t,x) \rangle \right|^2 = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x) \right|^2 = \left| \sum_k U^*_{\alpha k} e^{-iE_k t + i p_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\, L = \frac{E_k^2 - p_k^2}{E_k + p_k}\, L = \frac{m_k^2}{E_k + p_k}\, L \simeq \frac{m_k^2}{2E}\, L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* \, e^{-i m_k^2 L / 2E} \, U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L/E) &= \delta_{\alpha\beta} - 4 \sum_{k>j} \mathrm{Re}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E} \right) \\ &\quad + 2 \sum_{k>j} \mathrm{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$

Neutrinos and Antineutrinos

Antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\mathrm{CP}} \nu^{\mathrm{CP}}_{\alpha L} = \sum_k U^*_{\alpha k} \nu^{\mathrm{CP}}_{kL}$

States: $|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\mathrm{Re} \sum_{k>j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E} \right)$$

CPT, CP and T Symmetries

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\implies$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

is invariant under CPT: $\quad U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$ $\boxed{\text{CPT} \quad \Rightarrow \quad A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}$

$$A^{\text{CP}}_{\alpha\beta}(L,E) = 2\text{Re} \sum_{k>j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right) - 2\text{Re} \sum_{k>j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog invariants: $\boxed{J_{\alpha\beta;kj} = \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right]}$

violation of CP symmetry depends only on Dirac phases
(three neutrinos: $J_{\alpha\beta;kj} = \pm c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$)

$\left\langle A^{\text{CP}}_{\alpha\beta} \right\rangle = 0 \Longrightarrow$ observation of CP violation needs measurement of oscillations

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\begin{aligned} \text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\ &= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} = A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}} \end{aligned}$$

$$\boxed{A^{\text{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \, \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

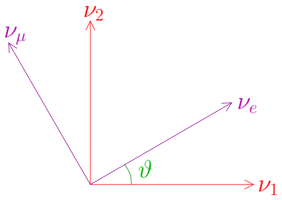
violation of T symmetry depends only on Dirac phases

$\left\langle A^{\text{T}}_{\alpha\beta} \right\rangle = 0 \Longrightarrow$ observation of T violation needs measurement of oscillations

Two-Neutrino Oscillations

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k} \, |\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta \, |\nu_1\rangle + \sin\vartheta \, |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta \, |\nu_1\rangle + \cos\vartheta \, |\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

two-neutrino mixing transition probability

$$\alpha \neq \beta \qquad\qquad \alpha, \beta = e, \mu, \tau$$

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\beta}(L, E) &= \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{m}]}{E[\text{MeV}]}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}\right)
\end{aligned}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \, \frac{E \, [\text{MeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{m} = 2.47 \, \frac{E \, [\text{GeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{km}$$

Types of Experiments

Two-Neutrino Mixing

$$\boxed{P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$$

observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

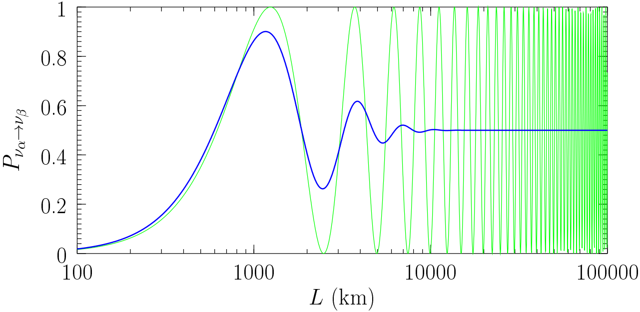
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\,\sin^2 2\vartheta\,\left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

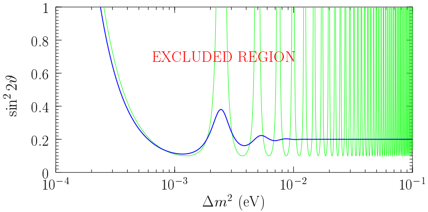


$\Delta m^2 = 10^{-3}\,\text{eV}$ $\quad$ $\sin^2 2\vartheta = 1$ $\quad$ $\langle E \rangle = 1\,\text{GeV}$ $\quad$ $\Delta E = 0.2\,\text{GeV}$

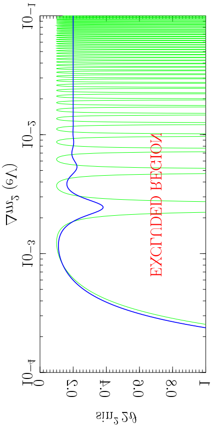
$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \rangle = \frac{1}{2}\,\sin^2 2\vartheta\,\left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha \neq \beta)$$

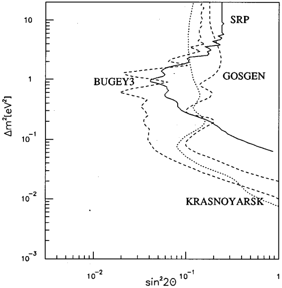
$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E \right] \qquad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L,E) \rangle \leq P^{\text{max}}_{\nu_\alpha \to \nu_\beta} \quad \Longrightarrow \quad \sin^2 2\vartheta \leq \frac{2 \, P^{\text{max}}_{\nu_\alpha \to \nu_\beta}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E} \right) \phi(E) \, \mathrm{d}E}$$

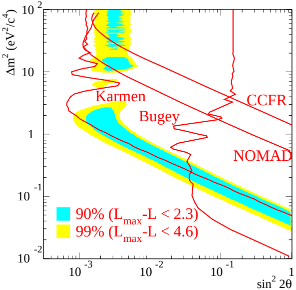


rotate and mirror

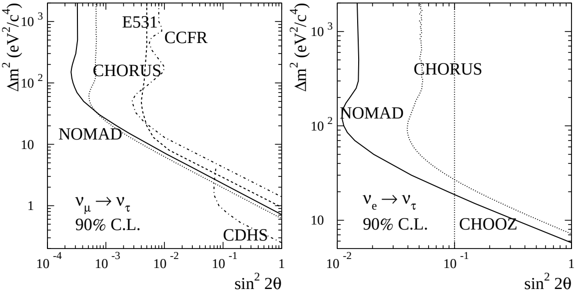




Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$

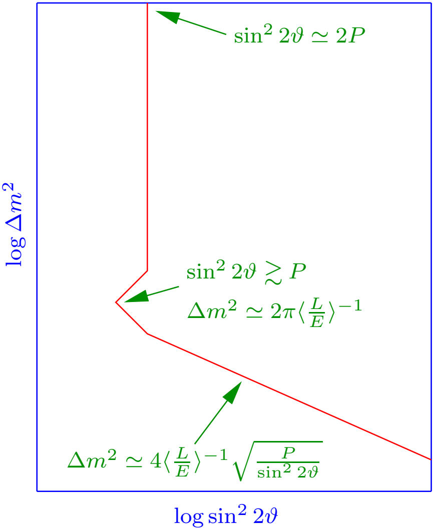


Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\tau$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau$

