

# Neutrino Physics

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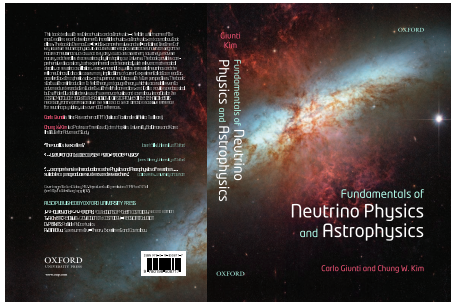
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# Part I

## Theory of Neutrino Masses and Mixing

# Dirac Neutrino Masses

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## Dirac Mass

- ▶ Dirac Equation:  $(i\cancel{\partial} - m)\nu(x) = 0$  ( $\cancel{\partial} \equiv \gamma^\mu \partial_\mu$ )
- ▶ Dirac Lagrangian:  $\mathcal{L}(x) = \bar{\nu}(x)(i\cancel{\partial} - m)\nu(x)$
- ▶ Chiral decomposition:  $\nu_L \equiv \frac{1 - \gamma^5}{2}\nu$ ,  $\nu_R \equiv \frac{1 + \gamma^5}{2}\nu$   
 $\nu = \nu_L + \nu_R$

$$\mathcal{L} = \bar{\nu}_L i\cancel{\partial}\nu_L + \bar{\nu}_R i\cancel{\partial}\nu_R - m(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$$

- ▶ In SM only  $\nu_L \implies$  no Dirac mass
- ▶ Oscillation experiments have shown that **neutrinos are massive**
- ▶ Simplest extension of the SM: add  $\nu_R$

# Higgs Mechanism in SM

▶ SM: fermion masses are generated through the Higgs mechanism

▶ Higgs Doublet:  $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$

▶ Higgs Lagrangian:  $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi)$

▶ Higgs Potential:  $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

▶  $\mu^2 < 0, \lambda > 0 \implies V(\Phi) = \lambda \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$ , with  $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$

▶ Vacuum:  $V_{\min}$  for  $\Phi^\dagger \Phi = \frac{v^2}{2} \implies \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

▶ Spontaneous Symmetry Breaking:  $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$

▶ Unitary Gauge:  $\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$



# Dirac Lepton Masses

$$L_L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \ell_R \quad \nu_R$$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = -y^\ell \bar{L}_L \Phi \ell_R - y^\nu \bar{L}_L \tilde{\Phi} \nu_R + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\tau_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{H,L} = & -\frac{y^\ell}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \ell_R \\ & -\frac{y^\nu}{\sqrt{2}} (\bar{\nu}_L \quad \bar{\ell}_L) \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix} \nu_R + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_{H,L} = -y^\ell \frac{v}{\sqrt{2}} \bar{\ell}_L \ell_R - y^\nu \frac{v}{\sqrt{2}} \bar{\nu}_L \nu_R$$

$$- \frac{y^\ell}{\sqrt{2}} \bar{\ell}_L \ell_R H - \frac{y^\nu}{\sqrt{2}} \bar{\nu}_L \nu_R H + \text{H.c.}$$

$$m_\ell = y^\ell \frac{v}{\sqrt{2}}$$

$$m_\nu = y^\nu \frac{v}{\sqrt{2}}$$

$$g_{\ell H} = \frac{y^\ell}{\sqrt{2}} = \frac{m_\ell}{v}$$

$$g_{\nu H} = \frac{y^\nu}{\sqrt{2}} = \frac{m_\nu}{v}$$

# Three-Generations Dirac Neutrino Masses

$L'_{eL} \equiv \begin{pmatrix} \nu'_{eL} \\ \ell'_{eL} \equiv e'_L \end{pmatrix}$	$L'_{\mu L} \equiv \begin{pmatrix} \nu'_{\mu L} \\ \ell'_{\mu L} \equiv \mu'_L \end{pmatrix}$	$L'_{\tau L} \equiv \begin{pmatrix} \nu'_{\tau L} \\ \ell'_{\tau L} \equiv \tau'_L \end{pmatrix}$
$\ell'_{eR} \equiv e'_R$	$\ell'_{\mu R} \equiv \mu'_R$	$\ell'_{\tau R} \equiv \tau'_R$
$\nu'_{eR}$	$\nu'_{\mu R}$	$\nu'_{\tau R}$

Lepton-Higgs Yukawa Lagrangian

$$\mathcal{L}_{H,L} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[ Y_{\alpha\beta}^{l\ell} \overline{L'_{\alpha L}} \Phi \ell'_{\beta R} + Y_{\alpha\beta}^{l\nu} \overline{L'_{\alpha L}} \tilde{\Phi} \nu'_{\beta R} \right] + \text{H.c.}$$

Unitary Gauge

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \tilde{\Phi} = i\tau_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \sum_{\alpha,\beta=e,\mu,\tau} \left[ Y_{\alpha\beta}^{l\ell} \overline{\ell'_{\alpha L}} \ell'_{\beta R} + Y_{\alpha\beta}^{l\nu} \overline{\nu'_{\alpha L}} \nu'_{\beta R} \right] + \text{H.c.}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell'_L} Y^{l\ell} \ell'_R + \overline{\nu'_L} Y^{l\nu} \nu'_R \right] + \text{H.c.}$$

$$\ell'_L \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad \ell'_R \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}$$

$$Y^{l\ell} \equiv \begin{pmatrix} Y_{ee}^{l\ell} & Y_{e\mu}^{l\ell} & Y_{e\tau}^{l\ell} \\ Y_{\mu e}^{l\ell} & Y_{\mu\mu}^{l\ell} & Y_{\mu\tau}^{l\ell} \\ Y_{\tau e}^{l\ell} & Y_{\tau\mu}^{l\ell} & Y_{\tau\tau}^{l\ell} \end{pmatrix}$$

$$Y^{l\nu} \equiv \begin{pmatrix} Y_{ee}^{l\nu} & Y_{e\mu}^{l\nu} & Y_{e\tau}^{l\nu} \\ Y_{\mu e}^{l\nu} & Y_{\mu\mu}^{l\nu} & Y_{\mu\tau}^{l\nu} \\ Y_{\tau e}^{l\nu} & Y_{\tau\mu}^{l\nu} & Y_{\tau\tau}^{l\nu} \end{pmatrix}$$

$$M^{l\ell} = \frac{v}{\sqrt{2}} Y^{l\ell}$$

$$M^{l\nu} = \frac{v}{\sqrt{2}} Y^{l\nu}$$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell}'_L Y^{\ell\ell} \ell'_R + \overline{\nu}'_L Y^{\nu\nu} \nu'_R \right] + \text{H.c.}$$

Diagonalization of  $Y^{\ell\ell}$  and  $Y^{\nu\nu}$  with **unitary**  $V_L^\ell, V_R^\ell, V_L^\nu, V_R^\nu$

$$\ell'_L = V_L^\ell \ell_L \quad \ell'_R = V_R^\ell \ell_R \quad \nu'_L = V_L^\nu \nu_L \quad \nu'_R = V_R^\nu \nu_R$$

Kinetic terms are invariant under unitary transformations of the fields

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell}_L V_L^{\ell\dagger} Y^{\ell\ell} V_R^\ell \ell_R + \overline{\nu}_L V_L^{\nu\dagger} Y^{\nu\nu} V_R^\nu \nu_R \right] + \text{H.c.}$$

$$V_L^{\ell\dagger} Y^{\ell\ell} V_R^\ell = Y^\ell \quad Y_{\alpha\beta}^\ell = y_\alpha^\ell \delta_{\alpha\beta} \quad (\alpha, \beta = e, \mu, \tau)$$

$$V_L^{\nu\dagger} Y^{\nu\nu} V_R^\nu = Y^\nu \quad Y_{kj}^\nu = y_k^\nu \delta_{kj} \quad (k, j = 1, 2, 3)$$

Real and Positive  $y_\alpha^\ell, y_k^\nu$

$$V_L^\dagger Y' V_R = Y \quad \Leftrightarrow \quad Y' = V_L Y V_R^\dagger$$

$$2N^2 \quad N^2 \quad N \quad N^2$$

$$18 \quad 9 \quad 3 \quad 9$$

# Massive Chiral Lepton Fields

$\ell_L = V_L^{\ell\dagger} \ell'_L \equiv \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$	$\ell_R = V_R^{\ell\dagger} \ell'_R \equiv \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$
$\mathbf{n}_L = V_L^{\nu\dagger} \nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$	$\mathbf{n}_R = V_R^{\nu\dagger} \nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{H,L} &= - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \overline{\ell}_L Y^\ell \ell_R + \overline{\mathbf{n}}_L Y^\nu \mathbf{n}_R \right] + \text{H.c.} \\ &= - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \sum_{\alpha=e,\mu,\tau} y_\alpha^\ell \overline{\ell}_{\alpha L} \ell_{\alpha R} + \sum_{k=1}^3 y_k^\nu \overline{\nu}_{kL} \nu_{kR} \right] + \text{H.c.} \end{aligned}$$

# Massive Dirac Lepton Fields

$$l_\alpha \equiv l_{\alpha L} + l_{\alpha R} \quad (\alpha = e, \mu, \tau)$$

$$\nu_k = \nu_{kL} + \nu_{kR} \quad (k = 1, 2, 3)$$

$$\mathcal{L}_{H,L} = - \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^l \nu}{\sqrt{2}} \bar{l}_\alpha l_\alpha - \sum_{k=1}^3 \frac{y_k^\nu \nu}{\sqrt{2}} \bar{\nu}_k \nu_k \quad \text{Mass Terms}$$

$$- \sum_{\alpha=e,\mu,\tau} \frac{y_\alpha^l}{\sqrt{2}} \bar{l}_\alpha l_\alpha H - \sum_{k=1}^3 \frac{y_k^\nu}{\sqrt{2}} \bar{\nu}_k \nu_k H \quad \text{Lepton-Higgs Couplings}$$

Charged Lepton and Neutrino Masses

$$m_\alpha = \frac{y_\alpha^l \nu}{\sqrt{2}} \quad (\alpha = e, \mu, \tau) \quad m_k = \frac{y_k^\nu \nu}{\sqrt{2}} \quad (k = 1, 2, 3)$$

Lepton-Higgs coupling  $\propto$  Lepton Mass



# Quantization

$$\nu_k(x) = \int \frac{d^3 p}{(2\pi)^3 2E_k} \sum_{h=\pm 1} \left[ a_k^{(h)}(p) u_k^{(h)}(p) e^{-ip \cdot x} + b_k^{(h)\dagger}(p) v_k^{(h)}(p) e^{ip \cdot x} \right]$$

$$p^0 = E_k = \sqrt{\vec{p}^2 + m_k^2} \quad \begin{aligned} (\not{p} - m_k) u_k^{(h)}(p) &= 0 \\ (\not{p} + m_k) v_k^{(h)}(p) &= 0 \end{aligned}$$

$$\frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} u_k^{(h)}(p) = h u_k^{(h)}(p)$$

$$\frac{\vec{p} \cdot \vec{\Sigma}}{|\vec{p}|} v_k^{(h)}(p) = -h v_k^{(h)}(p)$$

$$\{a_k^{(h)}(p), a_k^{(h')\dagger}(p')\} = \{b_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = (2\pi)^3 2E_k \delta^3(\vec{p} - \vec{p}') \delta_{hh'}$$

$$\{a_k^{(h)}(p), a_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), a_k^{(h')\dagger}(p')\} = 0$$

$$\{b_k^{(h)}(p), b_k^{(h')}(p')\} = \{b_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$

$$\{a_k^{(h)}(p), b_k^{(h')}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')\dagger}(p')\} = 0$$

$$\{a_k^{(h)}(p), b_k^{(h')\dagger}(p')\} = \{a_k^{(h)\dagger}(p), b_k^{(h')}(p')\} = 0$$

# Mixing

## Charged-Current Weak Interaction Lagrangian

$$\mathcal{L}_1^{(CC)} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \text{H.c.}$$

Weak Charged Current:  $j_W^\rho = j_{W,L}^\rho + j_{W,Q}^\rho$

### Leptonic Weak Charged Current

$$j_{W,L}^\rho = \sum_{\alpha=e,\mu,\tau} \bar{\nu}'_\alpha \gamma^\rho (1 - \gamma^5) \ell'_\alpha = 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}'_{\alpha L} \gamma^\rho \ell'_{\alpha L} = 2 \bar{\nu}'_L \gamma^\rho \ell'_L$$

$$\ell'_L = V_L^\ell \ell_L \quad \nu'_L = V_L^\nu \mathbf{n}_L$$

$$j_{W,L}^\rho = 2 \bar{\mathbf{n}}_L V_L^{\nu\dagger} \gamma^\rho V_L^\ell \ell_L = 2 \bar{\mathbf{n}}_L V_L^{\nu\dagger} V_L^\ell \gamma^\rho \ell_L = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

### Mixing Matrix

$$U^\dagger = V_L^{\nu\dagger} V_L^\ell \quad U = V_L^{\ell\dagger} V_L^\nu$$

► **Definition:** Left-Handed Flavor Neutrino Fields

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

► They allow us to write the **Leptonic Weak Charged Current** as in the SM:

$$j_{W,L}^\rho = 2 \overline{\nu}_L \gamma^\rho \ell_L = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^\rho \ell_{\alpha L}$$

► Each **left-handed flavor neutrino field** is associated with the corresponding **charged lepton field** which describes a massive charged lepton ( $e, \mu, \tau$ ).

► In practice **left-handed flavor neutrino fields** are useful for calculations in the SM approximation of massless neutrinos (**interactions**).

► If neutrino masses must be taken into account, it is necessary to use

$$j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} U_{\alpha k}^* \overline{\nu}_{kL} \gamma^\rho \ell_{\alpha L}$$

# Flavor Lepton Numbers

Flavor Neutrino Fields are useful for defining  
Flavor Lepton Numbers  
as in the SM

	$L_e$	$L_\mu$	$L_\tau$		$L_e$	$L_\mu$	$L_\tau$
$(\nu_e, e^-)$	+1	0	0	$(\nu_e^c, e^+)$	-1	0	0
$(\nu_\mu, \mu^-)$	0	+1	0	$(\nu_\mu^c, \mu^+)$	0	-1	0
$(\nu_\tau, \tau^-)$	0	0	+1	$(\nu_\tau^c, \tau^+)$	0	0	-1

$$L = L_e + L_\mu + L_\tau$$

Standard Model: Lepton numbers are conserved

- ▶ **Leptonic Weak Charged Current** is invariant under the global U(1) gauge transformations

$$l_{\alpha L} \rightarrow e^{i\varphi_\alpha} l_{\alpha L} \quad \nu_{\alpha L} \rightarrow e^{i\varphi_\alpha} \nu_{\alpha L} \quad (\alpha = e, \mu, \tau)$$

- ▶ If neutrinos are massless (SM), Noether's theorem implies that there is, for each flavor, a conserved current:

$$j_\alpha^\rho = \overline{\nu_{\alpha L}} \gamma^\rho \nu_{\alpha L} + \overline{l_\alpha} \gamma^\rho l_\alpha \quad \partial_\mu j_\alpha^\mu = 0$$

and a conserved charge:

$$L_\alpha = \int d^3x j_\alpha^0(x) \quad \partial_0 L_\alpha = 0$$

$$\begin{aligned} :L_\alpha: &= \int \frac{d^3p}{(2\pi)^3 2E} \left[ a_{\nu_\alpha}^{(-)\dagger}(p) a_{\nu_\alpha}^{(-)}(p) - b_{\nu_\alpha}^{(+)\dagger}(p) b_{\nu_\alpha}^{(+)}(p) \right] \\ &+ \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{l_\alpha}^{(h)\dagger}(p) a_{l_\alpha}^{(h)}(p) - b_{l_\alpha}^{(h)\dagger}(p) b_{l_\alpha}^{(h)}(p) \right] \end{aligned}$$

▶ Lepton-Higgs Yukawa Lagrangian:

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \left[ \sum_{\alpha=e,\mu,\tau} y_{\alpha}^{\ell} \overline{l_{\alpha L}} l_{\alpha R} + \sum_{k=1}^3 y_k^{\nu} \overline{\nu_{kL}} \nu_{kR} \right] + \text{H.c.}$$

▶ Mixing:  $\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \iff \nu_{kL} = \sum_{\alpha=e,\mu,\tau} U_{\alpha k}^* \nu_{\alpha L}$

$$\mathcal{L}_{H,L} = - \left( \frac{v+H}{\sqrt{2}} \right) \sum_{\alpha=e,\mu,\tau} \left[ y_{\alpha}^{\ell} \overline{l_{\alpha L}} l_{\alpha R} + \overline{\nu_{\alpha L}} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \right] + \text{H.c.}$$

▶ Invariant for

$$l_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha L}, \quad \nu_{\alpha L} \rightarrow e^{i\varphi_{\alpha}} \nu_{\alpha L}$$

$$l_{\alpha R} \rightarrow e^{i\varphi_{\alpha}} l_{\alpha R}, \quad \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR} \rightarrow e^{i\varphi_{\alpha}} \sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$$

▶ But kinetic part of neutrino Lagrangian is not invariant

$$\mathcal{L}_{\text{kinetic}}^{(\nu)} = \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} i \not{\partial} \nu_{\alpha L} + \sum_{k=1}^3 \overline{\nu_{kR}} i \not{\partial} \nu_{kR}$$

because  $\sum_{k=1}^3 U_{\alpha k} y_k^{\nu} \nu_{kR}$  is not a unitary combination of the  $\nu_{kR}$ 's

$$\mathcal{L}_{\text{mass}}^{\text{D}} = - \begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee}^{\text{D}} & m_{e\mu}^{\text{D}} & m_{e\tau}^{\text{D}} \\ m_{\mu e}^{\text{D}} & m_{\mu\mu}^{\text{D}} & m_{\mu\tau}^{\text{D}} \\ m_{\tau e}^{\text{D}} & m_{\tau\mu}^{\text{D}} & m_{\tau\tau}^{\text{D}} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \text{H.c.}$$

$L_e, L_\mu, L_\tau$  are not conserved

$L$  is conserved:  $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

# Total Lepton Number

- ▶ Dirac neutrino masses violate conservation of Flavor Lepton Numbers
- ▶ Total Lepton Number is conserved, because Lagrangian is invariant under the global U(1) gauge transformations

$$\nu_{kL} \rightarrow e^{i\varphi} \nu_{kL}, \quad \nu_{kR} \rightarrow e^{i\varphi} \nu_{kR} \quad (k = 1, 2, 3)$$

$$l_{\alpha L} \rightarrow e^{i\varphi} l_{\alpha L}, \quad l_{\alpha R} \rightarrow e^{i\varphi} l_{\alpha R} \quad (\alpha = e, \mu, \tau)$$

- ▶ From Noether's theorem:

$$j^\rho = \sum_{k=1}^3 \bar{\nu}_k \gamma^\rho \nu_k + \sum_{\alpha=e,\mu,\tau} \bar{l}_\alpha \gamma^\rho l_\alpha \quad \partial_\rho j^\rho = 0$$

$$\text{Conserved charge: } L_\alpha = \int d^3x j_\alpha^0(x) \quad \partial_0 L_\alpha = 0$$

$$\begin{aligned} :L: &= \sum_{k=1}^3 \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{\nu_k}^{(h)\dagger}(p) a_{\nu_k}^{(h)}(p) - b_{\nu_k}^{(h)\dagger}(p) b_{\nu_k}^{(h)}(p) \right] \\ &+ \sum_{\alpha=e,\mu,\tau} \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a_{l_\alpha}^{(h)\dagger}(p) a_{l_\alpha}^{(h)}(p) - b_{l_\alpha}^{(h)\dagger}(p) b_{l_\alpha}^{(h)}(p) \right] \end{aligned}$$



# Mixing Matrix

▶ Leptonic Weak Charged Current:  $j_{W,L}^\rho = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$

$$\text{▶ } U = V_L^{\ell\dagger} V_L^\nu = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

▶ Unitary  $N \times N$  matrix depends on  $N^2$  independent real parameters

$$N = 3 \quad \Rightarrow \quad \begin{array}{ll} \frac{N(N-1)}{2} = 3 & \text{Mixing Angles} \\ \frac{N(N+1)}{2} = 6 & \text{Phases} \end{array}$$

▶ Not all phases are physical observables

▶ Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

▶ Weak Charged Current: 
$$j_{W,L}^\rho = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{kL} U_{\alpha k}^* \gamma^\rho l_{\alpha L}$$

- ▶ Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations

$$\nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \quad l_\alpha \rightarrow e^{i\varphi_\alpha} l_\alpha \quad (\alpha = e, \mu, \tau)$$

- ▶ Performing this transformation, the Charged Current becomes

$$j_{W,L}^\rho = 2 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{kL} e^{-i\varphi_k} U_{\alpha k}^* e^{i\varphi_\alpha} \gamma^\rho l_{\alpha L}$$

$$j_{W,L}^\rho = 2 \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_1 \sum_{k=1}^3 \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{kL} \underbrace{e^{-i(\varphi_k - \varphi_1)}}_{N-1=2} U_{\alpha k}^* \underbrace{e^{i(\varphi_\alpha - \varphi_e)}}_{N-1=2} \gamma^\rho l_{\alpha L}$$

- ▶ There are  $1 + (N - 1) + (N - 1) = 2N - 1 = 5$  arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- ▶  $2N - 1$  and not  $2N$  phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant  $\iff$  conservation of Total Lepton Number.

- ▶ The mixing matrix contains

$$\frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2} = 1 \quad \text{Physical Phase}$$

- ▶ It is convenient to express the  $3 \times 3$  unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

# Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = R_{23} W_{13} R_{12}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13} \leq 2\pi$$

3 Mixing Angles  $\vartheta_{12}$ ,  $\vartheta_{23}$ ,  $\vartheta_{13}$  and 1 Phase  $\delta_{13}$

# CP Violation

- ▶  $U = U^* \iff$  CP symmetry
- ▶ General conditions for CP violation (14 conditions):
  1. No two charged leptons or two neutrinos are degenerate in mass (6 conditions)
  2. No mixing angle is equal to 0 or  $\pi/2$  (6 conditions)
  3. The physical phase is different from 0 or  $\pi$  (2 conditions)
- ▶ These 14 conditions are combined into the single condition  $\det C \neq 0$

$$C = -i [M^{\nu\nu} M^{\nu\nu\dagger}, M^{le} M^{le\dagger}]$$

$$\det C = -2 J \left( m_{\nu_2}^2 - m_{\nu_1}^2 \right) \left( m_{\nu_3}^2 - m_{\nu_1}^2 \right) \left( m_{\nu_3}^2 - m_{\nu_2}^2 \right) \\ \left( m_{\mu}^2 - m_e^2 \right) \left( m_{\tau}^2 - m_e^2 \right) \left( m_{\tau}^2 - m_{\mu}^2 \right)$$

- ▶ Jarlskog invariant:  $J = \Im [U_{\mu 3} U_{e 2} U_{\mu 2}^* U_{e 3}^*]$

[C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C 29 (1985) 491]

[O. W. Greenberg, Phys. Rev. D 32 (1985) 1841]

[I. Dunitz, O. W. Greenberg, Dan-di Wu, Phys. Rev. Lett. 55 (1985) 2935]

## Example: $\vartheta_{12} = 0$

$$U = R_{23}R_{13}W_{12}$$

$$W_{12} = \begin{pmatrix} \cos \vartheta_{12} & \sin \vartheta_{12} e^{-i\delta_{12}} & 0 \\ -\sin \vartheta_{12} e^{-i\delta_{12}} & \cos \vartheta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\vartheta_{12} = 0 \quad \Rightarrow \quad W_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \mathbf{1}$$

real mixing matrix

$$U = R_{23}R_{13}$$

## Example: $\vartheta_{13} = \pi/2$

$$U = R_{23} W_{13} R_{12}$$

$$W_{13} = \begin{pmatrix} \cos \vartheta_{13} & 0 & \sin \vartheta_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -\sin \vartheta_{13} e^{i\delta_{13}} & 0 & \cos \vartheta_{13} \end{pmatrix}$$

$$\vartheta_{13} = \pi/2 \quad \Rightarrow \quad W_{13} = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -e^{i\delta_{13}} & 0 & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & 0 \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta_{13}} & 0 \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix}$$

$$\lambda_{\mu 1} - \lambda_{\mu 2} = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi \quad \lambda_{\tau 1} - \lambda_{\mu 1} = \lambda_{\tau 2} - \lambda_{\mu 2} \pm \pi$$

$$\nu_k \rightarrow e^{i\varphi_k} \nu_k \quad (k = 1, 2, 3), \quad \ell_\alpha \rightarrow e^{i\varphi_\alpha} \ell_\alpha \quad (\alpha = e, \mu, \tau)$$

$$U \rightarrow \begin{pmatrix} e^{-i\varphi_e} & 0 & 0 \\ 0 & e^{-i\varphi_\mu} & 0 \\ 0 & 0 & e^{-i\varphi_\tau} \end{pmatrix} \begin{pmatrix} 0 & 0 & e^{-i\delta_{13}} \\ |U_{\mu 1}| e^{i\lambda_{\mu 1}} & |U_{\mu 2}| e^{i\lambda_{\mu 2}} & 0 \\ |U_{\tau 1}| e^{i\lambda_{\tau 1}} & |U_{\tau 2}| e^{i\lambda_{\tau 2}} & 0 \end{pmatrix} \begin{pmatrix} e^{i\varphi_1} & 0 & 0 \\ 0 & e^{i\varphi_2} & 0 \\ 0 & 0 & e^{i\varphi_3} \end{pmatrix}$$

$$U = \begin{pmatrix} 0 & 0 & e^{i(-\delta_{13}-\varphi_e+\varphi_3)} \\ |U_{\mu 1}| e^{i(\lambda_{\mu 1}-\varphi_\mu+\varphi_1)} & |U_{\mu 2}| e^{i(\lambda_{\mu 2}-\varphi_\mu+\varphi_2)} & 0 \\ |U_{\tau 1}| e^{i(\lambda_{\tau 1}-\varphi_\tau+\varphi_1)} & |U_{\tau 2}| e^{i(\lambda_{\tau 2}-\varphi_\tau+\varphi_2)} & 0 \end{pmatrix}$$

$$\varphi_1 = 0 \quad \varphi_\mu = \lambda_{\mu 1} \quad \varphi_\tau = \lambda_{\tau 1} \quad \varphi_2 = \varphi_\mu - \lambda_{\mu 2} = \lambda_{\mu 1} - \lambda_{\mu 2}$$

$$\varphi_2 = \varphi_\tau - \lambda_{\tau 2} \pm \pi = \lambda_{\tau 1} - \lambda_{\tau 2} \pm \pi = \lambda_{\mu 1} - \lambda_{\mu 2} \quad \text{OK!}$$

$$U = \begin{pmatrix} 0 & 0 & \pm 1 \\ |U_{\mu 1}| & |U_{\mu 2}| & 0 \\ |U_{\tau 1}| & -|U_{\tau 2}| & 0 \end{pmatrix}$$



Example:  $m_{\nu_2} = m_{\nu_3}$

$$j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$U = R_{12} R_{13} W_{23} \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L W_{23}^\dagger R_{13}^\dagger R_{12}^\dagger \gamma^\rho \ell_L$$

$$W_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \vartheta_{23} & \sin \vartheta_{23} e^{-i\delta_{23}} \\ 0 & -\sin \vartheta_{23} e^{-i\delta_{23}} & \cos \vartheta_{23} \end{pmatrix}$$

$$W_{23} \mathbf{n}'_L = \mathbf{n}'_L \quad R_{12} R_{13} = U' \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}}'_L U'^\dagger \gamma^\rho \ell_L$$

$\nu_2$  and  $\nu_3$  are indistinguishable

$$\text{drop the prime} \quad \Rightarrow \quad j_{W,L}^\rho = 2 \overline{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L$$

$$\text{real mixing matrix} \quad U = R_{12} R_{13}$$

# Jarlskog Invariant

$$J = \Im \left[ U_{\mu 3} U_{e 2} U_{\mu 2}^* U_{e 3}^* \right]$$

- ▶ All the imaginary parts of the rephasing-invariant quartic products  $U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*$  are equal up to a sign:

$$\Im \left[ U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] = \pm J$$

- ▶ In the standard parameterization

$$\begin{aligned} J &= c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13} \\ &= \frac{1}{8} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos \vartheta_{13} \sin 2\vartheta_{13} \sin \delta_{13} \end{aligned}$$

- ▶ The Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way

# Maximal CP Violation

- ▶ Maximal CP violation is defined as the case in which  $|J|$  has its maximum possible value

$$|J|_{\max} = \frac{1}{6\sqrt{3}}$$

- ▶ In the standard parameterization it is obtained for

$$\vartheta_{12} = \vartheta_{23} = \pi/4, \quad s_{13} = 1/\sqrt{3}, \quad \sin \delta_{13} = \pm 1$$

- ▶ This case is called **Trimaximal Mixing**. All the absolute values of the elements of the mixing matrix are equal to  $1/\sqrt{3}$ :

$$U = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \mp \frac{i}{\sqrt{3}} \\ -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{2} \mp \frac{i}{2\sqrt{3}} & -\frac{1}{2} \mp \frac{i}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & \mp i \\ -e^{\pm i\pi/6} & e^{\mp i\pi/6} & 1 \\ e^{\mp i\pi/6} & -e^{\pm i\pi/6} & 1 \end{pmatrix}$$

# GIM Mechanism

[S.L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2 (1970) 1285]

- ▶ The unitarity of  $V_L^\ell$ ,  $V_R^\ell$  and  $V_L^\nu$  implies that the expression of the neutral weak current in terms of the lepton fields with definite masses is the same as that in terms of the primed lepton fields:

$$\begin{aligned}j_{Z,L}^\rho &= 2 g_L^\nu \overline{\nu_L'} \gamma^\rho \nu_L' + 2 g_L^\ell \overline{\ell_L'} \gamma^\rho \ell_L' + 2 g_R^\ell \overline{\ell_R'} \gamma^\rho \ell_R' \\ &= 2 g_L^\nu \overline{\mathbf{n}_L} V_L^{\nu\dagger} \gamma^\rho V_L^\nu \mathbf{n}_L + 2 g_L^\ell \overline{\ell_L} V_L^{\ell\dagger} \gamma^\rho V_L^\ell \ell_L + 2 g_R^\ell \overline{\ell_R} V_R^{\ell\dagger} \gamma^\rho V_R^\ell \ell_R \\ &= 2 g_L^\nu \overline{\mathbf{n}_L} \gamma^\rho \mathbf{n}_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R\end{aligned}$$

- ▶ The unitarity of  $U$  implies the same expression for the neutral weak current in terms of the flavor neutrino fields  $\nu_L = U \mathbf{n}_L$ :

$$\begin{aligned}j_{Z,L}^\rho &= 2 g_L^\nu \overline{\nu_L} U \gamma^\rho U^\dagger \nu_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R \\ &= 2 g_L^\nu \overline{\nu_L} \gamma^\rho \nu_L + 2 g_L^\ell \overline{\ell_L} \gamma^\rho \ell_L + 2 g_R^\ell \overline{\ell_R} \gamma^\rho \ell_R\end{aligned}$$

# Lepton Numbers Violating Processes

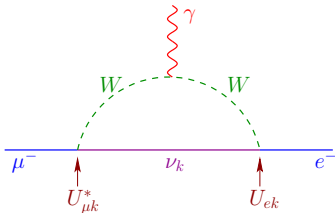
Dirac mass term allows  $L_e, L_\mu, L_\tau$  violating processes

Example:  $\mu^\pm \rightarrow e^\pm + \gamma, \quad \mu^\pm \rightarrow e^\pm + e^+ + e^-$

$$\mu^- \rightarrow e^- + \gamma$$

$\sum_k U_{\mu k}^* U_{ek} = 0 \implies$  only part of  $\nu_k$  propagator  $\propto m_k$  contributes

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \frac{3\alpha}{32\pi} \underbrace{\left| \sum_k U_{\mu k}^* U_{ek} \frac{m_k^2}{m_W^2} \right|^2}_{\text{BR}}$$



Suppression factor:  $\frac{m_k}{m_W} \lesssim 10^{-11}$  for  $m_k \lesssim 1 \text{ eV}$

$$(\text{BR})_{\text{the}} \lesssim 10^{-47}$$

$$(\text{BR})_{\text{exp}} \lesssim 10^{-11}$$

# Majorana Neutrino Masses

- Dirac Neutrino Masses
- Majorana Neutrino Masses
  - Two-Component Theory of a Massless Neutrino
  - Majorana Equation
  - Majorana Lagrangian
  - Majorana Antineutrino Jargon
  - Lepton Number
  - CP Symmetry
  - No Majorana Neutrino Mass in the SM
  - Effective Majorana Mass
  - Mixing of Three Majorana Neutrinos
  - Mixing Matrix
- Dirac-Majorana Mass Term

# Two-Component Theory of a Massless Neutrino

[L. Landau, Nucl. Phys. 3 (1957) 127], [T.D. Lee, C.N. Yang, Phys. Rev. 105 (1957) 1671], [A. Salam, Nuovo Cim. 5 (1957) 299]

- ▶ Dirac Equation:  $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- ▶ Chiral components of a Fermion Field:  $\psi = \psi_L + \psi_R$
- ▶ The equations for the Chiral components are coupled by the mass:

$$i\gamma^\mu\partial_\mu\psi_L = m\psi_R$$

$$i\gamma^\mu\partial_\mu\psi_R = m\psi_L$$

- ▶ They are decoupled for a massless fermion: **Weyl Equations** (1929)

$$i\gamma^\mu\partial_\mu\psi_L = 0$$

$$i\gamma^\mu\partial_\mu\psi_R = 0$$

- ▶ A massless fermion can be described by a single chiral field  $\psi_L$  or  $\psi_R$  (Weyl Spinor).

- ▶  $\psi_L$  and  $\psi_R$  have only two independent components: in the chiral representation

$$\psi_L = \begin{pmatrix} 0 \\ \chi_L \end{pmatrix} \equiv \begin{pmatrix} 0 \\ 0 \\ \chi_{L1} \\ \chi_{L2} \end{pmatrix} \quad \psi_R = \begin{pmatrix} \chi_R \\ 0 \end{pmatrix} \equiv \begin{pmatrix} \chi_{R1} \\ \chi_{R2} \\ 0 \\ 0 \end{pmatrix}$$

- ▶ The possibility to describe a physical particle with a Weyl spinor was rejected by Pauli in 1933 because it leads to the violation of parity
- ▶ The discovery of parity violation in 1956-57 invalidated Pauli's reasoning, opening the possibility to describe massless particles with Weyl spinor fields  $\implies$  Two-component Theory of a Massless Neutrino (1957)
- ▶  $V - A$  Charged-Current Weak Interactions  $\implies \nu_L$
- ▶ In the 1960s, the Two-component Theory of a Massless Neutrino was incorporated in the SM through the assumption of the absence of  $\nu_R$



# Majorana Equation

- ▶ Can a two-component spinor describe a massive fermion? **Yes!** (E. Majorana, 1937)
- ▶ Trick:  $\psi_R$  and  $\psi_L$  are not independent.
- ▶ The relation connecting  $\psi_R$  and  $\psi_L$  must be compatible with the Dirac equation:

$$i\gamma^\mu \partial_\mu \psi_L = m \psi_R \qquad i\gamma^\mu \partial_\mu \psi_R = m \psi_L$$

- ▶ The two equations must be two ways of writing the same equation for one independent field, say  $\psi_L$ .
- ▶ Consider  $i\gamma^\mu \partial_\mu \psi_R = m \psi_L$
- ▶ Take the Hermitian conjugate and multiply on the right with  $\gamma^0$ :  
 $-i\partial_\mu \psi_R^\dagger \gamma^{\mu\dagger} \gamma^0 = m \overline{\psi_L}$
- ▶  $\gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu \implies -i\partial_\mu \overline{\psi_R} \gamma^\mu = m \overline{\psi_L}$
- ▶ Transpose and multiply on the left with  $C$  ( $C \gamma_\mu^T C^{-1} = -\gamma_\mu$ )  $\implies$   
 $i\gamma^\mu \partial_\mu C \overline{\psi_R}^T = m C \overline{\psi_L}^T$

▶  $C\overline{\psi}_L^T$  is right-handed and  $C\overline{\psi}_R^T$  is left-handed

▶  $i\gamma^\mu\partial_\mu C\overline{\psi}_R^T = mC\overline{\psi}_L^T$  has the same structure as  $i\gamma^\mu\partial_\mu\psi_L = m\psi_R$

▶ We can consider them as identical by setting

$$\psi_R = \xi C\overline{\psi}_L^T \quad \text{with} \quad |\xi|^2 = 1$$

▶  $\xi$  is unphysical phase factor which can be eliminated by rephasing

$$\psi_L \rightarrow \xi^{1/2} \psi_L \quad \Longrightarrow \quad \boxed{\psi_R = C\overline{\psi}_L^T}$$

▶ Majorana Equation:  $\boxed{i\gamma^\mu\partial_\mu\psi_L = mC\overline{\psi}_L^T}$

▶ The field  $\psi = \psi_L + \psi_R = \psi_L + C\overline{\psi}_L^T$  is called Majorana Field

▶ Majorana Condition:  $\boxed{\psi = C\overline{\psi}^T}$

▶ A Majorana Field has only two independent components

▶ Chiral representation:  $\psi = \begin{pmatrix} i\sigma^2\chi_L^* \\ \chi_L \end{pmatrix}$

- ▶ Charge Conjugation:  $\psi_L^C = C \overline{\psi_L}^T$
- ▶ Majorana Field:  $\psi = \psi_L + \psi_L^C$       Majorana Condition:  $\psi = \psi^C$
- ▶ The Majorana condition implies the equality of particle and antiparticle
- ▶ Only neutral fermions can be Majorana particles
- ▶ Dirac equation for fermion with charge  $q$  coupled to electromagnetic field  $A_\mu$ :

$$(i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu - m)\psi = 0 \quad \text{(particle)}$$

$$(i\gamma^\mu \partial_\mu + q\gamma^\mu A_\mu - m)\psi^C = 0 \quad \text{(antiparticle)}$$

If  $q \neq 0$ ,  $\psi$  and  $\psi^C$  obey different equations and the Majorana equality cannot be imposed

- ▶ For a Majorana field, the electromagnetic current vanishes identically:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi^C}\gamma^\mu\psi^C = -\psi^T C^\dagger \gamma^\mu C \overline{\psi}^T = \overline{\psi} C \gamma^{\mu T} C^\dagger \psi = -\overline{\psi}\gamma^\mu\psi = 0$$

# Majorana Lagrangian

- ▶ Let us consider first the Dirac Lagrangian

$$\mathcal{L}^D = \bar{\nu} (i\partial - m) \nu = \bar{\nu}_L i\partial \nu_L + \bar{\nu}_R i\partial \nu_R - m (\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$$

- ▶ In order to write a Majorana Mass Term using  $\nu_L$  alone, we make the substitution  $\nu_R \rightarrow \nu_L^C = C \bar{\nu}_L^T$

- ▶ Majorana Lagrangian:

$$\mathcal{L}^M = \frac{1}{2} \left[ \bar{\nu}_L i\partial \nu_L + \bar{\nu}_L^C i\partial \nu_L^C - m (\bar{\nu}_L^C \nu_L + \bar{\nu}_L \nu_L^C) \right]$$

- ▶ The overall factor 1/2 avoids double counting in the derivation of the due to the fact that  $\nu_L^C$  and  $\bar{\nu}_L$  are not independent ( $\nu_L^C = C \bar{\nu}_L^T$ )

$$\mathcal{L}^M = \bar{\nu}_L i\partial \nu_L - \frac{m}{2} \left( -\nu_L^T C^\dagger \nu_L + \bar{\nu}_L C \bar{\nu}_L^T \right)$$