Anatomy of Exclusion Plots



- $\Delta m^2 \gg \langle L/E \rangle^{-1}$

  $$P \simeq \frac{1}{2} \sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P$$

- $\text{Min} \left\langle \cos\left(\frac{\Delta m^2 L}{2E} \right) \right\rangle \geq -1$

  $$\sin^2 2\vartheta = \frac{2\,P}{1 - \text{Min}\left\langle \cos\left(\frac{\Delta m^2 L}{2E} \right) \right\rangle} \geq P$$

  $$\Delta m^2 \simeq 2\pi \langle L/E \rangle^{-1}$$

- $\Delta m^2 \ll 2\pi \langle L/E \rangle^{-1}$

  $$\cos\left(\frac{\Delta m^2 L}{2E} \right) \simeq 1 - \frac{1}{2} \left(\frac{\Delta m^2 L}{2E} \right)^2$$

  $$\Delta m^2 \simeq 4 \left\langle \frac{L}{E} \right\rangle^{-1} \sqrt{\frac{P}{\sin^2 2\vartheta}}$$

Question: Do Charged Leptons Oscillate?

- Mass is the only property which distinguishes e, μ, τ.
- The flavor of a charged lepton is defined by its mass!
- By definition, the flavor of a charged lepton cannot change.

$\boxed{\text{THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE}}$

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

Correct definition of Charged Lepton Oscillations

[Pakvasa, Nuovo Cim. Lett. 31 (1981) 497]

$$\xrightarrow[\nu_1]{} \boxed{\mathrm{P}} \xrightarrow[e,\mu,\tau]{\hspace{6em}} \boxed{\mathrm{D}} \xrightarrow[\nu_2]{}$$

Analogy

- Neutrino Oscillations: massive neutrinos propagate unchanged between production and detection, with a difference of mass (flavor) of the charged leptons involved in the production and detection processes.

- Charged-Lepton Oscillations: massive charged leptons propagate unchanged between production and detection, with a difference of mass of the neutrinos involved in the production and detection processes.

NO FLAVOR CONVERSION!

The propagating charged leptons must be ultrarelativistic, in order to be produced and detected coherently (if τ is not ultrarelativistic, only e and μ contribute to the phase).

Practical Problems

- The initial and final neutrinos must be massive neutrinos of known type: precise neutrino mass measurements.

- The energy of the propagating charged leptons must be extremely high, in order to have a measurable oscillation length

$$\frac{4\pi E}{(m_\mu^2 - m_e^2)} \simeq \frac{4\pi E}{m_\mu^2} \simeq 2 \times 10^{-11} \left(\frac{E}{\text{GeV}} \right) \text{cm}$$

detailed discussion: [Akhmedov, JHEP 09 (2007) 116, arXiv:0706.1216]

Neutrino Oscillations in Matter

Matter Effects

a flavor neutrino ν_α with momentum p is described by

$$|\nu_\alpha(p)\rangle = \sum_k U^*_{\alpha k} \, |\nu_k(p)\rangle$$

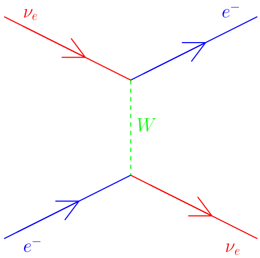
$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

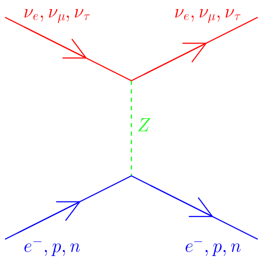
in matter $\quad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$

V_α = effective potential due to coherent interactions with the medium

forward elastic CC and NC scattering

Effective Potentials in Matter





$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}} \qquad \text{(common phase)}$$

$$V_e - V_\mu = V_{\text{CC}}$$

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}}$ $\qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Neutrino Flavors in Matter

Schrödinger picture: $$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu_\alpha(p,t)\rangle = \mathcal{H} |\nu_\alpha(p,t)\rangle \,, \qquad |\nu_\alpha(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes: $$\varphi_{\alpha\beta}(p,t) = \langle \nu_\beta(p) | \nu_\alpha(p,t) \rangle \,, \qquad \varphi_{\alpha\beta}(p,0) = \delta_{\alpha\beta}$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_{\alpha\beta}(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu_\alpha(p,t) \rangle = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\alpha(p,t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu_\alpha(p,t) \rangle$$

$$\begin{aligned} \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\alpha(p,t) \rangle &= \sum_\rho \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\rho(p) \rangle \underbrace{\langle \nu_\rho(p) | \nu_\alpha(p,t) \rangle}_{\varphi_{\alpha\rho}(p,t)} \\ &= \sum_\rho \sum_{k,j} U_{\beta k} \underbrace{\langle \nu_k(p) | \mathcal{H}_0 | \nu_j(p) \rangle}_{\delta_{kj} E_k} U_{\rho j}^* \, \varphi_{\alpha\rho}(p,t) \end{aligned}$$

$$\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\alpha(p,t) \rangle = \sum_\rho \underbrace{\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\rho(p) \rangle}_{\delta_{\beta\rho} V_\beta} \varphi_{\alpha\rho}(p,t) = V_\beta \, \varphi_{\alpha\beta}(p,t)$$

$$\boxed{ i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_{\alpha\beta} = \sum_\rho \left(\sum_k U_{\beta k} \, E_k \, U_{\rho k}^* + \delta_{\beta\rho} \, V_\beta \right) \varphi_{\alpha\rho} }$$