▶ Majorana Field:  $\nu = \nu_L + \nu_L^C$

▶ Majorana Condition:  $\nu^C = \nu$

▶ Majorana Lagrangian:  $\mathcal{L}^M = \frac{1}{2} \bar{\nu} (i\not{\partial} - m) \nu$

▶ The factor  $1/2$  distinguishes the Majorana Lagrangian from the Dirac Lagrangian

▶ Quantized Dirac Neutrino Field:

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a^{(h)}(p) u^{(h)}(p) e^{-ip \cdot x} + b^{(h)\dagger}(p) v^{(h)}(p) e^{ip \cdot x} \right]$$

▶ Quantized Majorana Neutrino Field [ $b^{(h)}(p) = a^{(h)}(p)$ ]

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[ a^{(h)}(p) u^{(h)}(p) e^{-ip \cdot x} + a^{(h)\dagger}(p) v^{(h)}(p) e^{ip \cdot x} \right]$$

▶ A Majorana field has half the degrees of freedom of a Dirac field

# Majorana Antineutrino Jargon

- ▶ A Majorana neutrino is the same as a Majorana antineutrino
- ▶ Neutrino interactions are described by the CC and NC Lagrangians

$$\mathcal{L}_{1,L}^{\text{CC}} = -\frac{g}{\sqrt{2}} \left( \bar{\nu}_L \gamma^\mu \ell_L W_\mu + \bar{\ell}_L \gamma^\mu \nu_L W_\mu^\dagger \right)$$

$$\mathcal{L}_{1,\nu}^{\text{NC}} = -\frac{g}{2 \cos \vartheta_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

- ▶ In practice, since detectable neutrinos are always ultrarelativistic, the neutrino mass can be neglected in interactions

- ▶ In interaction amplitudes we neglect corrections of order  $m/E$

▶ Dirac:  $\left\{ \begin{array}{l} \nu_L \left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed antineutrinos} \end{array} \right. \\ \bar{\nu}_L \left\{ \begin{array}{l} \text{destroys right-handed antineutrinos} \\ \text{creates left-handed neutrinos} \end{array} \right. \end{array} \right.$

▶ Majorana:  $\left\{ \begin{array}{l} \nu_L \left\{ \begin{array}{l} \text{destroys left-handed neutrinos} \\ \text{creates right-handed neutrinos} \end{array} \right. \\ \bar{\nu}_L \left\{ \begin{array}{l} \text{destroys right-handed neutrinos} \\ \text{creates left-handed neutrinos} \end{array} \right. \end{array} \right.$

- ▶ Common definitions:

Majorana neutrino with negative helicity  $\equiv$  neutrino

Majorana neutrino with positive helicity  $\equiv$  antineutrino

# Lepton Number

- ▶ The Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} m \left( \nu_L^T C^\dagger \nu_L + \nu_L^\dagger C \nu_L^* \right)$$

is not invariant under the global U(1) gauge transformation

$$\nu_L \rightarrow e^{i\varphi} \nu_L$$

$$\cancel{L = -1} \leftarrow \boxed{\nu^c = \nu} \rightarrow \cancel{L = +1}$$

- ▶ The **Total Lepton Number** is not conserved:  $\boxed{\Delta L = \pm 2}$
- ▶ However, the **Total Lepton Number** is conserved in interactions in the ultrarelativistic approximation of massless neutrinos
- ▶ Best process to find the violation of the **Total Lepton Number**:  
Neutrinoless Double- $\beta$  Decay

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + 2 e^- \quad (\beta\beta_{0\nu}^-)$$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z - 2) + 2 e^+ \quad (\beta\beta_{0\nu}^+)$$



# CP Symmetry

- ▶ Under a CP transformation

$$U_{\text{CP}} \nu_L(x) U_{\text{CP}}^{-1} = \xi_\nu^{\text{CP}} \gamma^0 \nu_L^{\text{C}}(x_{\text{P}})$$

$$U_{\text{CP}} \nu_L^{\text{C}}(x) U_{\text{CP}}^{-1} = -\xi_\nu^{\text{CP}*} \gamma^0 \nu_L(x_{\text{P}})$$

$$U_{\text{CP}} \bar{\nu}_L(x) U_{\text{CP}}^{-1} = \xi_\nu^{\text{CP}*} \bar{\nu}_L^{\text{C}}(x_{\text{P}}) \gamma^0$$

$$U_{\text{CP}} \bar{\nu}_L^{\text{C}}(x) U_{\text{CP}}^{-1} = -\xi_\nu^{\text{CP}} \bar{\nu}_L(x_{\text{P}}) \gamma^0$$

with  $|\xi_\nu^{\text{CP}}|^2 = 1$ ,  $x^\mu = (x^0, \vec{x})$ , and  $x_{\text{P}}^\mu = (x^0, -\vec{x})$

- ▶ The theory is CP-symmetric if there are values of the phase  $\xi_\nu^{\text{CP}}$  such that the Lagrangian transforms as

$$U_{\text{CP}} \mathcal{L}(x) U_{\text{CP}}^{-1} = \mathcal{L}(x_{\text{P}})$$

in order to keep invariant the action  $I = \int d^4x \mathcal{L}(x)$

► The Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{M}}(x) = -\frac{1}{2} m \left[ \overline{\nu_L^{\text{C}}}(x) \nu_L(x) + \overline{\nu_L}(x) \nu_L^{\text{C}}(x) \right]$$

transforms as

$$\begin{aligned} U_{\text{CP}} \mathcal{L}_{\text{mass}}^{\text{M}}(x) U_{\text{CP}}^{-1} = & -\frac{1}{2} m \left[ -(\xi_{\nu}^{\text{CP}})^2 \overline{\nu_L}(x_{\text{P}}) \nu_L^{\text{C}}(x_{\text{P}}) \right. \\ & \left. -(\xi_{\nu}^{\text{CP}*})^2 \overline{\nu_L^{\text{C}}}(x_{\text{P}}) \nu_L(x_{\text{P}}) \right] \end{aligned}$$

►  $U_{\text{CP}} \mathcal{L}_{\text{mass}}^{\text{M}}(x) U_{\text{CP}}^{-1} = \mathcal{L}_{\text{mass}}^{\text{M}}(x_{\text{P}})$  for  $\xi_{\nu}^{\text{CP}} = \pm i$

► The one-generation Majorana theory is CP-symmetric

► The Majorana case is different from the Dirac case, in which the CP phase  $\xi_{\nu}^{\text{CP}}$  is arbitrary

# No Majorana Neutrino Mass in the SM

- ▶ A Majorana Mass Term  $\propto [\nu_L^T C^\dagger \nu_L - \overline{\nu_L} C \overline{\nu_L}^T]$  involves only the neutrino left-handed chiral field  $\nu_L$ , which is present in the SM (one for each lepton generation)
- ▶ Eigenvalues of the weak isospin  $I$ , of its third component  $I_3$ , of the hypercharge  $Y$  and of the charge  $Q$  of the lepton and Higgs multiplets:

		$I$	$I_3$	$Y$	$Q = I_3 + \frac{Y}{2}$
lepton doublet	$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$-1$	$0$ $-1$
lepton singlet	$\ell_R$	$0$	$0$	$-2$	$-1$
Higgs doublet	$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	$1$ $0$

- ▶  $\nu_L^T C^\dagger \nu_L$  has  $I_3 = 1$  and  $Y = -2 \implies$  needed Higgs triplet with  $Y = 2$

# Effective Majorana Mass

- ▶ Dimensional analysis: Fermion Field  $\sim [E]^{3/2}$       Boson Field  $\sim [E]$
- ▶ Dimensionless action:  $I = \int d^4x \mathcal{L}(x) \implies \mathcal{L}(x) \sim [E]^4$
- ▶ Kinetic terms:  $\bar{\psi}i\partial\psi \sim [E]^4$ ,  $(\partial_\mu\phi)^\dagger \partial^\mu\phi \sim [E]^4$
- ▶ Mass terms:  $m\bar{\psi}\psi \sim [E]^4$ ,  $m^2\phi^\dagger\phi \sim [E]^4$
- ▶ CC weak interaction:  $g\bar{\nu}_L\gamma^\rho l_L W_\rho \sim [E]^4$
- ▶ Yukawa couplings:  $y\bar{L}_L\Phi l_R \sim [E]^4$
- ▶ Product of fields  $\mathcal{O}_d$  with energy dimension  $d \equiv \text{dim-}d$  operator
- ▶  $\mathcal{L}(\mathcal{O}_d) = C_{(\mathcal{O}_d)}\mathcal{O}_d \implies C_{(\mathcal{O}_d)} \sim [E]^{4-d}$
- ▶  $\mathcal{O}_{d>4}$  are not renormalizable

- ▶ SM Lagrangian includes all  $\mathcal{O}_{d \leq 4}$  invariant under  $SU(2)_L \times U(1)_Y$
- ▶ SM cannot be considered as the final theory of everything
- ▶ SM is an effective low-energy theory
- ▶ It is likely that SM is the low-energy product of the symmetry breaking of a high-energy unified theory
- ▶ It is plausible that at low-energy there are effective non-renormalizable  $\mathcal{O}_{d > 4}$  [S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]
- ▶ All  $\mathcal{O}_d$  must respect  $SU(2)_L \times U(1)_Y$ , because they are generated by the high-energy theory which must include the gauge symmetries of the SM in order to be effectively reduced to the SM at low energies
- ▶ Approach analogous to effective non-renormalizable four-fermion Fermi theory of weak interactions, which is a low-energy manifestation of the SM

- ▶  $\mathcal{O}_{d>4}$  is suppressed by a coefficient  $\mathcal{M}^{-(d-4)}$ , where  $\mathcal{M}$  is a heavy mass characteristic of the symmetry breaking scale of the high-energy unified theory:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}} \mathcal{O}_5 + \frac{g_6}{\mathcal{M}^2} \mathcal{O}_6 + \dots$$

- ▶ Analogy with  $\mathcal{L}_{\text{eff}}^{(\text{CC})} = -\frac{G_F}{\sqrt{2}} j_W^\dagger j_W^\mu$ :

$$\mathcal{O}_6 \rightarrow j_W^\dagger j_W^\mu \qquad \frac{g_6}{\mathcal{M}^2} \rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

- ▶  $\mathcal{M}^{-(d-4)}$  is a strong suppression factor which limits the observability of the low-energy effects of the new physics beyond the SM
- ▶ The difficulty to observe the effects of the effective low-energy non-renormalizable operators increase rapidly with their dimensionality
- ▶  $\mathcal{O}_5 \implies$  Majorana neutrino masses (Lepton number violation)
- ▶  $\mathcal{O}_6 \implies$  Baryon number violation (proton decay)

- ▶ Only one dim-5 operator:

$$\begin{aligned} \mathcal{O}_5 &= (L_L^T \tau_2 \Phi) C^\dagger (\Phi^T \tau_2 L_L) + \text{H.c.} \\ &= \frac{1}{2} (L_L^T C^\dagger \tau_2 \vec{\tau} L_L) \cdot (\Phi^T \tau_2 \vec{\tau} \Phi) + \text{H.c.} \end{aligned}$$

$$\mathcal{L}_5 = \frac{g_5}{2\mathcal{M}} (L_L^T C^\dagger \tau_2 \vec{\tau} L_L) \cdot (\Phi^T \tau_2 \vec{\tau} \Phi) + \text{H.c.}$$

- ▶ Electroweak Symmetry Breaking:  $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}}$   $\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

$$\mathcal{L}_5 \xrightarrow[\text{Breaking}]{\text{Symmetry}} \mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \frac{g_5 v^2}{\mathcal{M}} \nu_L^T C^\dagger \nu_L + \text{H.c.} \implies$$

$$m = \frac{g_5 v^2}{\mathcal{M}}$$

- ▶ The study of Majorana neutrino masses provides the most accessible low-energy window on new physics beyond the SM
- ▶  $m \propto \frac{v^2}{\mathcal{M}} \propto \frac{m_D^2}{\mathcal{M}}$  natural explanation of smallness of neutrino masses (special case: See-Saw Mechanism)
- ▶ Example:  $m_D \sim v \sim 10^2 \text{ GeV}$  and  $\mathcal{M} \sim 10^{15} \text{ GeV} \implies m \sim 10^{-2} \text{ eV}$



# Mixing of Three Majorana Neutrinos

$$\begin{aligned} \blacktriangleright \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \mathcal{L}_{\text{mass}}^M &= \frac{1}{2} \left( \nu'^T_L C^\dagger M^L \nu'_L - \overline{\nu}'_L M^{L\dagger} C \nu'^T_L \right) \\ &= \frac{1}{2} \sum_{\alpha, \beta=e, \mu, \tau} \left( \nu'^T_{\alpha L} C^\dagger M^L_{\alpha\beta} \nu'_{\beta L} - \overline{\nu}'_{\alpha L} M^{L*}_{\beta\alpha} C \nu'^T_{\beta L} \right) \end{aligned}$$

$\blacktriangleright$  In general, the matrix  $M^L$  is a complex symmetric matrix

$$\begin{aligned} \sum_{\alpha, \beta} \nu'^T_{\alpha L} C^\dagger M^L_{\alpha\beta} \nu'_{\beta L} &= - \sum_{\alpha, \beta} \nu'^T_{\beta L} M^L_{\alpha\beta} (C^\dagger)^T \nu'_{\alpha L} \\ &= \sum_{\alpha, \beta} \nu'^T_{\beta L} C^\dagger M^L_{\alpha\beta} \nu'_{\alpha L} = \sum_{\alpha, \beta} \nu'^T_{\alpha L} C^\dagger M^L_{\beta\alpha} \nu'_{\beta L} \end{aligned}$$

$$M^L_{\alpha\beta} = M^L_{\beta\alpha} \iff M^L = M^{L T}$$

- ▶  $\mathcal{L}_{\text{mass}}^M = \frac{1}{2} \left( \nu_L'^T C^\dagger M^L \nu_L' - \overline{\nu_L'} M^{L\dagger} C \nu_L'^T \right)$
- ▶ Diagonalization:  $\nu_L' = \mathbf{n}_L V_L^{\nu\dagger}$  with **unitary**  $V_L^\nu$
- ▶  $(V_L^\nu)^T M^L V_L^\nu = M$ ,  $M_{kj} = m_k \delta_{kj}$  ( $k, j = 1, 2, 3$ )
- ▶ Real and Positive  $m_k$

- ▶ Left-handed chiral fields with definite mass:  $\mathbf{n}_L = V_L^{\nu\dagger} \nu_L' = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$

$$\begin{aligned} \mathcal{L}_{\text{mass}}^M &= \frac{1}{2} \left( \mathbf{n}_L^T C^\dagger M \mathbf{n}_L - \overline{\mathbf{n}}_L M C \mathbf{n}_L^T \right) \\ &= \frac{1}{2} \sum_{k=1}^3 m_k \left( \nu_{kL}^T C^\dagger \nu_{kL} - \overline{\nu_{kL}} C \nu_{kL}^T \right) \end{aligned}$$

- ▶ Majorana fields of massive neutrinos:  $\nu_k = \nu_{kL} + \nu_{kL}^C$

$$\nu_k^C = \nu_k$$

- ▶  $\mathbf{n} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \implies \mathcal{L}^M = \frac{1}{2} \sum_{k=1}^3 \overline{\nu}_k (i\not{\partial} - m_k) \nu_k = \frac{1}{2} \overline{\mathbf{n}} (i\not{\partial} - M) \mathbf{n}$

# Mixing Matrix

- ▶ Leptonic Weak Charged Current:

$$j_{W,L}^\rho = 2 \bar{\mathbf{n}}_L U^\dagger \gamma^\rho \ell_L \quad \text{with} \quad U = V_L^{\ell\dagger} V_L^\nu$$

- ▶ Definition of the left-handed flavor neutrino fields:

$$\nu_L = U \mathbf{n}_L = V_L^{\ell\dagger} \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}$$

- ▶ Leptonic Weak Charged Current has the SM form

$$j_{W,L}^\rho = 2 \bar{\nu}_L \gamma^\rho \ell_L = 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^\rho \ell_{\alpha L}$$

- ▶ Important difference with respect to Dirac case:

Two additional CP-violating phases: Majorana phases

- ▶ The Majorana Mass Term  $\mathcal{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \sum_{k=1}^3 m_k \left( \nu_{kL}^T C^\dagger \nu_{kL} - \overline{\nu_{kL}} C \nu_{kL}^T \right)$  is not invariant under the global U(1) gauge transformations  

$$\nu_{kL} \rightarrow e^{i\varphi_k} \nu_{kL} \quad (k = 1, 2, 3)$$

- ▶ The left-handed massive neutrino fields cannot be rephased in order to eliminate the two phases that can be factorized on the right of the mixing matrix

$$U = U^{\text{D}} D^{\text{M}} \quad D^{\text{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- ▶  $U^{\text{D}}$  is analogous to a Dirac mixing matrix, with one Dirac phase
- ▶ Standard parameterization:

$$U^{\text{D}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

- ▶ Jarlskog invariant:  $J = c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13} \sin \delta_{13}$  as in the Dirac case

▶  $D^M = \text{diag}(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3})$ , but only two Majorana phases are physical

▶ All measurable quantities depend only on the differences of the Majorana phases

$$\ell_\alpha \rightarrow e^{i\varphi} \ell_\alpha \implies e^{i\lambda_k} \rightarrow e^{i(\lambda_k - \varphi)}$$

$e^{i(\lambda_k - \lambda_j)}$  remains constant

▶ Our convention:  $\lambda_1 = 0 \implies D^M = \text{diag}(1, e^{i\lambda_2}, e^{i\lambda_3})$

▶ CP is conserved if all the elements of each column of the mixing matrix are either real or purely imaginary:

$$\delta_{13} = 0 \text{ or } \pi \quad \text{and} \quad \lambda_k = 0 \text{ or } \pi/2 \text{ or } \pi \text{ or } 3\pi/2$$

# Dirac-Majorana Mass Term

- Dirac Neutrino Masses
- Majorana Neutrino Masses
- Dirac-Majorana Mass Term
  - One Generation
  - Real Mass Matrix
  - Maximal Mixing
  - Dirac Limit
  - Pseudo-Dirac Neutrinos
  - See-Saw Mechanism
  - Majorana Neutrino Mass?
  - Right-Handed Neutrino Mass Term
  - Singlet Majoron Model
  - Three-Generation Mixing
  - Number of Massive Neutrinos?