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $\quad E = p \quad t = x$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \varphi_{\alpha\beta}(p,x) = (p + V_{\text{NC}})\, \varphi_{\alpha\beta}(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \varphi_{\alpha\rho}(p,x)$$

$$\psi_{\alpha\beta}(p,x) = \varphi_{\alpha\beta}(p,x)\, e^{ipx + i \int_0^x V_{\text{NC}}(x')\,\mathrm{d}x'}$$

$$\Downarrow$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \psi_{\alpha\beta} = e^{ipx + i \int_0^x V_{\text{NC}}(x')\,\mathrm{d}x'} \left(-p - V_{\text{NC}} + i \frac{\mathrm{d}}{\mathrm{d}x} \right) \varphi_{\alpha\beta}$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \psi_{\alpha\beta} = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{\text{CC}} \right) \psi_{\alpha\rho}}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = \left|\varphi_{\alpha\beta}\right|^2 = \left|\psi_{\alpha\beta}\right|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\,\Psi_\alpha = \frac{1}{2E}\left(U\,\mathbb{M}^2\,U^\dagger + \mathbb{A}\right)\Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha\mu} \\ \psi_{\alpha\tau} \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} A_{\mathrm{CC}} &= 2EV_{\mathrm{CC}} \\ &= 2\sqrt{2}EG_{\mathrm{F}}N_e \end{aligned}$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U\,\mathbb{M}^2\,U^\dagger \xrightarrow{\text{matter}} U\,\mathbb{M}^2\,U^\dagger + 2\,E\,\underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering simplest case: two-neutrino mixing

$$\nu_e \to \nu_\mu \text{ transitions with } U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta\,m_1^2 + \sin^2\vartheta\,m_2^2 & \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) \\ \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) & \sin^2\vartheta\,m_1^2 + \cos^2\vartheta\,m_2^2 \end{pmatrix}$$

$$= \frac{1}{2}\,\underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\Sigma m^2} + \frac{1}{2}\begin{pmatrix} -\Delta m^2\cos 2\vartheta & \Delta m^2\sin 2\vartheta \\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta \end{pmatrix}$$

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$
i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2\sin 2\vartheta \\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix}
$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_{ee}(0)\\ \psi_{e\mu}(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

$$
\begin{aligned}
&P_{\nu_e\to\nu_\mu}(x) = |\psi_{e\mu}(x)|^2\\
&P_{\nu_e\to\nu_e}(x) = |\psi_{ee}(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x)
\end{aligned}
$$

Constant Matter Density

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x} = 0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix}\psi_{ee}\\ \psi_{e\mu}\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\underset{\substack{\uparrow\\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A^{\mathrm{R}}_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\mathrm{R}}_{e} = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_{ee}\\\psi_{e\mu}\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_{ee}\\\psi_{e\mu}\end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_{ee}(0)\\\psi_{e\mu}(0)\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1(0)\\\psi_2(0)\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

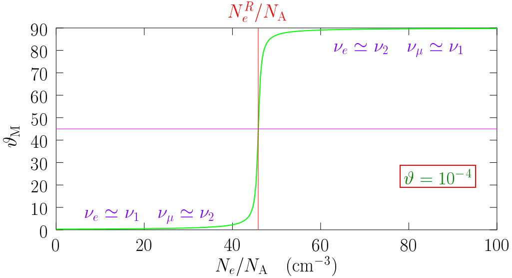
$$\psi_1(x) = \cos\vartheta_{\mathrm{M}}\exp\left(i\,\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

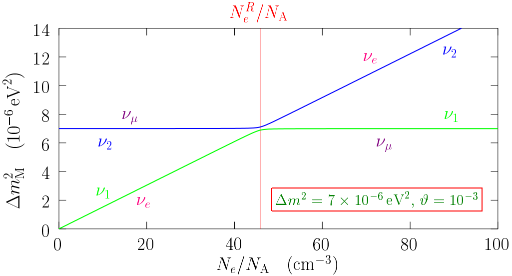
$$\psi_2(x) = \sin\vartheta_{\mathrm{M}}\exp\left(-i\,\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_{e\mu}(x)|^2 = |-\sin\vartheta_{\mathrm{M}}\psi_1(x) + \cos\vartheta_{\mathrm{M}}\psi_2(x)|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m^2_{\mathrm{M}}x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)





$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_M = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

$$\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E} \begin{pmatrix} -\Delta m^2_{\mathrm{M}} & 0 \\ 0 & \Delta m^2_{\mathrm{M}} \end{pmatrix} + \underset{\substack{\uparrow \\ \text{maximum near resonance}}}{\begin{pmatrix} 0 & -i \dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} \\ i \dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0 \end{pmatrix}} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} & -\sin\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} & \cos\vartheta^0_{\mathrm{M}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\vartheta^0_{\mathrm{M}} \\ \sin\vartheta^0_{\mathrm{M}} \end{pmatrix}$$

$$\begin{aligned} \psi_1(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{11} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{21} \right] \\ & \times \exp\left(i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \\ \psi_2(x) \simeq{} & \left[\cos\vartheta^0_{\mathrm{M}} \exp\left(i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{12} + \sin\vartheta^0_{\mathrm{M}} \exp\left(-i \int_0^{x_{\mathrm{R}}} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \mathcal{A}^{\mathrm{R}}_{22} \right] \\ & \times \exp\left(-i \int_{x_{\mathrm{R}}}^{x} \frac{\Delta m^2_{\mathrm{M}}(x')}{4E} \, \mathrm{d}x' \right) \end{aligned}$$

Averaged Survival Probability

$$\psi_{ee}(x) = \cos\vartheta_{\mathrm{M}}^{x}\,\psi_1(x) + \sin\vartheta_{\mathrm{M}}^{x}\,\psi_2(x)$$

neglect interference (averaged over energy spectrum)

$$\begin{aligned}\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_{ee}(x)\rangle|^2 = {} & \cos^2\vartheta_{\mathrm{M}}^{x}\,\cos^2\vartheta_{\mathrm{M}}^{0}\,|\mathcal{A}_{11}^{\mathrm{R}}|^2 + \cos^2\vartheta_{\mathrm{M}}^{x}\,\sin^2\vartheta_{\mathrm{M}}^{0}\,|\mathcal{A}_{21}^{\mathrm{R}}|^2 \\ & + \sin^2\vartheta_{\mathrm{M}}^{x}\,\cos^2\vartheta_{\mathrm{M}}^{0}\,|\mathcal{A}_{12}^{\mathrm{R}}|^2 + \sin^2\vartheta_{\mathrm{M}}^{x}\,\sin^2\vartheta_{\mathrm{M}}^{0}\,|\mathcal{A}_{22}^{\mathrm{R}}|^2\end{aligned}$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^{\mathrm{R}}|^2 = |\mathcal{A}_{21}^{\mathrm{R}}|^2 = P_{\mathrm{c}} \qquad |\mathcal{A}_{11}^{\mathrm{R}}|^2 = |\mathcal{A}_{22}^{\mathrm{R}}|^2 = 1 - P_{\mathrm{c}}$$

$P_{\mathrm{c}} \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathrm{c}}\right)\cos2\vartheta_{\mathrm{M}}^{0}\,\cos2\vartheta_{\mathrm{M}}^{x}}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_{\mathrm{c}} = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter:
$$\gamma = \left.\frac{\Delta m^2_{\mathrm{M}}/2E}{2|\mathrm{d}\vartheta_{\mathrm{M}}/\mathrm{d}x|}\right|_{\mathrm{R}} = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{\mathrm{d}\ln A_{\mathrm{CC}}}{\mathrm{d}x}\right|_{\mathrm{R}}}$$