# One Generation

If  $\nu_R$  exists, the most general mass term is the

Dirac-Majorana Mass Term

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = -m_{\text{D}} \bar{\nu}_R \nu_L + \text{H.c.} \quad \text{Dirac Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{L}} = \frac{1}{2} m_{\text{L}} \nu_L^T C^\dagger \nu_L + \text{H.c.} \quad \text{Majorana Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{R}} = \frac{1}{2} m_{\text{R}} \nu_R^T C^\dagger \nu_R + \text{H.c.} \quad \text{New Majorana Mass Term!}$$

- ▶ Column matrix of left-handed chiral fields:  $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ C \bar{\nu}_R^T \end{pmatrix}$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} N_L^T C^\dagger M N_L + \text{H.c.} \quad M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

- ▶ The Dirac-Majorana Mass Term has the structure of a Majorana Mass Term for two chiral neutrino fields coupled by the Dirac mass

- ▶ Diagonalization:  $n_L = U^\dagger N_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{Real } m_k \geq 0$$

- ▶  $\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \sum_{k=1,2} m_k \nu_{kL}^T C^\dagger \nu_{kL} + \text{H.c.} = -\frac{1}{2} \sum_{k=1,2} m_k \bar{\nu}_k \nu_k$

$$\nu_k = \nu_{kL} + \nu_{kL}^C$$

- ▶ Massive neutrinos are Majorana!

$$\nu_k = \nu_k^C$$



## Real Mass Matrix

▶ CP is conserved if the mass matrix is real:  $M = M^*$

▶  $M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$  we consider real and positive  $m_R$  and  $m_D$  and real  $m_L$

▶ A real symmetric mass matrix can be diagonalized with  $U = \mathcal{O} \rho$

$$\mathcal{O} = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \quad \rho_k^2 = \pm 1$$

▶  $\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}$   $\tan 2\vartheta = \frac{2m_D}{m_R - m_L}$

$$m'_{2,1} = \frac{1}{2} \left[ m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2} \right]$$

▶  $m'_1$  is negative if  $m_L m_R < m_D^2$

$$U^T M U = \rho^T \mathcal{O}^T M \mathcal{O} \rho = \begin{pmatrix} \rho_1^2 m'_1 & 0 \\ 0 & \rho_2^2 m'_2 \end{pmatrix} \implies \boxed{m_k = \rho_k^2 m'_k}$$

- $m'_2$  is always positive:

$$m_2 = m'_2 = \frac{1}{2} \left[ m_L + m_R + \sqrt{(m_L - m_R)^2 + 4 m_D^2} \right]$$

- If  $m_L m_R \geq m_D^2$ , then  $m'_1 \geq 0$  and  $\rho_1^2 = 1$

$$m_1 = \frac{1}{2} \left[ m_L + m_R - \sqrt{(m_L - m_R)^2 + 4 m_D^2} \right]$$

$$\rho_1 = 1 \text{ and } \rho_2 = 1 \implies U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

- If  $m_L m_R < m_D^2$ , then  $m'_1 < 0$  and  $\rho_1^2 = -1$

$$m_1 = \frac{1}{2} \left[ \sqrt{(m_L - m_R)^2 + 4 m_D^2} - (m_L + m_R) \right]$$

$$\rho_1 = i \text{ and } \rho_2 = 1 \implies U = \begin{pmatrix} i \cos \vartheta & \sin \vartheta \\ -i \sin \vartheta & \cos \vartheta \end{pmatrix}$$

- ▶ If  $\Delta m^2$  is small, there are oscillations between active  $\nu_a$  generated by  $\nu_L$  and sterile  $\nu_s$  generated by  $\nu_R^C$ :

$$P_{\nu_a \rightarrow \nu_s}(L, E) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

$$\Delta m^2 = m_2^2 - m_1^2 = (m_L + m_R) \sqrt{(m_L - m_R)^2 + 4m_D^2}$$

- ▶ It can be shown that the CP parity of  $\nu_k$  is  $\xi_k^{\text{CP}} = i\rho_k^2$ :

$$U_{\text{CP}} \nu_k(x) U_{\text{CP}}^{-1} = i\rho_k^2 \gamma^0 \nu_k(x_P)$$

- ▶ Special cases:

- ▶  $m_L = m_R \implies$  Maximal Mixing
- ▶  $m_L = m_R = 0 \implies$  Dirac Limit
- ▶  $|m_L|, m_R \ll m_D \implies$  Pseudo-Dirac Neutrinos
- ▶  $m_L = 0 \quad m_D \ll m_R \implies$  See-Saw Mechanism

# Maximal Mixing

$$m_L = m_R$$

$$\vartheta = \pi/4$$

$$m'_{2,1} = m_L \pm m_D$$

$$\left\{ \begin{array}{ll} \rho_1^2 = +1, & m_1 = m_L - m_D \quad \text{if } m_L \geq m_D \\ \rho_1^2 = -1, & m_1 = m_D - m_L \quad \text{if } m_L < m_D \end{array} \right.$$

$$m_2 = m_L + m_D$$

$$\underline{m_L < m_D}$$

$$\left\{ \begin{array}{l} \nu_{1L} = \frac{-i}{\sqrt{2}} (\nu_L - \nu_R^C) \\ \nu_{2L} = \frac{1}{\sqrt{2}} (\nu_L + \nu_R^C) \end{array} \right.$$

$$\left\{ \begin{array}{l} \nu_1 = \nu_{1L} + \nu_{1L}^C = \frac{-i}{\sqrt{2}} [(\nu_L + \nu_R) - (\nu_L^C + \nu_R^C)] \\ \nu_2 = \nu_{2L} + \nu_{2L}^C = \frac{1}{\sqrt{2}} [(\nu_L + \nu_R) + (\nu_L^C + \nu_R^C)] \end{array} \right.$$

## Dirac Limit

$$m_L = m_R = 0$$

$$\blacktriangleright m'_{2,1} = \pm m_D \implies \begin{cases} \rho_1^2 = -1, & m_1 = m_D \\ \rho_2^2 = +1, & m_2 = m_D \end{cases}$$

- $\blacktriangleright$  The two Majorana fields  $\nu_1$  and  $\nu_2$  can be combined to give one Dirac field:

$$\nu = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) = \nu_L + \nu_R$$

- $\blacktriangleright$  A Dirac field  $\nu$  can always be split in two Majorana fields:

$$\begin{aligned} \nu &= \frac{1}{2} \left[ (\nu - \nu^C) + (\nu + \nu^C) \right] \\ &= \frac{i}{\sqrt{2}} \left( -i \frac{\nu - \nu^C}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left( \frac{\nu + \nu^C}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} (i\nu_1 + \nu_2) \end{aligned}$$

- $\blacktriangleright$  A Dirac field is equivalent to two Majorana fields with the same mass and opposite CP parities

# Pseudo-Dirac Neutrinos

$$|m_L|, m_R \ll m_D$$

$$\blacktriangleright m'_{2,1} \simeq \frac{m_L + m_R}{2} \pm m_D$$

$$\blacktriangleright m'_1 < 0 \implies \rho_1^2 = -1 \implies m_{2,1} \simeq m_D \pm \frac{m_L + m_R}{2}$$

- ▶ The two massive Majorana neutrinos have opposite CP parities and are almost degenerate in mass
- ▶ The best way to reveal pseudo-Dirac neutrinos are active-sterile neutrino oscillations due to the small squared-mass difference

$$\Delta m^2 \simeq m_D (m_L + m_R)$$

- ▶ The oscillations occur with practically maximal mixing:

$$\tan 2\vartheta = \frac{2m_D}{m_R - m_L} \gg 1 \implies \vartheta \simeq \pi/4$$

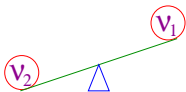
# See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

$$m_L = 0 \quad m_D \ll m_R$$

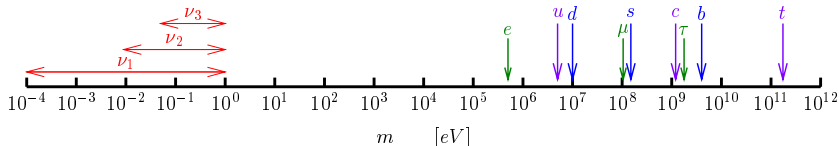
- ▶  $\mathcal{L}_{\text{mass}}^L$  is forbidden by SM symmetries  $\implies m_L = 0$
- ▶  $m_D \lesssim v \sim 100 \text{ GeV}$  is generated by SM Higgs Mechanism (protected by SM symmetries)
- ▶  $m_R$  is not protected by SM symmetries  $\implies m_R \sim \mathcal{M}_{\text{GUT}} \gg v$

$$\left. \begin{array}{l} m'_1 \simeq -\frac{m_D^2}{m_R} \\ m'_2 \simeq m_R \end{array} \right\} \implies \left\{ \begin{array}{l} \rho_1^2 = -1, \quad m_1 \simeq \frac{m_D^2}{m_R} \\ \rho_2^2 = +1, \quad m_2 \simeq m_R \end{array} \right.$$



- ▶ Natural explanation of smallness of neutrino masses
- ▶ Mixing angle is very small:  $\tan 2\vartheta = 2 \frac{m_D}{m_R} \ll 1$
- ▶  $\nu_1$  is composed mainly of active  $\nu_L$ :  $\nu_{1L} \simeq -i \nu_L$
- ▶  $\nu_2$  is composed mainly of sterile  $\nu_R$ :  $\nu_{2L} \simeq \nu_R^C$

# Majorana Neutrino Mass?



known natural explanation of smallness of  $\nu$  masses

New High Energy Scale  $\mathcal{M} \Rightarrow \left\{ \begin{array}{l} \text{See-Saw Mechanism (if } \nu_R \text{'s exist)} \\ \text{5-D Non-Renormaliz. Eff. Operator} \end{array} \right.$

both imply  $\left\{ \begin{array}{l} \text{Majorana } \nu \text{ masses} \iff |\Delta L| = 2 \iff \beta\beta_{0\nu} \text{ decay} \\ \text{see-saw type relation } m_\nu \sim \frac{\mathcal{M}_{EW}^2}{\mathcal{M}} \end{array} \right.$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model



# Right-Handed Neutrino Mass Term

Majorana mass term for  $\nu_R$  respects the  $SU(2)_L \times U(1)_Y$  Standard Model Symmetry!

$$\mathcal{L}_R^M = -\frac{1}{2} m \left( \overline{\nu_R^c} \nu_R + \overline{\nu_R} \nu_R^c \right)$$

Majorana mass term for  $\nu_R$  breaks Lepton number conservation!

Three possibilities:

- ▶ Lepton number can be explicitly broken
- ▶ Lepton number is spontaneously broken locally, with a massive vector boson coupled to the lepton number current
- ▶ Lepton number is spontaneously broken globally and a massless Goldstone boson appears in the theory (Majoron)

# Singlet Majoron Model

[Chikashige, Mohapatra, Peccei, Phys. Lett. B98 (1981) 265, Phys. Rev. Lett. 45 (1980) 1926]

$$\begin{aligned} \mathcal{L}_\Phi &= -y_d \left( \overline{L}_L \Phi \nu_R + \overline{\nu}_R \Phi^\dagger L_L \right) \xrightarrow{\langle \Phi \rangle \neq 0} -m_D (\overline{\nu}_L \nu_R + \overline{\nu}_R \nu_L) \\ \mathcal{L}_\eta &= -y_s \left( \eta \overline{\nu}_R^c \nu_R + \eta^\dagger \overline{\nu}_R \nu_R^c \right) \xrightarrow{\langle \eta \rangle \neq 0} -\frac{1}{2} m_R \left( \overline{\nu}_R^c \nu_R + \overline{\nu}_R \nu_R^c \right) \end{aligned}$$

$$\eta = 2^{-1/2} (\langle \eta \rangle + \rho + i\chi) \quad \mathcal{L}_{\text{mass}} = -\frac{1}{2} (\overline{\nu}_L^c \ \overline{\nu}_R) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

$$\begin{array}{c} m_R \\ \text{scale of } L \text{ violation} \end{array} \gg \begin{array}{c} m_D \\ \text{EW scale} \end{array} \implies \text{See-Saw: } \boxed{m_1 \simeq \frac{m_D^2}{m_R}}$$

$\rho$  = massive scalar,  $\chi$  = Majoron (massless pseudoscalar Goldstone boson)

The Majoron is weakly coupled to the light neutrino

$$\mathcal{L}_{\chi-\nu} = \frac{iy_s}{\sqrt{2}} \chi \left[ \overline{\nu}_2 \gamma^5 \nu_2 - \frac{m_D}{m_R} \left[ \overline{\nu}_2 \gamma^5 \nu_1 + \overline{\nu}_1 \gamma^5 \nu_2 \right] + \left( \frac{m_D}{m_R} \right)^2 \overline{\nu}_1 \gamma^5 \nu_1 \right]$$

# Three-Generation Mixing

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{L}} + \mathcal{L}_{\text{mass}}^{\text{R}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = - \sum_{s=1}^{N_S} \sum_{\alpha=e,\mu,\tau} \overline{\nu'_{sR}} M_{s\alpha}^{\text{D}} \nu'_{\alpha L} + \text{H.c.}$$

$$\mathcal{L}_{\text{mass}}^{\text{L}} = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu'_{\alpha L} C^\dagger M_{\alpha\beta}^{\text{L}} \nu'_{\beta L} + \text{H.c.}$$

$$\mathcal{L}_{\text{mass}}^{\text{R}} = \frac{1}{2} \sum_{s,s'=1}^{N_S} \nu'_{sR} C^\dagger M_{ss'}^{\text{R}} \nu'_{s'R} + \text{H.c.}$$

$$\mathbf{N}'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \\ \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'^{\text{IC}}_R \equiv \begin{pmatrix} \nu'_{1R} \\ \vdots \\ \nu'_{N_S R} \end{pmatrix}$$

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \frac{1}{2} \mathbf{N}'_L{}^T C^\dagger M^{\text{D+M}} \mathbf{N}'_L + \text{H.c.} \quad M^{\text{D+M}} = \begin{pmatrix} M^{\text{L}} & M^{\text{D}^T} \\ M^{\text{D}} & M^{\text{R}} \end{pmatrix}$$

- ▶ Diagonalization of the Dirac-Majorana Mass Term  $\implies$  massive Majorana neutrinos
- ▶ See-Saw Mechanism  $\implies$  sterile right-handed neutrinos have large masses and are decoupled from the low-energy phenomenology
- ▶ At low energy we have an effective mixing of three Majorana neutrinos

# Number of Massive Neutrinos?

$Z \rightarrow \nu \bar{\nu} \Rightarrow \nu_e \nu_\mu \nu_\tau$  active flavor neutrinos

mixing  $\Rightarrow \nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL} \quad \alpha = e, \mu, \tau \quad N \geq 3$   
no upper limit!

Mass Basis:	$\nu_1$	$\nu_2$	$\nu_3$	$\nu_4$	$\nu_5$	$\dots$
Flavor Basis:	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$\nu_{S1}$	$\nu_{S2}$	$\dots$
	ACTIVE			STERILE		

## STERILE NEUTRINOS

singlets of SM  $\Rightarrow$  no interactions!

active  $\rightarrow$  sterile transitions are possible if  $\nu_4, \dots$  are light (no see-saw)



disappearance of active neutrinos

## Part II

# Neutrino Oscillations in Vacuum and in Matter

# Neutrino Oscillations in Vacuum

- Neutrino Oscillations in Vacuum
  - Ultrarelativistic Approximation
  - Easy Example of Neutrino Production
  - Neutrino Oscillations in Vacuum
  - Neutrinos and Antineutrinos
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

# Ultrarelativistic Approximation

Only neutrinos with energy  $\gtrsim 0.1 \text{ MeV}$  are detectable!

Charged-Current Processes: Threshold

$$\begin{aligned} \nu + A &\rightarrow B + C \\ &\Downarrow \\ s &= 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ &\Downarrow \\ E_{\text{th}} &= \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2} \end{aligned}$$

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^- \quad E_{\text{th}} = 0.233 \text{ MeV}$$

$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^- \quad E_{\text{th}} = 0.81 \text{ MeV}$$

$$\bar{\nu}_e + p \rightarrow n + e^+ \quad E_{\text{th}} = 1.8 \text{ MeV}$$

$$\nu_\mu + n \rightarrow p + \mu^- \quad E_{\text{th}} = 110 \text{ MeV}$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \quad E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9 \text{ GeV}$$

Elastic Scattering Processes: Cross Section  $\propto$  Energy

$$\nu + e^- \rightarrow \nu + e^- \quad \sigma(E) \sim \sigma_0 E/m_e \quad \sigma_0 \sim 10^{-44} \text{ cm}^2$$

Background  $\implies E_{\text{th}} \simeq 5 \text{ MeV}$  (SK, SNO),  $0.25 \text{ MeV}$  (Borexino)

Laboratory and Astrophysical Limits  $\implies m_\nu \lesssim 1 \text{ eV}$



# Easy Example of Neutrino Production

$$\underline{\pi^+ \rightarrow \mu^+ + \nu_\mu}$$

$$\nu_\mu = \sum_k U_{\mu k} \nu_k$$

two-body decay  $\implies$  fixed kinematics

$$E_k^2 = p_k^2 + m_k^2$$

$$\pi \text{ at rest: } \begin{cases} p_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4 m_\pi^2} \\ E_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4 m_\pi^2} \end{cases}$$

$$0^{\text{th}} \text{ order: } m_k = 0 \implies p_k = E_k = E = \frac{m_\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \text{ MeV}$$

$$1^{\text{st}} \text{ order: } E_k \simeq E + \xi \frac{m_k^2}{2E}$$

$$p_k \simeq E - (1 - \xi) \frac{m_k^2}{2E}$$

$$\xi = \frac{1}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

# Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

[Bilenky, Pontecorvo, Nuovo Cim. Lett. 17 (1976) 569] [Bilenky, Pontecorvo, Phys. Rep. 41 (1978) 225]

Flavor Neutrino Production:  $j_{W,L}^\rho = 2 \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma^\rho \ell_{\alpha L}$

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}$$

Fields  $\bar{\nu}_{\alpha L} = \sum_k U_{\alpha k}^* \bar{\nu}_{kL} \Rightarrow |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$  States

$$|\nu_k(t, x)\rangle = e^{-iE_k t + ip_k x} |\nu_k\rangle \Rightarrow |\nu_\alpha(t, x)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \Rightarrow |\nu_\alpha(t, x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left( \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, x)} |\nu_\beta\rangle$$

Transition Probability

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t, x) = |\langle \nu_\beta | \nu_\alpha(t, x) \rangle|^2 = |\mathcal{A}_{\nu_\alpha \rightarrow \nu_\beta}(t, x)|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos  $\implies t \simeq x = L$  source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* e^{-im_k^2 L/2E} U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L/E) &= \delta_{\alpha\beta} - 4 \sum_{k>j} \text{Re} \left[ U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left( \frac{\Delta m_{kj}^2 L}{4E} \right) \\ &\quad + 2 \sum_{k>j} \text{Im} \left[ U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \sin \left( \frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$

# Neutrinos and Antineutrinos

Antineutrinos are described by CP-conjugated fields:

$$\nu^{\text{CP}} = \gamma^0 C \bar{\nu}^T = -C \nu^*$$

C  $\implies$  Particle  $\iff$  Antiparticle

P  $\implies$  Left-Handed  $\iff$  Right-Handed

Fields:  $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$

States:  $|\nu_{\alpha}\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_{\alpha}\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS     $U \iff U^*$     ANTINEUTRINOS

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

# CPT, CP and T Symmetries

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
  - CPT Symmetry
  - CP Symmetry
  - T Symmetry
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

# CPT Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$\text{CPT Asymmetries: } A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$\text{Local Quantum Field Theory} \implies A_{\alpha\beta}^{\text{CPT}} = 0 \quad \text{CPT Symmetry}$$

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$\text{is invariant under CPT: } U \leftrightarrow U^* \quad \alpha \leftrightarrow \beta$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$$

$$P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha}$$

(solar  $\nu_e$ , reactor  $\bar{\nu}_e$ , accelerator  $\nu_\mu$ )

# CP Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{\text{CP}} P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$$

CP Asymmetries:  $A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$  CPT  $\Rightarrow A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}$

$$A_{\alpha\beta}^{\text{CP}}(L, E) = 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) - 2\text{Re} \sum_{k>j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$A_{\alpha\beta}^{\text{CP}}(L, E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

Jarlskog invariants:

$$J_{\alpha\beta;kj} = \text{Im} \left[ U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right]$$

violation of CP symmetry depends only on Dirac phases  
(three neutrinos:  $J_{\alpha\beta;kj} = \pm c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13}$ )

$\langle A_{\alpha\beta}^{\text{CP}} \rangle = 0 \Rightarrow$  observation of CP violation needs measurement of oscillations

# T Symmetry

$$P_{\nu_\alpha \rightarrow \nu_\beta} \xrightarrow{T} P_{\nu_\beta \rightarrow \nu_\alpha}$$

T Asymmetries:  $A_{\alpha\beta}^T = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\nu_\beta \rightarrow \nu_\alpha}$

CPT  $\implies 0 = A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$   
 $= P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\nu_\beta \rightarrow \nu_\alpha} + P_{\nu_\beta \rightarrow \nu_\alpha} - P_{\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha}$   
 $= A_{\alpha\beta}^T + A_{\beta\alpha}^{\text{CP}} = A_{\alpha\beta}^T - A_{\alpha\beta}^{\text{CP}} \implies A_{\alpha\beta}^T = A_{\alpha\beta}^{\text{CP}}$

$$A_{\alpha\beta}^T(L, E) = 4 \sum_{k>j} J_{\alpha\beta;kj} \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

violation of T symmetry depends only on Dirac phases

$\langle A_{\alpha\beta}^T \rangle = 0 \implies$  observation of T violation needs measurement of oscillations