$A \propto x$ $\quad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

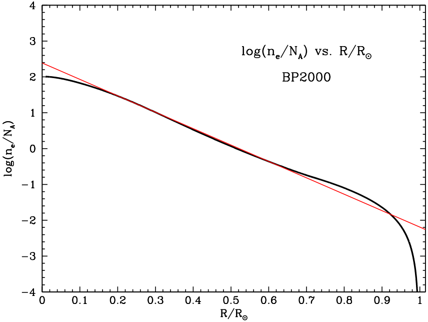
$A \propto 1/x$ $\quad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $\quad F = 1 - \tan^2\vartheta$ [Pizzochero, PRD 36 (1987) 2293] [Toshev, PLB 196 (1987) 170] [Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $N_e(x) \simeq N_e^{c} \exp\left(-\frac{x}{x_0}\right)$ $\quad N_e^{c} = 245\, N_A/\text{cm}^3$ $\quad x_0 = \frac{R_\odot}{10.54}$



$$\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right) \cos 2\vartheta_M^0 \, \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left|\frac{d\ln A_{CC}}{dx}\right|_R}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{CC} = 2\sqrt{2} E G_F N_e$$

practical prescription:

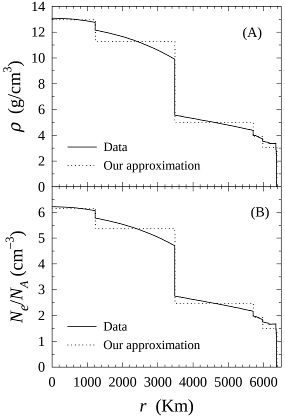
[Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } \left|d \ln A_{CC}/dx\right|_R & \text{for } x \le 0.904 R_\odot \\ \left|d \ln A_{CC}/dx\right|_R \to \frac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e\to\nu_e} = \overline{P}^{\text{sun}}_{\nu_e\to\nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e\to\nu_e}\right)\left(P^{\text{earth}}_{\nu_2\to\nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



[Giunti, Kim, Monteno, NP B 521 (1998) 3]

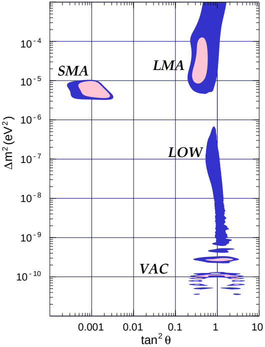
$P^{\text{earth}}_{\nu_2\to\nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

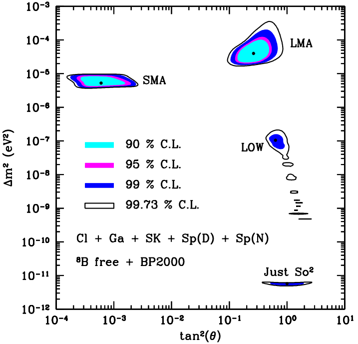
Wave functions of flavor neutrinos are joined at the boundaries of steps.

Phenomenology of Solar Neutrinos

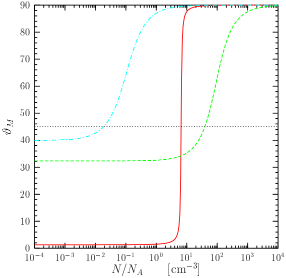
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\text{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



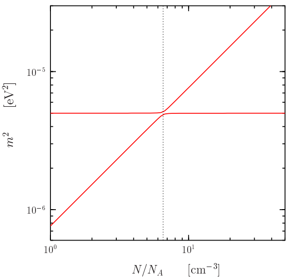
[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



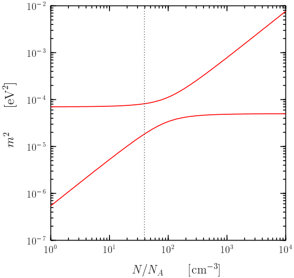
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\mathrm{eV}^2$
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\mathrm{eV}^2$
(typical LMA) $\tan^2\vartheta = 0.4$

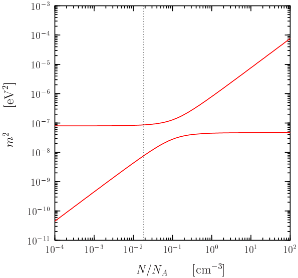
dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\mathrm{eV}^2$
(typical LOW) $\tan^2\vartheta = 0.7$



typical SMA

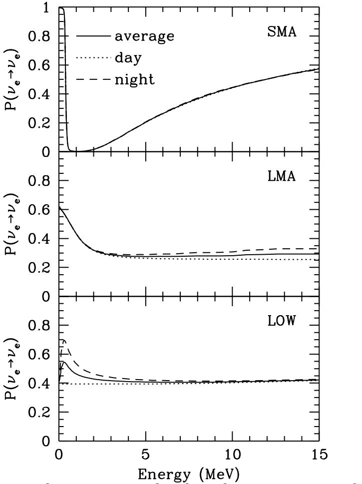


typical LMA



typical LOW

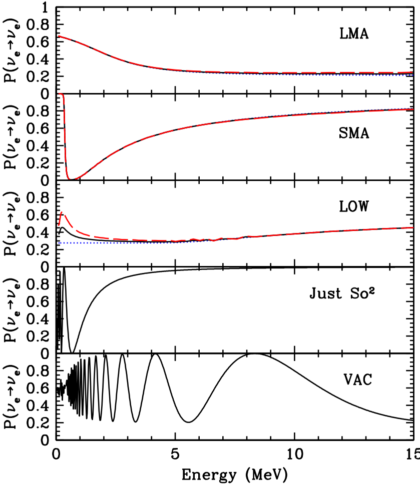
[Bahcall, Krastev, Smirnov, PRD 58 (1998) 096016]



SMA: $\Delta m^2 = 5.0 \times 10^{-6}\ \text{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$

LMA: $\Delta m^2 = 1.6 \times 10^{-5}\ \text{eV}^2$ $\sin^2 2\vartheta = 0.57$

LOW: $\Delta m^2 = 7.9 \times 10^{-8}\ \text{eV}^2$ $\sin^2 2\vartheta = 0.95$



[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]

LMA: $\Delta m^2 = 4.2 \times 10^{-5}\ \text{eV}^2$ $\tan^2 \vartheta = 0.26$

SMA: $\Delta m^2 = 5.2 \times 10^{-6}\ \text{eV}^2$ $\tan^2 \vartheta = 5.5 \times 10^{-4}$