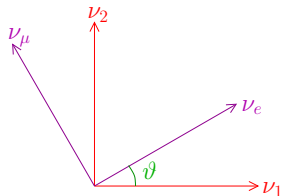


# Two-Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
  - Two-Neutrino Mixing and Oscillations
  - Types of Experiments
  - Average over Energy Resolution of the Detector
  - Anatomy of Exclusion Plots
- Neutrino Oscillations in Matter

# Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^2 U_{\alpha k} |\nu_k\rangle \quad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

$$\begin{aligned} |\nu_e\rangle &= \cos \vartheta |\nu_1\rangle + \sin \vartheta |\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin \vartheta |\nu_1\rangle + \cos \vartheta |\nu_2\rangle \end{aligned}$$

$$\Delta m^2 \equiv \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

Transition Probability:  $P_{\nu_e \rightarrow \nu_\mu} = P_{\nu_\mu \rightarrow \nu_e} = \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$

Survival Probabilities:  $P_{\nu_e \rightarrow \nu_e} = P_{\nu_\mu \rightarrow \nu_\mu} = 1 - P_{\nu_e \rightarrow \nu_\mu}$

## two-neutrino mixing transition probability

$$\alpha \neq \beta$$

$$\alpha, \beta = e, \mu, \tau$$

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) &= \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \\ &= \sin^2 2\vartheta \sin^2 \left( 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{m}]}{E [\text{MeV}]} \right) \\ &= \sin^2 2\vartheta \sin^2 \left( 1.27 \frac{\Delta m^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]} \right) \end{aligned}$$

## oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \frac{E [\text{MeV}]}{\Delta m^2 [\text{eV}^2]} \text{ m} = 2.47 \frac{E [\text{GeV}]}{\Delta m^2 [\text{eV}^2]} \text{ km}$$

# Types of Experiments

Two-Neutrino  
Mixing

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

observable if  
 $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 10 \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1 \text{ eV}^2$

Reactor:  $L \sim 10 \text{ m}$ ,  $E \sim 1 \text{ MeV}$

Accelerator:  $L \sim 1 \text{ km}$ ,  $E \gtrsim 0.1 \text{ GeV}$

ATM & LBL

$L/E \lesssim 10^4 \text{ eV}^{-2}$

Reactor:  $L \sim 1 \text{ km}$ ,  $E \sim 1 \text{ MeV}$  CHOOZ, PALO VERDE

Accelerator:  $L \sim 10^3 \text{ km}$ ,  $E \gtrsim 1 \text{ GeV}$  K2K, MINOS, CNGS

Atmospheric:  $L \sim 10^2 - 10^4 \text{ km}$ ,  $E \sim 0.1 - 10^2 \text{ GeV}$

$\Delta m^2 \gtrsim 10^{-4} \text{ eV}^2$  Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$L \sim 10^8 \text{ km}$ ,  $E \sim 0.1 - 10 \text{ MeV}$

$\frac{L}{E} \sim 10^{11} \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11} \text{ eV}^2$

Homestake, Kamiokande, GALLEX, SAGE,

Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW)  $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1$ ,  $10^{-8} \text{ eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4} \text{ eV}^2$

VLBL

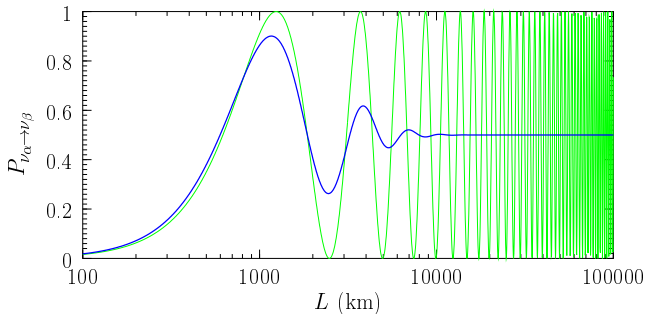
$L/E \lesssim 10^5 \text{ eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5} \text{ eV}^2$

Reactor:  $L \sim 10^2 \text{ km}$ ,  $E \sim 1 \text{ MeV}$

KamLAND

# Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) = \frac{1}{2} \sin^2 2\vartheta \left[ 1 - \cos \left( \frac{\Delta m^2 L}{2E} \right) \right]$$

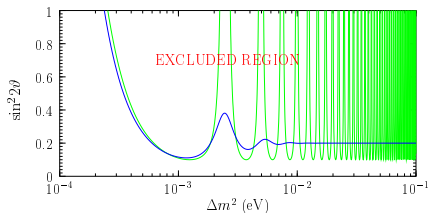


$$\Delta m^2 = 10^{-3} \text{ eV}^2 \quad \sin^2 2\vartheta = 1 \quad \langle E \rangle = 1 \text{ GeV} \quad \Delta E = 0.2 \text{ GeV}$$

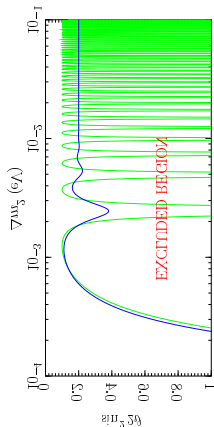
$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[ 1 - \int \cos \left( \frac{\Delta m^2 L}{2E} \right) \phi(E) dE \right] \quad (\alpha \neq \beta)$$

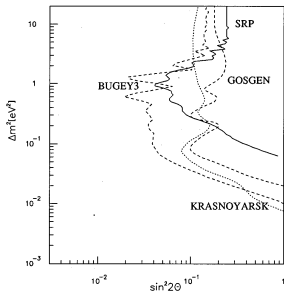
$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[ 1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) dE \right] \quad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) \rangle \leq P_{\nu_\alpha \rightarrow \nu_\beta}^{\max} \implies \sin^2 2\vartheta \leq \frac{2 P_{\nu_\alpha \rightarrow \nu_\beta}^{\max}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) dE}$$

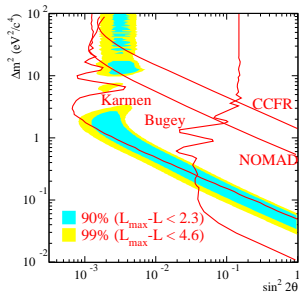


rotate  
and  
mirror

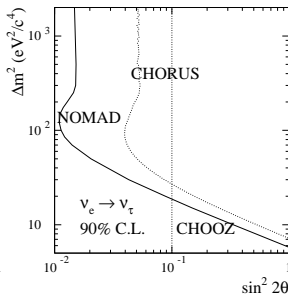
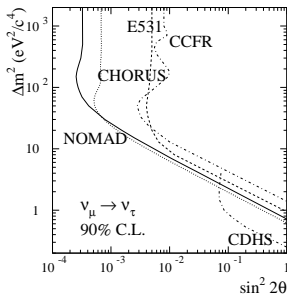




Reactor SBL Experiments:  $\bar{\nu}_e \rightarrow \bar{\nu}_e$

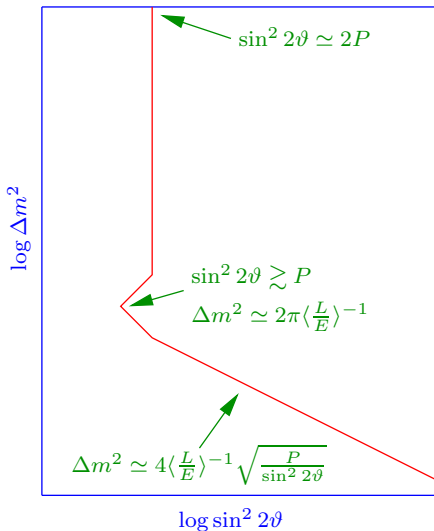


Accelerator SBL Experiments:  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$



Accelerator SBL Experiments:  $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$  and  $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$

# Anatomy of Exclusion Plots



▶  $\Delta m^2 \gg \langle L/E \rangle^{-1}$

$$P \simeq \frac{1}{2} \sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P$$

▶  $\text{Min} \left\langle \cos \left( \frac{\Delta m^2 L}{2E} \right) \right\rangle \geq -1$

$$\sin^2 2\vartheta = \frac{2P}{1 - \text{Min} \left\langle \cos \left( \frac{\Delta m^2 L}{2E} \right) \right\rangle} \geq P$$

$$\Delta m^2 \simeq 2\pi \langle L/E \rangle^{-1}$$

▶  $\Delta m^2 \ll 2\pi \langle L/E \rangle^{-1}$

$$\cos \left( \frac{\Delta m^2 L}{2E} \right) \simeq 1 - \frac{1}{2} \left( \frac{\Delta m^2 L}{2E} \right)^2$$

$$\Delta m^2 \simeq 4 \left\langle \frac{L}{E} \right\rangle^{-1} \sqrt{\frac{P}{\sin^2 2\vartheta}}$$



# Neutrino Oscillations in Matter

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter
  - Matter Effects
  - Effective Potentials in Matter
  - Evolution of Neutrino Flavors in Matter
  - Constant Matter Density
  - MSW Effect (Resonant Transitions in Matter)
  - Averaged Survival Probability
  - Crossing Probability
  - Solar Neutrinos
  - Electron Neutrino Regeneration in the Earth
  - Phenomenology of Solar Neutrinos

# Matter Effects

a flavor neutrino  $\nu_\alpha$  with momentum  $p$  is described by

$$|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* |\nu_k(p)\rangle$$

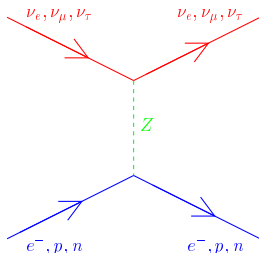
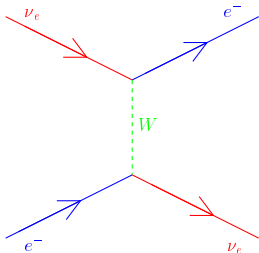
$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \quad E_k = \sqrt{p^2 + m_k^2}$$

in matter  $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \quad \mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$

$V_\alpha$  = effective potential due to coherent interactions with the medium

forward elastic CC and NC scattering

# Effective Potentials in Matter



$$V_{CC} = \sqrt{2} G_F N_e$$

$$V_{NC}^{(e^-)} = -V_{NC}^{(p)} \Rightarrow$$

$$V_{NC} = V_{NC}^{(n)} = -\frac{\sqrt{2}}{2} G_F N_n$$

$$V_e = V_{CC} + V_{NC} \quad V_\mu = V_\tau = V_{NC} \quad (\text{common phase})$$

$$V_e - V_\mu = V_{CC}$$

$$\text{antineutrinos:} \quad \bar{V}_{CC} = -V_{CC} \quad \bar{V}_{NC} = -V_{NC}$$

# Evolution of Neutrino Flavors in Matter

Schrödinger picture:  $i \frac{d}{dt} |\nu_\alpha(p, t)\rangle = \mathcal{H} |\nu_\alpha(p, t)\rangle, \quad |\nu_\alpha(p, 0)\rangle = |\nu_\alpha(p)\rangle$

flavor transition amplitudes:  $\varphi_{\alpha\beta}(p, t) = \langle \nu_\beta(p) | \nu_\alpha(p, t) \rangle, \quad \varphi_{\alpha\beta}(p, 0) = \delta_{\alpha\beta}$

$$i \frac{d}{dt} \varphi_{\alpha\beta}(p, t) = \langle \nu_\beta(p) | \mathcal{H} | \nu_\alpha(p, t) \rangle = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\alpha(p, t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu_\alpha(p, t) \rangle$$

$$\begin{aligned} \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\alpha(p, t) \rangle &= \sum_\rho \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\rho(p) \rangle \underbrace{\langle \nu_\rho(p) | \nu_\alpha(p, t) \rangle}_{\varphi_{\alpha\rho}(p, t)} \\ &= \sum_\rho \sum_{kj} U_{\beta k} \underbrace{\langle \nu_k(p) | \mathcal{H}_0 | \nu_j(p) \rangle}_{\delta_{kj} E_k} U_{\rho j}^* \varphi_{\alpha\rho}(p, t) \end{aligned}$$

$$\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\alpha(p, t) \rangle = \sum_\rho \underbrace{\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\rho(p) \rangle}_{\delta_{\beta\rho} V_\beta} \varphi_{\alpha\rho}(p, t) = V_\beta \varphi_{\alpha\beta}(p, t)$$

$$i \frac{d}{dt} \varphi_{\alpha\beta} = \sum_\rho \left( \sum_k U_{\beta k} E_k U_{\rho k}^* + \delta_{\beta\rho} V_\beta \right) \varphi_{\alpha\rho}$$

ultrarelativistic neutrinos:  $E_k = p + \frac{m_k^2}{2E}$      $E = p$      $t = x$

$$V_e = V_{CC} + V_{NC}$$

$$V_\mu = V_\tau = V_{NC}$$

$$i \frac{d}{dx} \varphi_{\alpha\beta}(p, x) = (p + V_{NC}) \varphi_{\alpha\beta}(p, x) + \sum_\rho \left( \sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{CC} \right) \varphi_{\alpha\rho}(p, x)$$

$$\psi_{\alpha\beta}(p, x) = \varphi_{\alpha\beta}(p, x) e^{ipx + i \int_0^x V_{NC}(x') dx'}$$



$$i \frac{d}{dx} \psi_{\alpha\beta} = e^{ipx + i \int_0^x V_{NC}(x') dx'} \left( -p - V_{NC} + i \frac{d}{dx} \right) \varphi_{\alpha\beta}$$

$$i \frac{d}{dx} \psi_{\alpha\beta} = \sum_\rho \left( \sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \delta_{\rho e} V_{CC} \right) \psi_{\alpha\rho}$$

$$P_{\nu_\alpha \rightarrow \nu_\beta} = |\varphi_{\alpha\beta}|^2 = |\psi_{\alpha\beta}|^2$$

evolution of flavor transition amplitudes in matrix form

$$i \frac{d}{dx} \Psi_\alpha = \frac{1}{2E} (U M^2 U^\dagger + A) \Psi_\alpha$$

$$\Psi_\alpha = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha \mu} \\ \psi_{\alpha \tau} \end{pmatrix} \quad M^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \quad A = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{aligned} A_{CC} &= 2EV_{CC} \\ &= 2\sqrt{2}EG_F N_e \end{aligned}$$

effective  
mass-squared  
matrix  
in vacuum

$$M_{\text{VAC}}^2 = U M^2 U^\dagger \xrightarrow{\text{matter}} U M^2 U^\dagger + 2E V = M_{\text{MAT}}^2$$

potential due to coherent  
forward elastic scattering

effective  
mass-squared  
matrix  
in matter

simplest case: two-neutrino mixing

$$\nu_e \rightarrow \nu_\mu \text{ transitions with } U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\begin{aligned} U M^2 U^\dagger &= \begin{pmatrix} \cos^2\vartheta m_1^2 + \sin^2\vartheta m_2^2 & \cos\vartheta \sin\vartheta (m_2^2 - m_1^2) \\ \cos\vartheta \sin\vartheta (m_2^2 - m_1^2) & \sin^2\vartheta m_1^2 + \cos^2\vartheta m_2^2 \end{pmatrix} \\ &= \frac{1}{2} \Sigma m^2 + \frac{1}{2} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \end{aligned}$$

↑

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2$$

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix}$$

$$\text{initial } \nu_e \implies \begin{pmatrix} \psi_{ee}(0) \\ \psi_{e\mu}(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P_{\nu_e \rightarrow \nu_\mu}(x) = |\psi_{e\mu}(x)|^2$$
$$P_{\nu_e \rightarrow \nu_e}(x) = |\psi_{ee}(x)|^2 = 1 - P_{\nu_e \rightarrow \nu_\mu}(x)$$



## Constant Matter Density

$$i \frac{d}{dx} \begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{CC} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix}$$

$$\frac{dA_{CC}}{dx} = 0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \begin{pmatrix} \cos \vartheta_M & \sin \vartheta_M \\ -\sin \vartheta_M & \cos \vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[ \frac{A_{CC}}{4E} + \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

↑  
irrelevant common phase

## Effective Mixing Angle in Matter

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

## Effective Squared-Mass Difference

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Resonance ( $\vartheta_M = \pi/4$ )

$$A_{CC}^R = \Delta m^2 \cos 2\vartheta \quad \Rightarrow \quad N_e^R = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2}EG_F}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_M & \sin\vartheta_M \\ -\sin\vartheta_M & \cos\vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\vartheta_M & -\sin\vartheta_M \\ \sin\vartheta_M & \cos\vartheta_M \end{pmatrix} \begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix}$$

$$\nu_e \rightarrow \nu_\mu \Rightarrow \begin{pmatrix} \psi_{ee}(0) \\ \psi_{e\mu}(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} \begin{pmatrix} \cos\vartheta_M \\ \sin\vartheta_M \end{pmatrix}$$

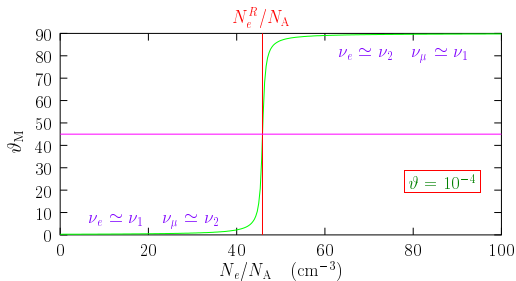
$$\psi_1(x) = \cos\vartheta_M \exp\left(i \frac{\Delta m_M^2 x}{4E}\right)$$

$$\psi_2(x) = \sin\vartheta_M \exp\left(-i \frac{\Delta m_M^2 x}{4E}\right)$$

$$P_{\nu_e \rightarrow \nu_\mu}(x) = |\psi_{e\mu}(x)|^2 = |-\sin\vartheta_M \psi_1(x) + \cos\vartheta_M \psi_2(x)|^2$$

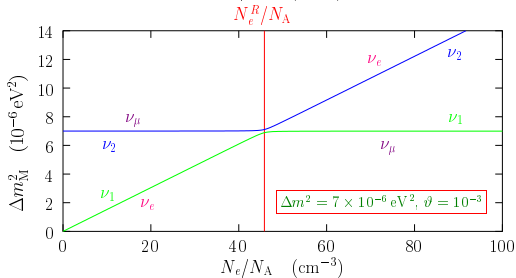
$$P_{\nu_e \rightarrow \nu_\mu}(x) = \sin^2 2\vartheta_M \sin^2\left(\frac{\Delta m_M^2 x}{4E}\right)$$

# MSW Effect (Resonant Transitions in Matter)



$$\begin{aligned}\nu_e &= \cos\vartheta_M \nu_1 + \sin\vartheta_M \nu_2 \\ \nu_\mu &= -\sin\vartheta_M \nu_1 + \cos\vartheta_M \nu_2\end{aligned}$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \frac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$



$$\Delta m_M^2 = \left[ (\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2 \right]^{1/2}$$

$$\begin{pmatrix} \psi_{ee} \\ \psi_{e\mu} \end{pmatrix} = \begin{pmatrix} \cos\vartheta_M & \sin\vartheta_M \\ -\sin\vartheta_M & \cos\vartheta_M \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i \frac{d}{dx} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[ \underbrace{\frac{A_{CC}}{4E}}_{\text{irrelevant common phase}} + \frac{1}{4E} \begin{pmatrix} -\Delta m_M^2 & 0 \\ 0 & \Delta m_M^2 \end{pmatrix} + \begin{pmatrix} 0 & -i \frac{d\vartheta_M}{dx} \\ i \frac{d\vartheta_M}{dx} & 0 \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

irrelevant common phase

maximum near resonance

$$\begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix} = \begin{pmatrix} \cos\vartheta_M^0 & -\sin\vartheta_M^0 \\ \sin\vartheta_M^0 & \cos\vartheta_M^0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\vartheta_M^0 \\ \sin\vartheta_M^0 \end{pmatrix}$$

$$\psi_1(x) \simeq \left[ \cos\vartheta_M^0 \exp\left(i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx'\right) \mathcal{A}_{11}^R + \sin\vartheta_M^0 \exp\left(-i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx'\right) \mathcal{A}_{21}^R \right] \\ \times \exp\left(i \int_{x_R}^x \frac{\Delta m_M^2(x')}{4E} dx'\right)$$

$$\psi_2(x) \simeq \left[ \cos\vartheta_M^0 \exp\left(i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx'\right) \mathcal{A}_{12}^R + \sin\vartheta_M^0 \exp\left(-i \int_0^{x_R} \frac{\Delta m_M^2(x')}{4E} dx'\right) \mathcal{A}_{22}^R \right] \\ \times \exp\left(-i \int_{x_R}^x \frac{\Delta m_M^2(x')}{4E} dx'\right)$$