LOW: $\Delta m^2 = 7.6 \times 10^{-8}\ \text{eV}^2$ $\tan^2 \vartheta = 0.72$

Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\ \text{eV}^2$ $\tan^2 \vartheta = 1.0$

VAC: $\Delta m^2 = 1.4 \times 10^{-10}\ \text{eV}^2$ $\tan^2 \vartheta = 0.38$

In Neutrino Oscillations Dirac = Majorana

Evolution of Amplitudes: $$\frac{d\nu_\alpha}{dt} = \frac{1}{2E}\sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \nu_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad D M^2 = M^2 D \quad \Longrightarrow \quad D M^2 D^\dagger = M^2$$

$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$

Part III

Phenomenology of Three-Neutrino Mixing

Phenomenology of Three-Neutrino Oscillations

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Reactor
$\bar{\nu}_e$ disappearance

$$\left.\begin{array}{c} \begin{pmatrix} \text{Homestake} \\ \text{Kamiokande} \\ \text{GALLEX/GNO} \\ \text{SAGE} \\ \text{Super-Kamiokande} \end{pmatrix} \\ \text{(KamLAND)} \end{array}\right\} \xrightarrow{2\sigma} \left\{ \begin{array}{l} \Delta m^2_{\text{SOL}} = 7.92\,(1 \pm 0.09) \times 10^{-5}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{SOL}} = 0.314\,\left(1^{+0.18}_{-0.15}\right) \end{array}\right.$$

[Fogli et al, PPNP 57 (2006) 742, hep-ph/0506083]

Atmospheric
$\nu_\mu \to \nu_\tau$

Accelerator
ν_μ disappearance

$$\left.\begin{array}{c} \begin{pmatrix} \text{Kamiokande} \\ \text{IMB} \\ \text{Super-Kamiokande} \\ \text{MACRO} \\ \text{Soudan-2} \end{pmatrix} \\ \text{(K2K \& MINOS)} \end{array}\right\} \xrightarrow{2\sigma} \left\{ \begin{array}{l} \Delta m^2_{\text{ATM}} = 2.6\,\left(1^{+0.14}_{-0.15}\right) \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{ATM}} = 0.45\,\left(1^{+0.35}_{-0.20}\right) \end{array}\right.$$

[Fogli et al, hep-ph/0608060]

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\quad \vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\,\nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

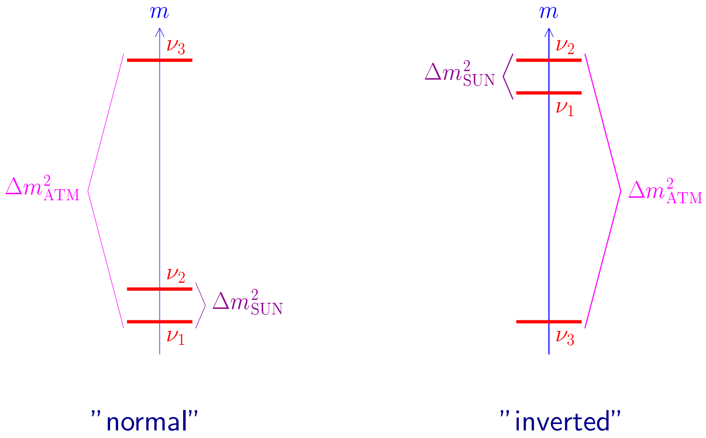
three flavor fields: ν_e, ν_μ, ν_τ

three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq 8.0 \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

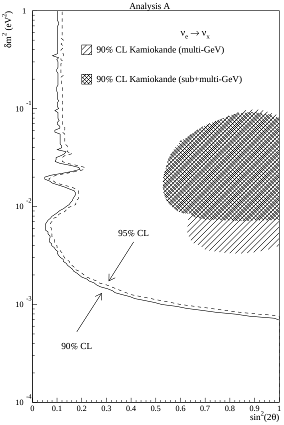
$$\begin{array}{rl} \text{SOL} \rightarrow & \\ U = & \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ & \qquad\qquad\qquad\quad \uparrow \\ & \qquad\qquad\qquad \text{ATM} \end{array}$$

CHOOZ: $\begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1-|U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 < 5 \times 10^{-2} \ (99.73\% \text{ C.L.})}$$

[Fogli et al., PRD 66 (2002) 093008]



[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2\vartheta_{\text{SOL}} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2\vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, C.G, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\text{CHOOZ} + \text{SK} + \text{MINOS} \Longrightarrow \sin^2 \vartheta_{\text{CHOOZ}} = 0.008^{+0.023}_{-0.008} \ @ \ 2\sigma$$

[Fogli et al, hep-ph/0608060]

Bilarge Mixing

$$\boxed{|U_{e3}|^2 \ll 1}$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Longrightarrow \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \quad\;\; = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \Longrightarrow \vartheta_A \simeq \frac{\pi}{4} \Longrightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\text{Solar } \nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{CC}^{SNO}}{\Phi_{\nu_e}^{SSM}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6\,\text{MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \Longrightarrow U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

[Harrison, Perkins, Scott, hep-ph/0202074]

Global Fit of Oscillation Data: Bilarge Mixing

$$\Delta m^2_{21} = 7.92\,(1 \pm 0.09) \times 10^{-5}\,\text{eV}^2 \qquad \sin^2\vartheta_{12} = 0.314\left(1^{+0.18}_{-0.15}\right)$$

$$|\Delta m^2_{31}| = 2.6\left(1^{+0.14}_{-0.15}\right) \times 10^{-3}\,\text{eV}^2 \qquad \sin^2\vartheta_{23} = 0.45\left(1^{+0.35}_{-0.20}\right)$$

$$\sin^2\vartheta_{13} = 0.008^{+0.023}_{-0.008}$$

[Fogli et al, hep-ph/0608060]

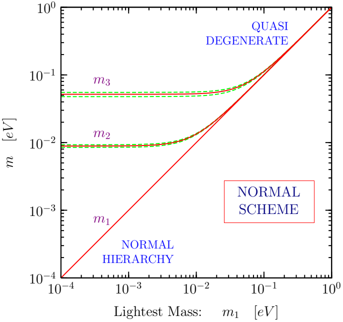
$$|U|_{\text{bf}} \simeq \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.37-0.47 & 0.58-0.65 & 0.67 \\ 0.32-0.43 & 0.52-0.59 & 0.74 \end{pmatrix}$$

$$|U|_{2\sigma} \simeq \begin{pmatrix} 0.78-0.86 & 0.51-0.61 & 0.00-0.18 \\ 0.21-0.57 & 0.41-0.74 & 0.59-0.78 \\ 0.19-0.56 & 0.39-0.72 & 0.62-0.80 \end{pmatrix}$$