# Averaged Survival Probability

$$\psi_{ee}(x) = \cos\vartheta_M^x \psi_1(x) + \sin\vartheta_M^x \psi_2(x)$$

neglect interference (averaged over energy spectrum)

$$\begin{aligned}\overline{P}_{\nu_e \rightarrow \nu_e}(x) = |\langle \psi_{ee}(x) \rangle|^2 &= \cos^2\vartheta_M^x \cos^2\vartheta_M^0 |\mathcal{A}_{11}^R|^2 + \cos^2\vartheta_M^x \sin^2\vartheta_M^0 |\mathcal{A}_{21}^R|^2 \\ &+ \sin^2\vartheta_M^x \cos^2\vartheta_M^0 |\mathcal{A}_{12}^R|^2 + \sin^2\vartheta_M^x \sin^2\vartheta_M^0 |\mathcal{A}_{22}^R|^2\end{aligned}$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^R|^2 = |\mathcal{A}_{21}^R|^2 = P_c \qquad |\mathcal{A}_{11}^R|^2 = |\mathcal{A}_{22}^R|^2 = 1 - P_c$$

$P_c \equiv$  crossing probability

$$\overline{P}_{\nu_e \rightarrow \nu_e}(x) = \frac{1}{2} + \left( \frac{1}{2} - P_c \right) \cos 2\vartheta_M^0 \cos 2\vartheta_M^x$$

[Parke, PRL 57 (1986) 1275]

# Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: 
$$\gamma = \frac{\Delta m_M^2/2E}{2|d\vartheta_M/dx|} \Big|_R = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta} \Big| \frac{d \ln A_{cc}}{dx} \Big|_R$$

$A \propto x$        $F = 1$  (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$        $F = (1 - \tan^2 \vartheta)^2 / (1 + \tan^2 \vartheta)$  [Kuo, Pantaleone, PRD 39 (1989) 1930]

[Pizzochero, PRD 36 (1987) 2293]

$A \propto \exp(-x)$        $F = 1 - \tan^2 \vartheta$  [Toshev, PLB 196 (1987) 170]

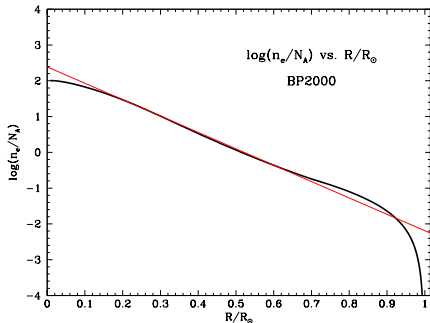
[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

# Solar Neutrinos

SUN:  $N_e(x) \simeq N_e^c \exp\left(-\frac{x}{x_0}\right)$

$$N_e^c = 245 N_A / \text{cm}^3 \quad x_0 = \frac{R_\odot}{10.54}$$



$$\overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left| \frac{d \ln A_{cc}}{dx} \right|_R}$$

$$F = 1 - \tan^2 \vartheta$$

$$A_{cc} = 2\sqrt{2}EG_F N_e$$

practical prescription:

[Lisi et al., PRD 63 (2001) 093002]

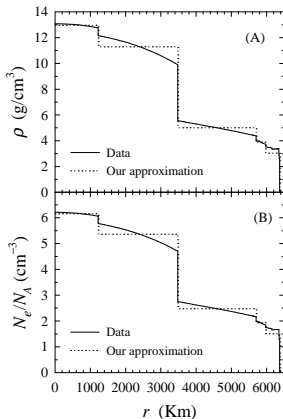
$$\left\{ \begin{array}{ll} \text{numerical } \left| \frac{d \ln A_{cc}}{dx} \right|_R & \text{for } x \leq 0.904 R_\odot \\ \left| \frac{d \ln A_{cc}}{dx} \right|_R \rightarrow \frac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{array} \right.$$



# Electron Neutrino Regeneration in the Earth

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun+earth}} = \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} + \frac{\left(1 - 2\overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}}\right) \left(P_{\nu_2 \rightarrow \nu_e}^{\text{earth}} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



$P_{\nu_2 \rightarrow \nu_e}^{\text{earth}}$  is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

Wave functions of flavor neutrinos are joined at the boundaries of steps.

[Giunti, Kim, Monteno, NP B 521 (1998) 3]

# Phenomenology of Solar Neutrinos

LMA (Large Mixing Angle):

LOW (LOW  $\Delta m^2$ ):

SMA (Small Mixing Angle):

QVO (Quasi-Vacuum Oscillations):

VAC (VACuum oscillations):

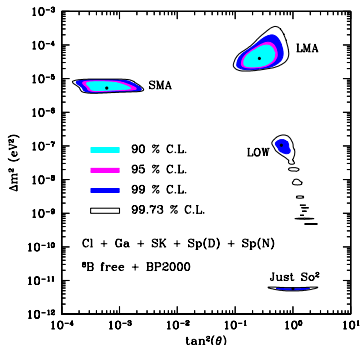
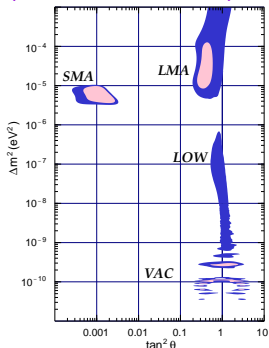
$$\Delta m^2 \sim 5 \times 10^{-5} \text{ eV}^2, \quad \tan^2 \vartheta \sim 0.8$$

$$\Delta m^2 \sim 7 \times 10^{-8} \text{ eV}^2, \quad \tan^2 \vartheta \sim 0.6$$

$$\Delta m^2 \sim 5 \times 10^{-6} \text{ eV}^2, \quad \tan^2 \vartheta \sim 10^{-3}$$

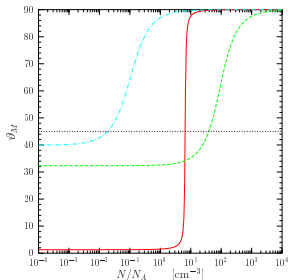
$$\Delta m^2 \sim 10^{-9} \text{ eV}^2, \quad \tan^2 \vartheta \sim 1$$

$$\Delta m^2 \lesssim 5 \times 10^{-10} \text{ eV}^2, \quad \tan^2 \vartheta \sim 1$$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]

[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



**solid line:**  
(typical SMA)

$$\Delta m^2 = 5 \times 10^{-6} \text{ eV}^2$$

$$\tan^2 \vartheta = 5 \times 10^{-4}$$

**dashed line:**  
(typical LMA)

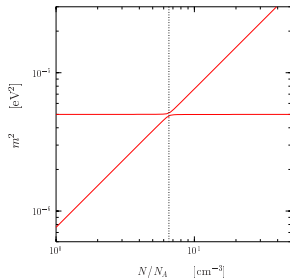
$$\Delta m^2 = 7 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta = 0.4$$

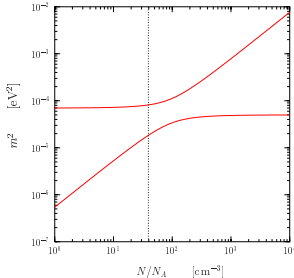
**dash-dotted line:**  
(typical LOW)

$$\Delta m^2 = 8 \times 10^{-8} \text{ eV}^2$$

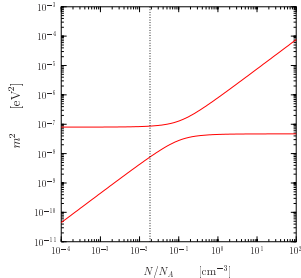
$$\tan^2 \vartheta = 0.7$$



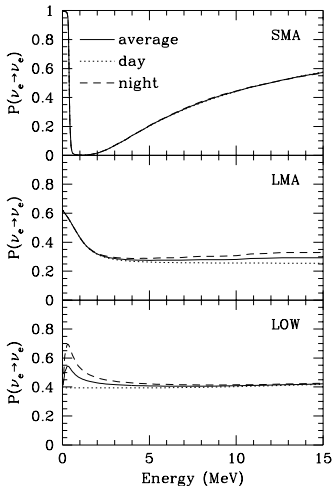
typical SMA



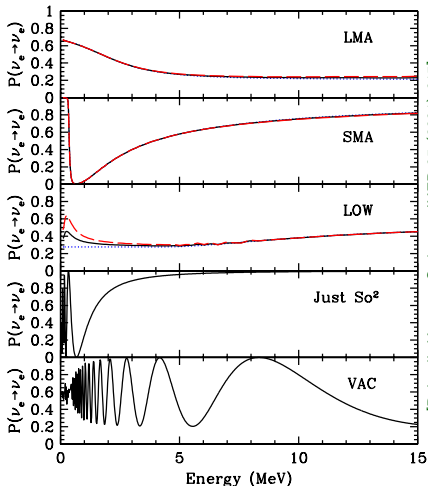
typical LMA



typical LOW



SMA:  $\Delta m^2 = 5.0 \times 10^{-6} \text{ eV}^2$   $\sin^2 2\theta = 3.5 \times 10^{-3}$   
 LMA:  $\Delta m^2 = 1.6 \times 10^{-5} \text{ eV}^2$   $\sin^2 2\theta = 0.57$   
 LOW:  $\Delta m^2 = 7.9 \times 10^{-8} \text{ eV}^2$   $\sin^2 2\theta = 0.95$



LMA:  $\Delta m^2 = 4.2 \times 10^{-5} \text{ eV}^2$   $\tan^2 \theta = 0.26$   
 SMA:  $\Delta m^2 = 5.2 \times 10^{-6} \text{ eV}^2$   $\tan^2 \theta = 5.5 \times 10^{-4}$   
 LOW:  $\Delta m^2 = 7.6 \times 10^{-8} \text{ eV}^2$   $\tan^2 \theta = 0.72$   
 Just  $\text{So}^2$ :  $\Delta m^2 = 5.5 \times 10^{-12} \text{ eV}^2$   $\tan^2 \theta = 1.0$   
 VAC:  $\Delta m^2 = 1.4 \times 10^{-10} \text{ eV}^2$   $\tan^2 \theta = 0.38$

# In Neutrino Oscillations Dirac = Majorana

Evolution of Amplitudes: 
$$\frac{d\nu_\alpha}{dt} = \frac{1}{2E} \sum_\beta (UM^2U^\dagger + 2EV)_{\alpha\beta} \nu_\beta$$

difference: 
$$\left\{ \begin{array}{ll} \text{Dirac:} & U^{(D)} \\ \text{Majorana:} & U^{(M)} = U^{(D)} D(\lambda) \end{array} \right.$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & e^{i\lambda_{21}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{i\lambda_{N1}} \end{pmatrix} \Rightarrow D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \dots & 0 \\ 0 & m_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_N^2 \end{pmatrix} \Rightarrow DM^2 = M^2D \Rightarrow DM^2D^\dagger = M^2$$

$$U^{(M)} M^2 (U^{(M)})^\dagger = U^{(D)} D M^2 D^\dagger (U^{(D)})^\dagger = U^{(D)} M^2 (U^{(D)})^\dagger$$

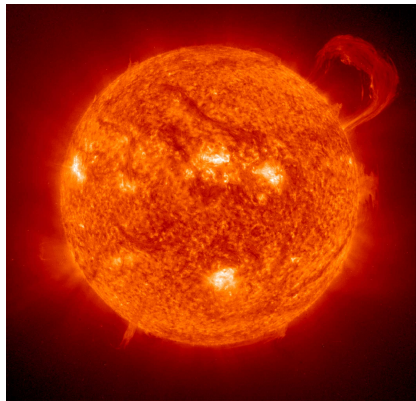
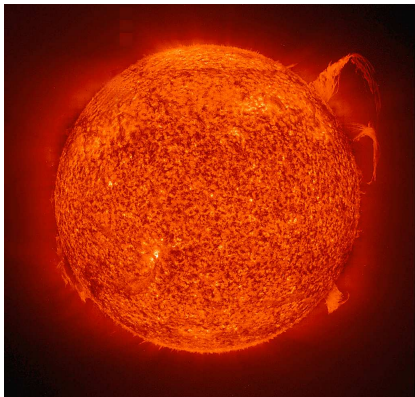
## Part III

# Experimental Results and Theoretical Implications

# Solar Neutrinos and KamLAND

- Solar Neutrinos and KamLAND
  - The Sun
  - Standard Solar Model (SSM)
  - Homestake
  - Gallium Experiments
    - SAGE: Soviet-American Gallium Experiment
    - GALLEX: GALLium EXperiment
  - GNO: Gallium Neutrino Observatory
  - Kamiokande
  - Super-Kamiokande
  - SNO: Sudbury Neutrino Observatory
  - KamLAND
  - Combined Fit of Solar and Reactor Neutrino Data
  - Sterile Neutrinos in Solar Neutrino Flux?
  - Determination of Solar Neutrino Fluxes
  - Details of Solar Neutrino Oscillations
  - Future

# The Sun

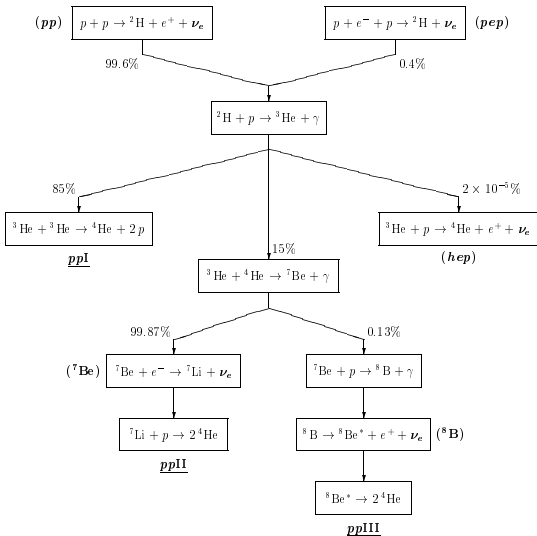


Extreme ultraviolet Imaging Telescope (EIT) 304 Å images of the Sun  
emission in this spectral line (He II) shows the upper chromosphere  
at a temperature of about 60,000 K

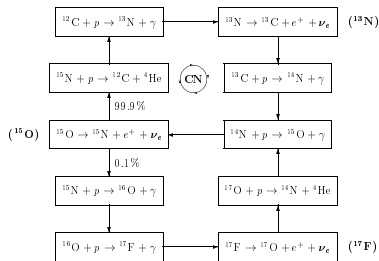
[The Solar and Heliospheric Observatory (SOHO), <http://sohowww.nascom.nasa.gov/>]



# Standard Solar Model (SSM)

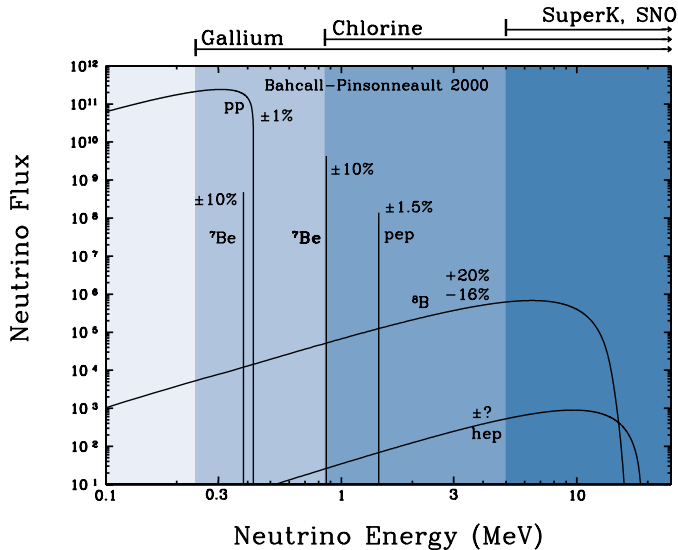


## pp chain and CNO cycle

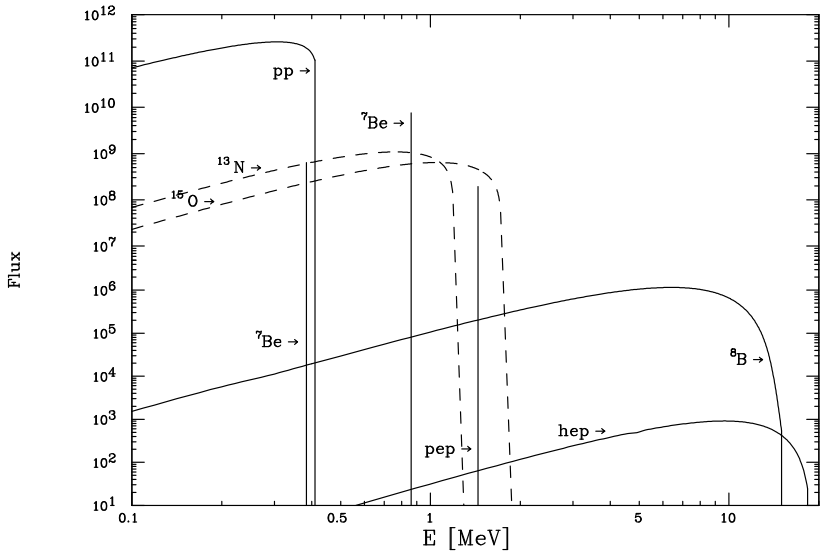


## Bahcall SSMs

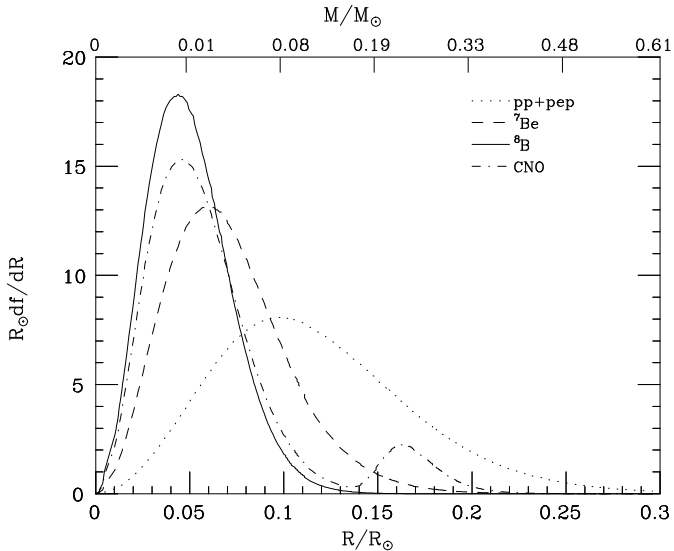
[J.N. Bahcall, <http://www.sns.ias.edu/~jnb>]