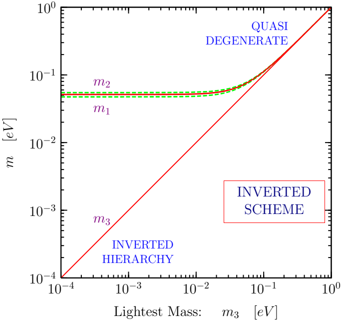
future: measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\mathrm{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\mathrm{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\mathrm{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\mathrm{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\mathrm{ATM}}^2} \simeq 5 \times 10^{-2}\,\mathrm{eV}$

Tritium Beta-Decay

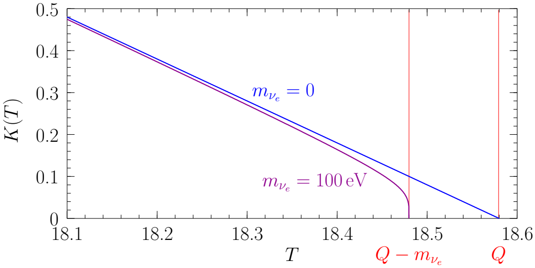
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$$

Mainz & Troitsk

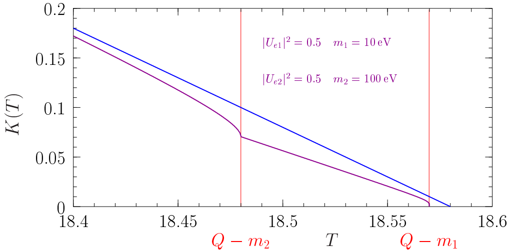
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters $\left(\sum_k |U_{ek}|^2 = 1 \right)$

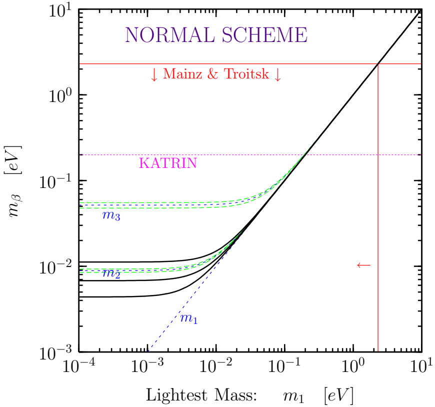
if experiment is not sensitive to masses ($m_k \ll Q - T$)

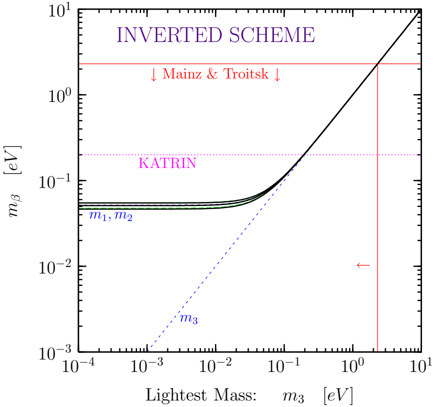
effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$





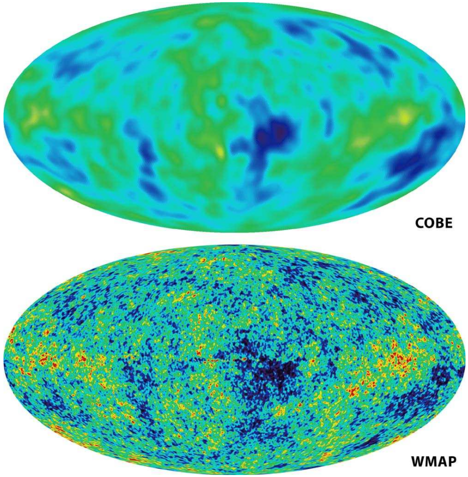
Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses

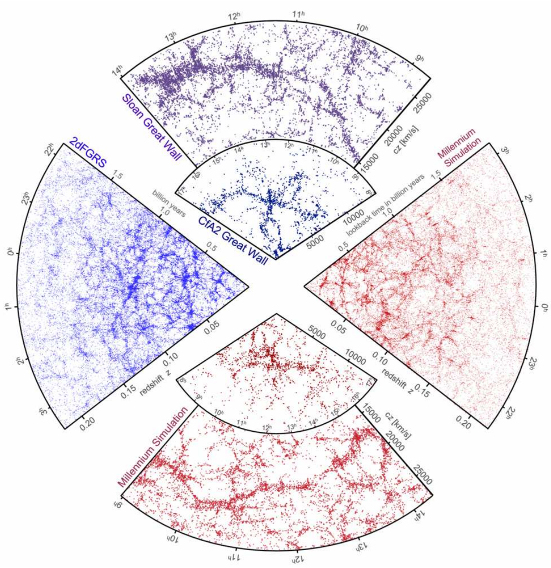
- Phenomenology of Three-Neutrino Oscillations
- Absolute Scale of Neutrino Masses
- Tritium Beta-Decay
- Cosmological Bound on Neutrino Masses
  - WMAP (Wilkinson Microwave Anisotropy Probe)
  - Galaxy Redshift Surveys
  - Lyman-alpha Forest
  - Relic Neutrinos
  - Power Spectrum of Density Fluctuations
- Neutrinoless Double-Beta Decay
- Conclusions

WMAP (Wilkinson Microwave Anisotropy Probe)



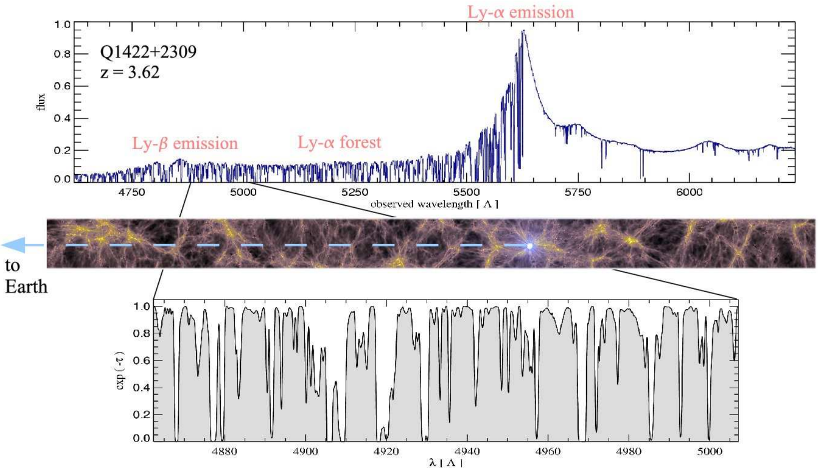
[WMAP, http://map.gsfc.nasa.gov]