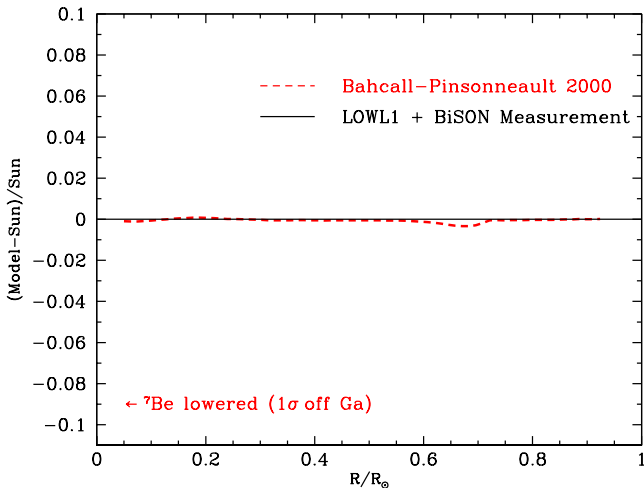
[J.N. Bahcall, <http://www.sns.ias.edu/~jnb>]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[Castellani, Degl'Innocenti, Fiorentini, Lissia, Ricci, Phys. Rept. 281 (1997) 309, astro-ph/9606180]



[J.N. Bahcall, <http://www.sns.ias.edu/~jnb>]

predicted versus measured sound speed

the rms fractional difference between the calculated and the measured sound speeds is 0.10% for all solar radii between between  $0.05 R_{\odot}$  and  $0.95 R_{\odot}$  and is 0.08% for the deep interior region,  $r < 0.25 R_{\odot}$ , in which neutrinos are produced

# Homestake

$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$  [Pontecorvo (1946), Alvarez (1949)] radiochemical experiment

Homestake Gold Mine (South Dakota)

1478 m deep, 4200 m.w.e.  $\Rightarrow \Phi_\mu \simeq 4 \text{ m}^{-2} \text{ day}^{-1}$

steel tank, 6.1 m diameter, 14.6 m long ( $6 \times 10^5$  liters)

615 tons of tetrachloroethylene ( $\text{C}_2\text{Cl}_4$ ),  $2.16 \times 10^{30}$  atoms of  ${}^{37}\text{Cl}$  (133 tons)

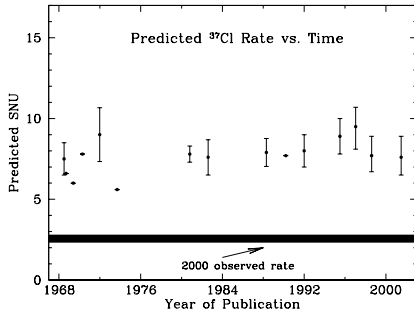
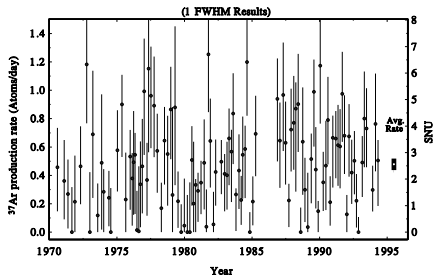
energy threshold:  $E_{\text{th}}^{\text{Cl}} = 0.814 \text{ MeV} \Rightarrow {}^8\text{B}, {}^7\text{Be}, \text{pep}, \text{hep}, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F}$

1970–1994, 108 extractions  $\Rightarrow \frac{R_{\text{Cl}}^{\text{exp}}}{R_{\text{Cl}}^{\text{SSM}}} = 0.34 \pm 0.03$  [APJ 496 (1998) 505]

$$R_{\text{Cl}}^{\text{exp}} = 2.56 \pm 0.23 \text{ SNU}$$

$$R_{\text{Cl}}^{\text{SSM}} = 7.6_{-1.1}^{+1.3} \text{ SNU}$$

$$1 \text{ SNU} = 10^{-36} \text{ events atom}^{-1} \text{ s}^{-1}$$



# Gallium Experiments

## SAGE, GALLEX, GNO

radiochemical experiments



threshold:  $E_{\text{th}}^{\text{Ga}} = 0.233 \text{ MeV} \implies pp, {}^7\text{Be}, {}^8\text{B}, pep, hep, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F}$

$$\text{SAGE+GALLEX+GNO} \implies \frac{R_{\text{Ga}}^{\text{exp}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.56 \pm 0.03$$

$$R_{\text{Ga}}^{\text{exp}} = 72.4 \pm 4.7 \text{ SNU}$$

$$R_{\text{Ga}}^{\text{SSM}} = 128_{-7}^{+9} \text{ SNU}$$

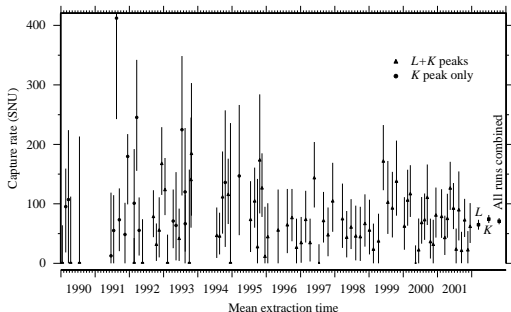
# SAGE: Soviet-American Gallium Experiment

Baksan Neutrino Observatory, northern Caucasus

50 tons of metallic  $^{71}\text{Ga}$ , 2000 m deep, 4700 m.w.e.  $\Rightarrow \Phi_{\mu} \simeq 2.6 \text{ m}^{-2} \text{ day}^{-1}$

detector test:  $^{51}\text{Cr}$  Source:  $R = 0.95_{-0.10}^{+0.11}_{-0.05}$  [PRC 59 (1999) 2246]

1990 – 2001  $\Rightarrow \frac{R_{\text{Ga}}^{\text{SAGE}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.54 \pm 0.05$  [astro-ph/0204245]





# GALLEX: GALLium EXperiment

Gran Sasso Underground Laboratory, Italy, overhead shielding: 3300 m.w.e.

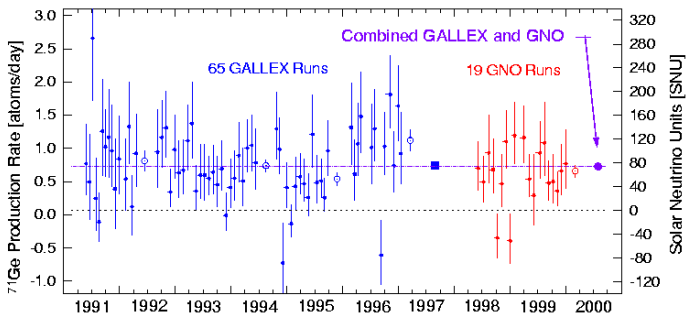
30.3 tons of gallium in 101 tons of gallium chloride ( $\text{GaCl}_3\text{-HCl}$ ) solution

May 1991 – Jan 1997  $\implies \frac{R_{\text{Ga}}^{\text{GALLEX}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.61 \pm 0.06$  [PLB 477 (1999) 127]

# GNO: Gallium Neutrino Observatory

continuation of GALLEX: 30.3 tons of gallium

May 1998 – Jan 2000  $\Rightarrow$   $\frac{R_{\text{Ga}}^{\text{GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.51 \pm 0.08$  [PLB 490 (2000) 16]



$$\frac{R_{\text{Ga}}^{\text{GALLEX+GNO}}}{R_{\text{Ga}}^{\text{SSM}}} = 0.58 \pm 0.05$$

# Kamiokande

water Cherenkov detector  $\nu + e^- \rightarrow \nu + e^-$

Sensitive to  $\nu_e, \nu_\mu, \nu_\tau$ , but  $\sigma(\nu_e) \simeq 6 \sigma(\nu_{\mu,\tau})$

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

3000 tons of water, 680 tons fiducial volume, 948 PMTs

threshold:  $E_{\text{th}}^{\text{Kam}} \simeq 6.75 \text{ MeV} \implies {}^8\text{B}, \text{hep}$

Jan 1987 – Feb 1995 (2079 days)

$$\frac{R_{\nu_e}^{\text{Kam}}}{R_{\nu_e}^{\text{SSM}}} = 0.55 \pm 0.08 \quad [\text{PRL } 77 \text{ (1996) } 1683]$$

# Super-Kamiokande

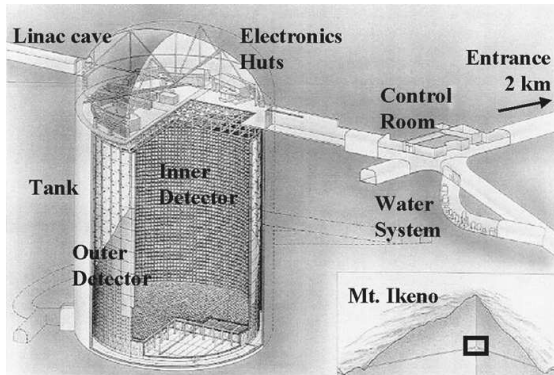
continuation of Kamiokande

50 ktons of water, 22.5 ktons fiducial volume, 11146 PMTs

threshold:  $E_{\text{th}}^{\text{Kam}} \simeq 4.75 \text{ MeV} \implies {}^8\text{B}, \text{ hep}$

1996 – 2001 (1496 days)

$$\frac{R_{\nu_e}^{\text{SK}}}{R_{\nu_e}^{\text{SSM}}} = 0.465 \pm 0.015 \quad [\text{SK, PLB 539 (2002) 179}]$$



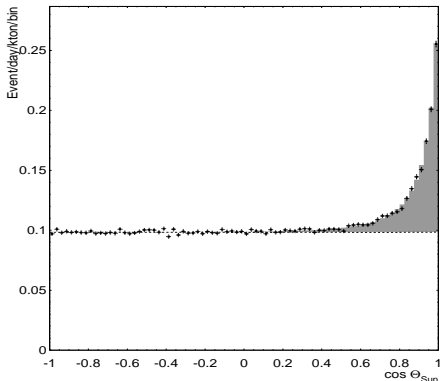
the Super-Kamiokande underground water Cherenkov detector  
located near Higashi-Mozumi, Gifu Prefecture, Japan  
access is via a 2 km long truck tunnel

[R. J. Wilkes, SK, hep-ex/0212035]

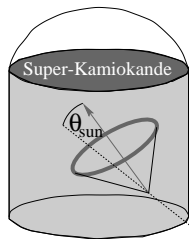
## Super-Kamiokande $\cos \theta_{\text{sun}}$ distribution

the points represent observed data, the histogram shows the best-fit signal (shaded) plus background, the horizontal dashed line shows the estimated background

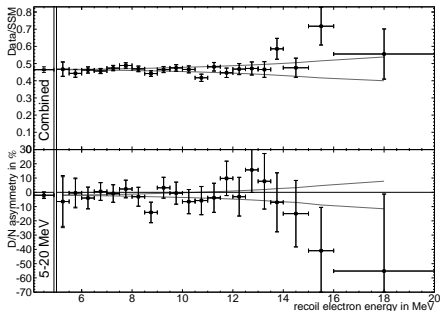
the peak at  $\cos \theta_{\text{sun}} = 1$  is due to solar neutrinos



[Smy, hep-ex/0208004]



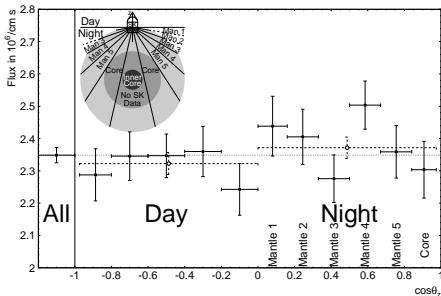
## Super-Kamiokande energy spectrum normalized to BP2000 SSM



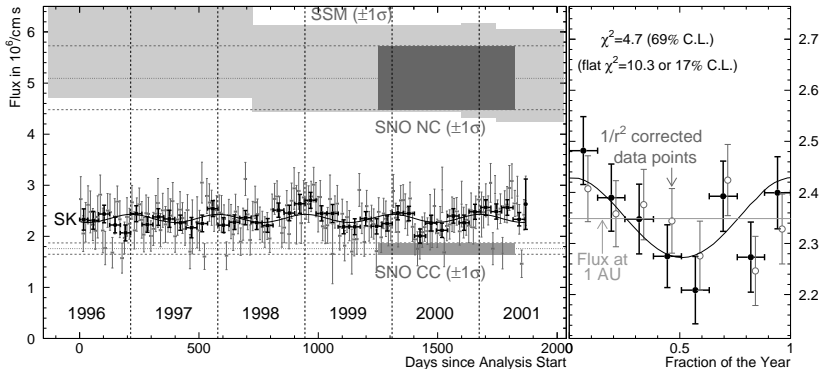
Day-Night asymmetry  
as a function of energy

[Smy, hep-ex/0208004]

## solar zenith angle ( $\theta_z$ ) dependence of Super-Kamiokande data



## Time variation of the Super-Kamiokande data



The gray data points are measured every 10 days.

The black data points are measured every 1.5 months.

The black line indicates the expected annual 7% flux variation.

The right-hand panel combines the 1.5 month bins to search for yearly variations.

The gray data points (open circles) are obtained from the black data points by subtracting the expected 7% variation.

[Smy, hep-ex/0208004]



# SNO: Sudbury Neutrino Observatory

water Cherenkov detector, Sudbury, Ontario, Canada

1 kton of  $D_2O$ , 9456 20-cm PMTs

2073 m underground, 6010 m.w.e.



$$\left. \begin{array}{l} \text{CC threshold: } E_{\text{th}}^{\text{SNO}}(\text{CC}) \simeq 8.2 \text{ MeV} \\ \text{NC threshold: } E_{\text{th}}^{\text{SNO}}(\text{NC}) \simeq 2.2 \text{ MeV} \\ \text{ES threshold: } E_{\text{th}}^{\text{SNO}}(\text{ES}) \simeq 7.0 \text{ MeV} \end{array} \right\} \Rightarrow {}^8\text{B, hep}$$

$D_2O$  phase: 1999 – 2001

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.35 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.01 \pm 0.13$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.47 \pm 0.05$$

[PRL 89 (2002) 011301]

$NaCl$  phase: 2001 – 2002

$$\frac{R_{\text{CC}}^{\text{SNO}}}{R_{\text{CC}}^{\text{SSM}}} = 0.31 \pm 0.02$$

$$\frac{R_{\text{NC}}^{\text{SNO}}}{R_{\text{NC}}^{\text{SSM}}} = 1.03 \pm 0.09$$

$$\frac{R_{\text{ES}}^{\text{SNO}}}{R_{\text{ES}}^{\text{SSM}}} = 0.44 \pm 0.06$$

[nucl-ex/0309004]

$$\Phi_{\nu_e}^{\text{SNO}} = 1.76 \pm 0.11 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

$$\Phi_{\nu_{\mu}, \nu_{\tau}}^{\text{SNO}} = 5.41 \pm 0.66 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$$

SNO solved  
solar neutrino problem



Neutrino Physics  
(April 2002)

[SNO, PRL 89 (2002) 011301, nucl-ex/0204008]

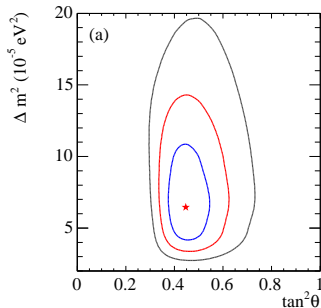
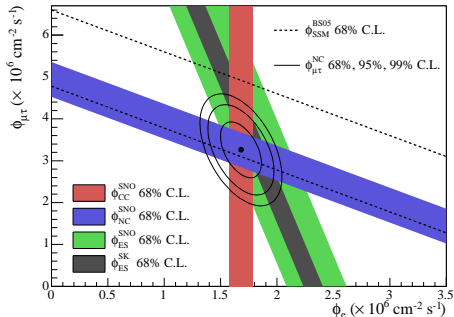
$\nu_e \rightarrow \nu_{\mu}, \nu_{\tau}$  oscillations



Large Mixing Angle solution

$$\Delta m^2 \simeq 7 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \vartheta \simeq 0.45$$



[SNO, PRC 72 (2005) 055502, nucl-ex/0502021]

# KamLAND

## Kamioka Liquid scintillator Anti-Neutrino Detector

### long-baseline reactor $\bar{\nu}_e$ experiment

Kamioka mine (200 km west of Tokyo), 1000 m underground, 2700 m.w.e.

6.7% of flux from one reactor at 88 km

average distance from reactors: 180 km 79% of flux from 26 reactors at 138–214 km

14.3% of flux from other reactors at >295 km

1 kt liquid scintillator detector:  $\bar{\nu}_e + p \rightarrow e^+ + n$ , energy threshold:  $E_{\text{th}}^{\bar{\nu}_e p} = 1.8 \text{ MeV}$

data taking: 4 March – 6 October 2002, 145.1 days (162 ton yr)

expected number of reactor neutrino events (no osc.):

$$N_{\text{expected}}^{\text{KamLAND}} = 86.8 \pm 5.6$$

expected number of background events:

$$N_{\text{background}}^{\text{KamLAND}} = 0.95 \pm 0.99$$

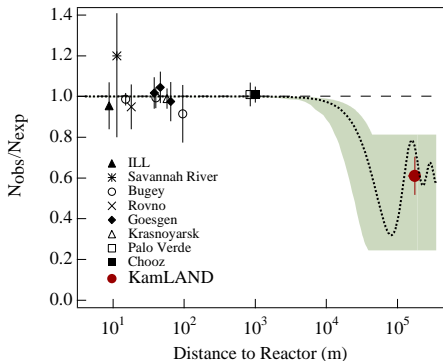
observed number of neutrino events:

$$N_{\text{observed}}^{\text{KamLAND}} = 54$$

$$\frac{N_{\text{observed}}^{\text{KamLAND}} - N_{\text{background}}^{\text{KamLAND}}}{N_{\text{expected}}^{\text{KamLAND}}} = 0.611 \pm 0.085 \pm 0.041$$

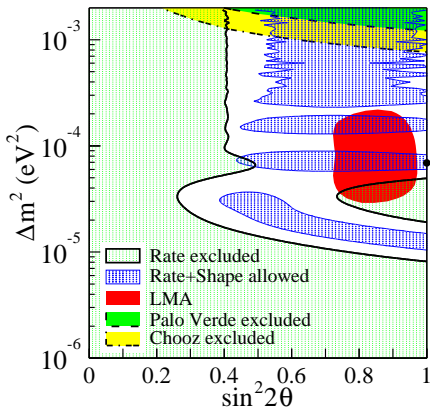
99.95% C.L. evidence  
of  $\bar{\nu}_e$  disappearance

confirmation of LMA (December 2002)



Shade: 95% C.L. LMA

Curve: 
$$\begin{cases} \Delta m^2 = 5.5 \times 10^{-5} \text{ eV}^2 \\ \sin^2 2\vartheta = 0.83 \end{cases}$$

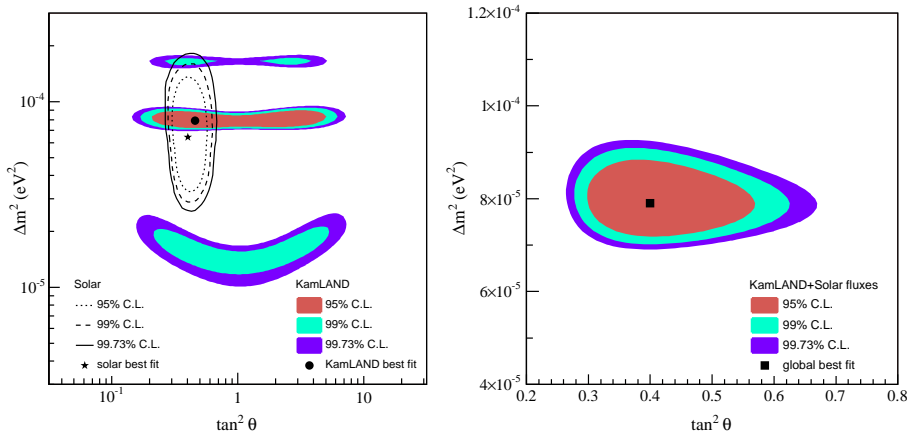


95% C.L.