Galaxy Redshift Surveys



[Springel, Frenk, White, astro-ph/0604561]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67\ \AA$, $\lambda_\beta^0 = 1025.72\ \AA$, $\lambda_\gamma^0 = 972.54\ \AA$

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

$$\overbrace{\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4}}^{\text{weak interactions freeze out}} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

Relic Neutrinos: $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$

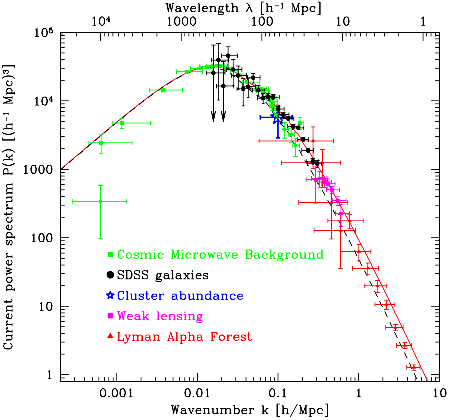
number density: $n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$

density contribution: $\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)$ $\Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\,m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu\,h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \Longrightarrow \quad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_M^0 = 0.28\,,\ h = 0.72\,,\ \Omega_B^0/\Omega_M^0 = 0.16)$

Dashed Curve: $\displaystyle\sum_{k=1}^{3} m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \rangle = \int \frac{d^3 k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026\,\sqrt{\frac{m_\nu}{1\,\text{eV}}}\,\sqrt{\Omega_m}\,h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP, AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SNIa $\Longrightarrow$ ΛCDM

$$T_0 = 13.7 \pm 0.1\,\text{Gyr} \qquad h = 0.71^{+0.04}_{-0.03}$$

$$\Omega_0 = 1.02 \pm 0.02 \qquad \Omega_B h^2 = 0.0224 \pm 0.0009 \qquad \Omega_M h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_\nu h^2 < 0.0076 \quad (95\%\ \text{conf.}) \quad \Longrightarrow \quad \boxed{\sum_{k=1}^{3} m_k < 0.71\,\text{eV}}$$

WMAP, astro-ph/0603449

Flat ΛCDM (WMAP+HST: $\Omega_0 = 1.010^{+0.016}_{-0.009}$, $\Omega_\Lambda = 0.72 \pm 0.04$)

$$\sum_{k=1}^{3} m_k < \begin{cases} 2.0\,\text{eV} & \text{WMAP} \\ 0.91\,\text{eV} & \text{WMAP+SDSS} \\ 0.87\,\text{eV} & \text{WMAP+2dFGRS} \\ 0.68\,\text{eV} & \text{CMB+LSS+SNIa} \end{cases} \quad (95\%\ \text{conf.})$$

Goobar, Hannestad, Mortsell, Tu, JCAP 0606 (2006) 019, astro-ph/0602155

Flat ΛCDM

$$\sum_{k=1}^{3} m_k < \begin{cases} 0.70\,\text{eV} & \text{CMB+LSS+SNIa} \\ 0.48\,\text{eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.27\,\text{eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \qquad (95\%\ \text{conf.})$$

Seljak, Slosar, McDonald, astro-ph/0604335

Flat ΛCDM CMB+LSS+SNIa+BAO+Lyα

$$\sum_{k=1}^{3} m_k < 0.17\,\text{eV} \qquad (95\%\ \text{conf.})$$

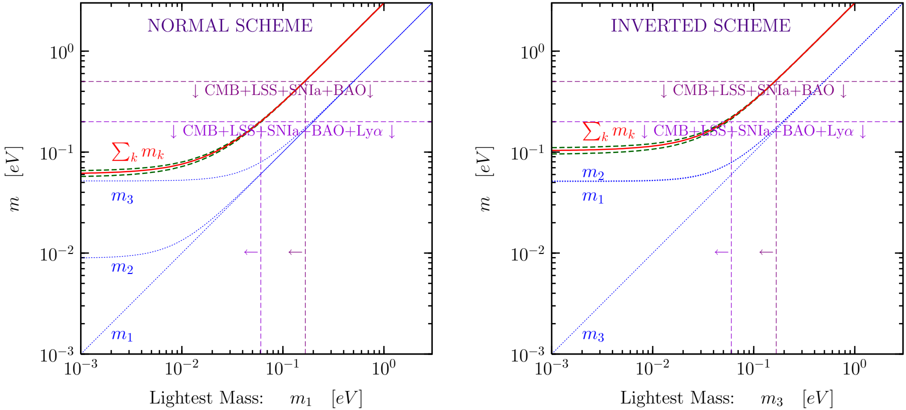
Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, hep-ph/0608060

Flat ΛCDM

$$\sum_{k=1}^{3} m_k < \begin{cases} 0.75\,\text{eV} & \text{CMB+LSS+SNIa} \\ 0.58\,\text{eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.17\,\text{eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \qquad (95\%\ \text{conf.})$$

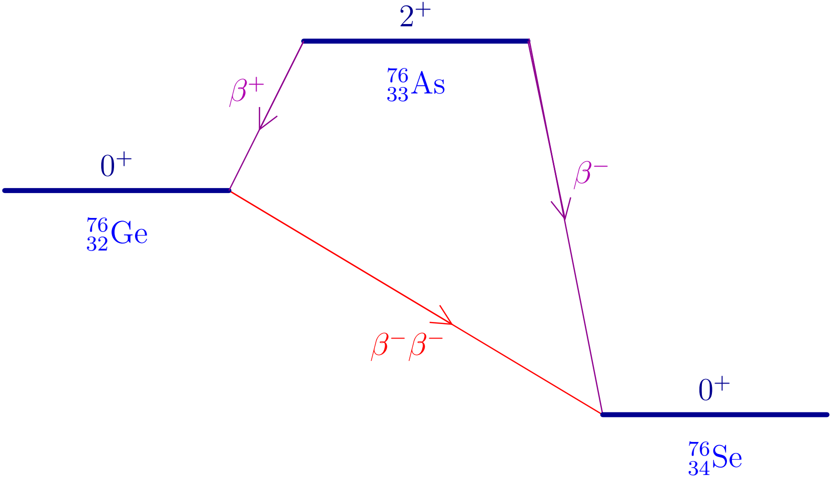
$$\sum_{k=1}^{3} m_k \lesssim 0.5\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO}$$

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO+Ly}\alpha$$



FUTURE: IF $\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Experimental Bounds