[KamLAND, PRL 90 (2003) 021802, hep-ex/0212021]

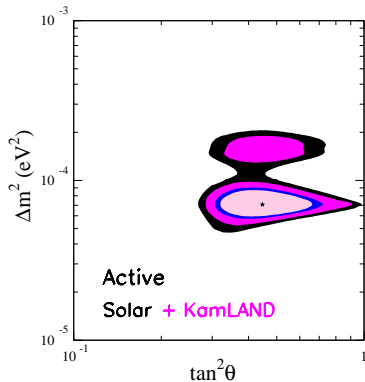
# Combined Fit of Solar and Reactor Neutrino Data

[KamLAND, hep-ex/0406035]



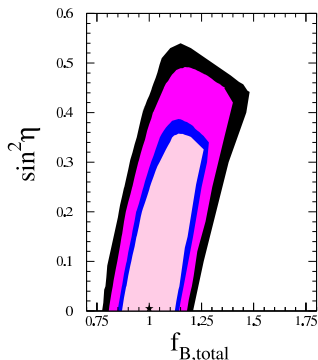
Best Fit:  $\Delta m^2 = 0.82_{-0.5}^{+0.6} \times 10^{-5} \text{ eV}^2$        $\tan^2 \vartheta = 0.40_{-0.07}^{+0.09}$

# Sterile Neutrinos in Solar Neutrino Flux?



90%, 95%, 99%, 99.73% ( $3\sigma$ ) C.L.

[Bahcall, Gonzalez-Garcia, Pena-Garay, JHEP 0302 (2003) 009]



$$\nu_e \rightarrow \cos \eta \nu_a + \sin \eta \nu_s$$

$$\sin^2 \eta < 0.52 (3\sigma)$$

$$f_{B,\text{total}} = \frac{\Phi_{8B}}{\Phi_{8B}^{\text{SSM}}} = 1.00 \pm 0.06$$

# Determination of Solar Neutrino Fluxes

[Bahcall, Peña-Garay, hep-ph/0305159]

fit of solar and KamLAND neutrino data with fluxes as free parameters

+ luminosity constraint

$$\sum_r \alpha_r \Phi_r = K_{\odot} \quad (r = pp, pep, hep, {}^7\text{Be}, {}^8\text{B}, {}^{13}\text{N}, {}^{15}\text{O}, {}^{17}\text{F})$$
$$K_{\odot} \equiv \mathcal{L}_{\odot}/4\pi(1\text{a.u.})^2 = 8.534 \times 10^{11} \text{ MeV cm}^{-2} \text{ s}^{-1}$$

solar constant

$$\Delta m^2 = 7.3_{-0.6}^{+0.4} \text{ eV}^2 \quad \tan^2 \vartheta = 0.42_{-0.06}^{+0.08} \left( \begin{smallmatrix} +0.39 \\ -0.19 \end{smallmatrix} \right)$$

$$\frac{\Phi_{8\text{B}}}{\Phi_{8\text{B}}^{\text{SSM}}} = 1.01_{-0.06}^{+0.06} \left( \begin{smallmatrix} +0.22 \\ -0.17 \end{smallmatrix} \right)$$

moderate uncertainty  
will improve with new SNO  
NC data (salt phase)

$$\frac{\Phi_{7\text{Be}}}{\Phi_{7\text{Be}}^{\text{SSM}}} = 0.97_{-0.54}^{+0.28} \left( \begin{smallmatrix} +0.85 \\ -0.97 \end{smallmatrix} \right)$$

large uncertainty  
needs  ${}^7\text{Be}$  experiment  
(KamLAND, Borexino?)

$$\frac{\Phi_{pp}}{\Phi_{pp}^{\text{SSM}}} = 1.02_{-0.02}^{+0.02} \left( \begin{smallmatrix} +0.07 \\ -0.07 \end{smallmatrix} \right)$$

small uncertainty

$$\text{CNO luminosity: } \mathcal{L}_{\text{CNO}}/\mathcal{L}_{\odot} = 0.0_{-0.0}^{+2.8} \left( \begin{smallmatrix} +7.3 \\ -0.0 \end{smallmatrix} \right)$$

[Bahcall, Gonzalez-Garcia, Peña-Garay, PRL 90 (2003) 131301]

# Details of Solar Neutrino Oscillations

best fit of reactor + solar neutrino data:  $\Delta m^2 \simeq 7 \times 10^{-5} \text{ eV}^2$   $\tan^2 \vartheta \simeq 0.4$

$$\overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} = \frac{1}{2} + \left( \frac{1}{2} - P_c \right) \cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2 \vartheta}\right)} \quad \gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E \cos 2\vartheta \left| \frac{d \ln A}{dx} \right|_R} \quad F = 1 - \tan^2 \vartheta$$

$$A_{\text{CC}} \simeq 2\sqrt{2}EG_F N_e^c \exp\left(-\frac{x}{x_0}\right) \implies \left| \frac{d \ln A}{dx} \right| \simeq \frac{1}{x_0} = \frac{10.54}{R_\odot} \simeq 3 \times 10^{-15} \text{ eV}$$

$$\tan^2 \vartheta \simeq 0.4 \implies \sin^2 2\vartheta \simeq 0.82, \cos 2\vartheta \simeq 0.43 \quad \gamma \simeq 2 \times 10^4 \left( \frac{E}{\text{MeV}} \right)^{-1}$$

$$\gamma \gg 1 \implies P_c \ll 1 \implies \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun,LMA}} \simeq \frac{1}{2} + \frac{1}{2} \cos 2\vartheta_M^0 \cos 2\vartheta$$



$$\cos 2\vartheta_M^0 = \frac{\Delta m^2 \cos 2\vartheta - A_{CC}^0}{\sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC}^0)^2 + (\Delta m^2 \sin 2\vartheta)^2}}$$

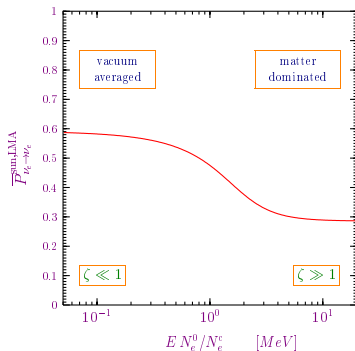
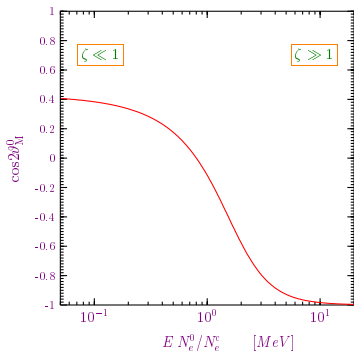
critical parameter [Bahcall, Peña-Garay, hep-ph/0305159]

$$\zeta = \frac{A_{CC}^0}{\Delta m^2 \cos 2\vartheta} = \frac{2\sqrt{2}EG_F N_e^0}{\Delta m^2 \cos 2\vartheta} \simeq 1.2 \left( \frac{E}{\text{MeV}} \right) \left( \frac{N_e^0}{N_e^c} \right)$$

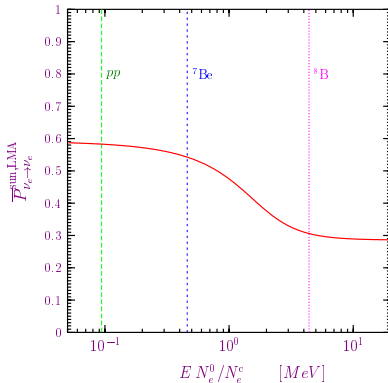
$$\zeta \ll 1 \Rightarrow \vartheta_M^0 \simeq \vartheta \Rightarrow \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq 1 - \frac{1}{2} \sin^2 2\vartheta$$

$$\zeta \gg 1 \Rightarrow \vartheta_M^0 \simeq \pi/2 \Rightarrow \overline{P}_{\nu_e \rightarrow \nu_e}^{\text{sun}} \simeq \sin^2 \vartheta$$

vacuum averaged  
survival probability  
matter dominated  
survival probability



$$\begin{aligned} \langle E \rangle_{pp} \simeq 0.27 \text{ MeV}, \quad \langle r_0 \rangle_{pp} \simeq 0.1 R_\odot &\implies \langle E N_e^0 / N_e^c \rangle_{pp} \simeq 0.094 \text{ MeV} \\ E_{7\text{Be}} \simeq 0.86 \text{ MeV}, \quad \langle r_0 \rangle_{7\text{Be}} \simeq 0.06 R_\odot &\implies \langle E N_e^0 / N_e^c \rangle_{7\text{Be}} \simeq 0.46 \text{ MeV} \\ \langle E \rangle_{8\text{B}} \simeq 6.7 \text{ MeV}, \quad \langle r_0 \rangle_{8\text{B}} \simeq 0.04 R_\odot &\implies \langle E N_e^0 / N_e^c \rangle_{8\text{B}} \simeq 4.4 \text{ MeV} \end{aligned}$$



each neutrino experiment is mainly sensitive to one flux  
 each neutrino experiment is mainly sensitive to  $\vartheta$   
 accurate  $pp$  experiment can improve determination of  $\vartheta$

[Bahcall, Peña-Garay, hep-ph/0305159]

# Future

## Goals of Future Solar Neutrino Experiments

[Bahcall, Peña-Garay, hep-ph/0305159]

- ▶ Improve the determination of  $\vartheta$
- ▶ Accurate measure of solar neutrino fluxes
- ▶ Discover or constraint subdominant neutrino conversion mechanisms

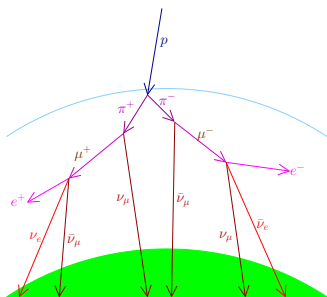
## Precise Determination of $\Delta m^2$ with New Reactor Experiment

LMA:  $L \simeq 70 - 80$  km

# Atmospheric and LBL Oscillation Experiments

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
  - Atmospheric Neutrinos
  - Super-Kamiokande Up-Down Asymmetry
  - Fit of Super-Kamiokande Atmospheric Data
  - Kamiokande, Soudan-2, MACRO and MINOS
  - K2K
  - MINOS
  - Sterile Neutrinos in Atmospheric Neutrino Flux?
- Phenomenology of Three-Neutrino Mixing
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay

# Atmospheric Neutrinos



$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2 \quad \text{at } E \lesssim 1 \text{ GeV}$$

uncertainty on ratios:  $\sim 5\%$

uncertainty on fluxes:  $\sim 30\%$

ratio of ratios

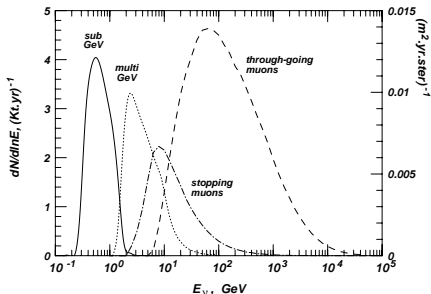
$$R \equiv \frac{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{data}}}{[N(\nu_\mu + \bar{\nu}_\mu)/N(\nu_e + \bar{\nu}_e)]_{\text{MC}}}$$

$$R_{\text{sub-GeV}}^K = 0.60 \pm 0.07 \pm 0.05$$

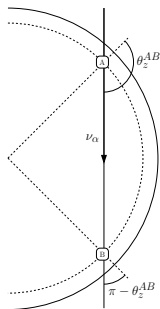
[Kamiokande, PLB 280 (1992) 146]

$$R_{\text{multi-GeV}}^K = 0.57 \pm 0.08 \pm 0.07$$

[Kamiokande, PLB 335 (1994) 237]



# Super-Kamiokande Up-Down Asymmetry



$E_\nu \gtrsim 1 \text{ GeV} \Rightarrow$  isotropic flux of cosmic rays

$$\phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\pi - \theta_z^{AB}) \quad \phi_{\nu_\alpha}^{(A)}(\theta_z^{AB}) = \phi_{\nu_\alpha}^{(B)}(\theta_z^{AB})$$

$$\Downarrow$$

$$\phi_{\nu_\alpha}^{(A)}(\theta_z) = \phi_{\nu_\alpha}^{(A)}(\pi - \theta_z)$$

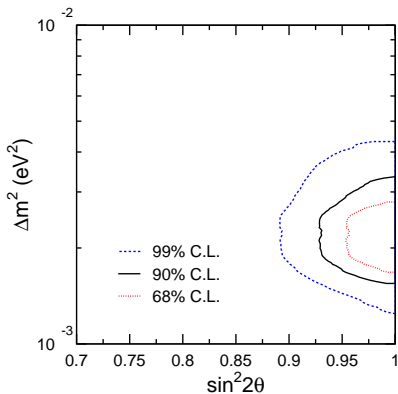
(December 1998)

$$A_{\nu_\mu}^{\text{up-down}}(\text{SK}) = \left( \frac{N_{\nu_\mu}^{\text{up}} - N_{\nu_\mu}^{\text{down}}}{N_{\nu_\mu}^{\text{up}} + N_{\nu_\mu}^{\text{down}}} \right) = -0.296 \pm 0.048 \pm 0.01$$

[Super-Kamiokande, Phys. Rev. Lett. 81 (1998) 1562, hep-ex/9807003]

6 $\sigma$  MODEL INDEPENDENT EVIDENCE OF  $\nu_\mu$  DISAPPEARANCE!

# Fit of Super-Kamiokande Atmospheric Data



Best Fit: 
$$\begin{cases} \nu_\mu \rightarrow \nu_\tau \\ \Delta m^2 = 2.1 \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta = 1.0 \end{cases}$$

1489.2 live-days (Apr 1996 – Jul 2001)

[Super-Kamiokande, PRD 71 (2005) 112005, hep-ex/0501064]

Measure of  $\nu_\tau$  CC Int. is Difficult:

- ▶  $E_{\text{th}} = 3.5 \text{ GeV} \Rightarrow \sim 20 \text{ events/yr}$
- ▶  $\tau$ -Decay  $\Rightarrow$  Many Final States

$\nu_\tau$ -Enriched Sample

$$N_{\nu_\tau}^{\text{the}} = 78 \pm 26 @ \Delta m^2 = 2.4 \times 10^{-3} \text{ eV}^2$$

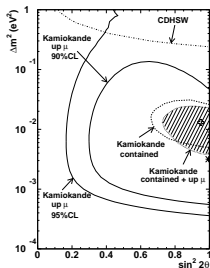
$$N_{\nu_\tau}^{\text{exp}} = 138_{-58}^{+50}$$

$$N_{\nu_\tau} > 0 @ 2.4\sigma$$

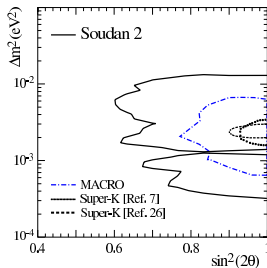
[Super-Kamiokande, PRL 97(2006) 171801, hep-ex/0607059]

future: CNGS  $\nu_\mu \rightarrow \nu_\tau$  (OPERA)

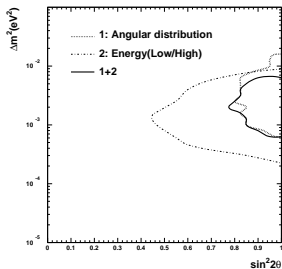
# Kamiokande, Soudan-2, MACRO and MINOS



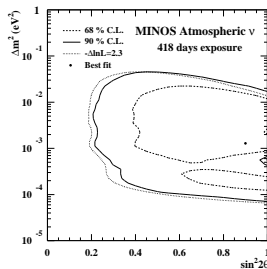
[Kamiokande, hep-ex/9806038]



[Soudan 2, hep-ex/0507068]



[MACRO, hep-ex/0304037]

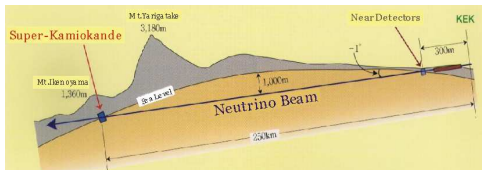


[MINOS, hep-ex/0512036]



# K2K

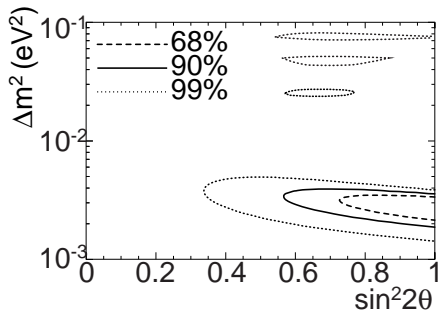
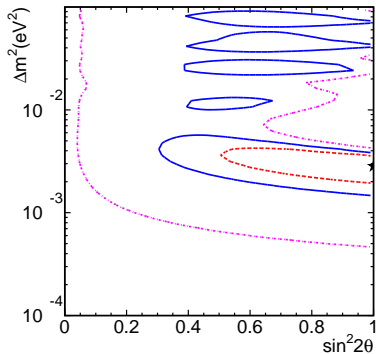
confirmation of atmospheric allowed region (June 2002)



KEK to Kamioka  
(Super-Kamiokande)

250 km

$\nu_\mu \rightarrow \nu_\mu$



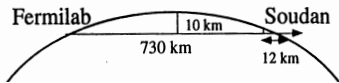
[K2K, PRL 94 (2005) 081802, hep-ex/0411038]

[K2K, Phys. Rev. Lett. 90 (2003) 041801]

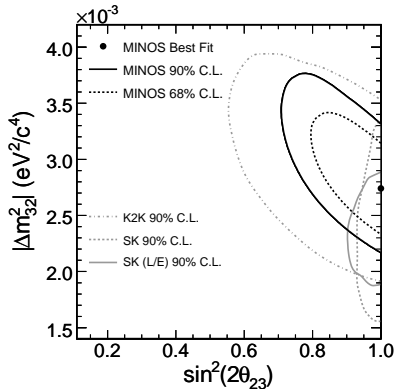
# MINOS

May 2005 – Feb 2006

<http://www-nu.fermi.fnal.gov/>



Near Detector: 1 km



$\nu_\mu \rightarrow \nu_\mu$

$$\Delta m^2 = 2.74^{+0.44}_{-0.26} \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta > 0.87 @ 68\% CL$$

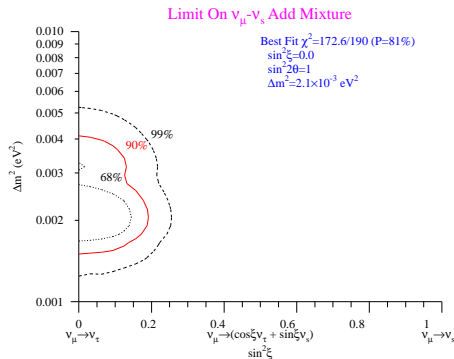
[MINOS, PRL 97 (2006) 191801, hep-ex/0607088]

# Sterile Neutrinos in Atmospheric Neutrino Flux?

## Nature of atmospheric Oscillation

Mode	Best fit	$\Delta\chi^2$	$\sigma$
$\nu_\mu - \nu_\tau$	$\sin^2 2\theta = 1.00$ ; $\Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2$	0.0	0.0
$\nu_\mu - \nu_e$	$\sin^2 2\theta = 0.97$ ; $\Delta m^2 = 5.0 \times 10^{-3} \text{eV}^2$	79.3	8.9
$\nu_\mu - \nu_s$	$\sin^2 2\theta = 0.96$ ; $\Delta m^2 = 3.6 \times 10^{-3} \text{eV}^2$	19.0	4.4
LxE	$\sin^2 2\theta = 0.90$ ; $\alpha = 5.3 \times 10^{-4}$	67.1	8.2
$\nu_\mu$ Decay	$\cos^2 \theta = 0.47$ ; $\alpha = 3.0 \times 10^{-3} \text{eV}^2$	81.1	9.0
$\nu_\mu$ Decay to $\nu_s$	$\cos^2 \theta = 0.33$ ; $\alpha = 1.1 \times 10^{-2} \text{eV}^2$	14.1	3.8

[Smy (SK), Moriond 2002]



[Nakaya (SK), hep-ex/0209036]