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

CUORICINO (^{130}Te) [PRL 95 (2005) 142501]

$$\boxed{T^{0\nu}_{1/2} > 1.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.2 - 1.1\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

NEMO 3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES

$$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$$

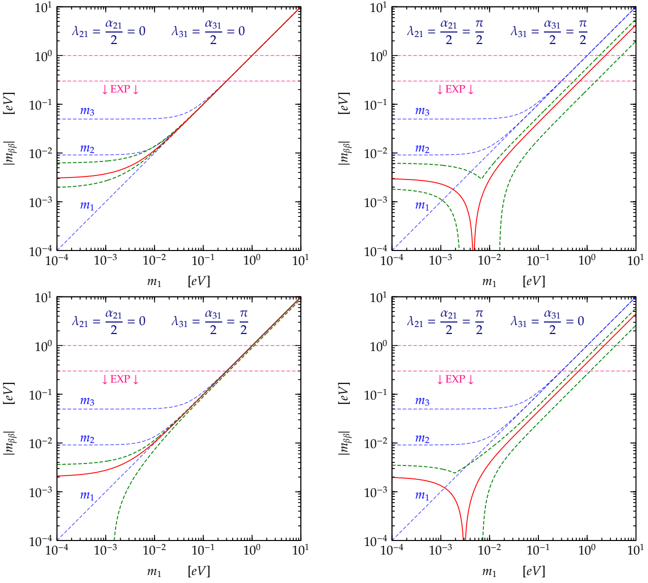
Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$

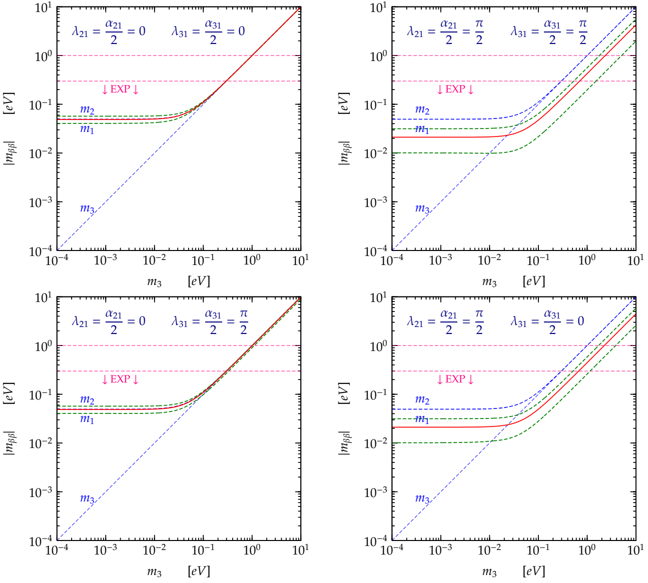
CP conservation

$$\alpha_{21} = 0 \, , \, \pi \qquad \alpha_{31} = 0 \, , \, \pi$$

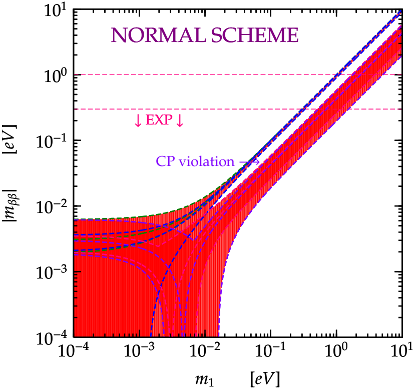
CP Conservation: Normal Scheme

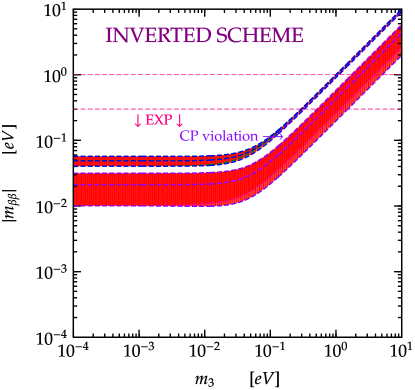


CP Conservation: Inverted Scheme



$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_{21}} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_{31}} \, m_3$$



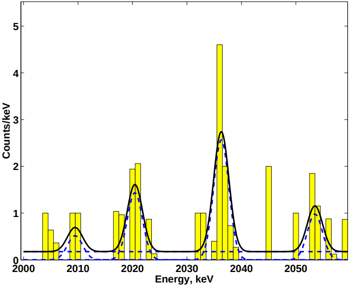


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

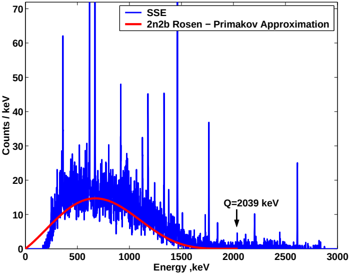
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$T_{1/2}^{0\nu\,\mathrm{bf}} = 1.19 \times 10^{25}\,\mathrm{y} \quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\mathrm{y}\,(3\sigma) \quad 4.2\sigma$ evidence



pulse-shape selected spectrum



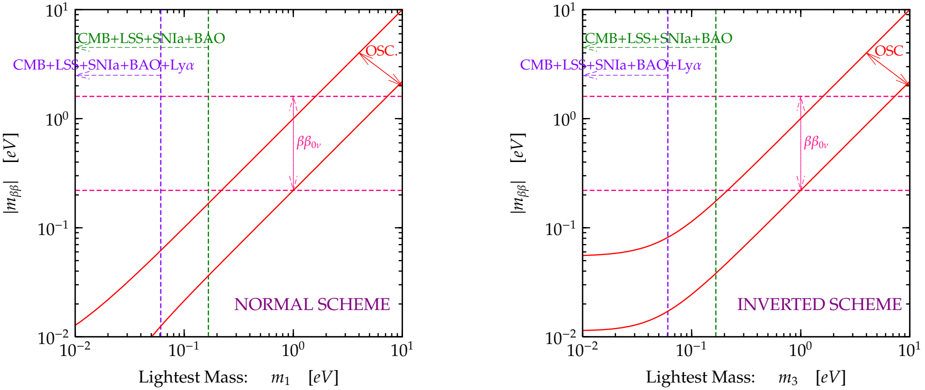
3.8σ evidence [PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Longrightarrow \quad 0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$$

if confirmed, very exciting (Majorana ν and large mass scale)

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ ($\sim 3\sigma$ range)



tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (atm. ν, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why only $|U_{e3}|^2 \ll 1$?

Improve uncertainties in calculation of $\mathcal{M}_{0\nu}$!

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy

Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale

Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements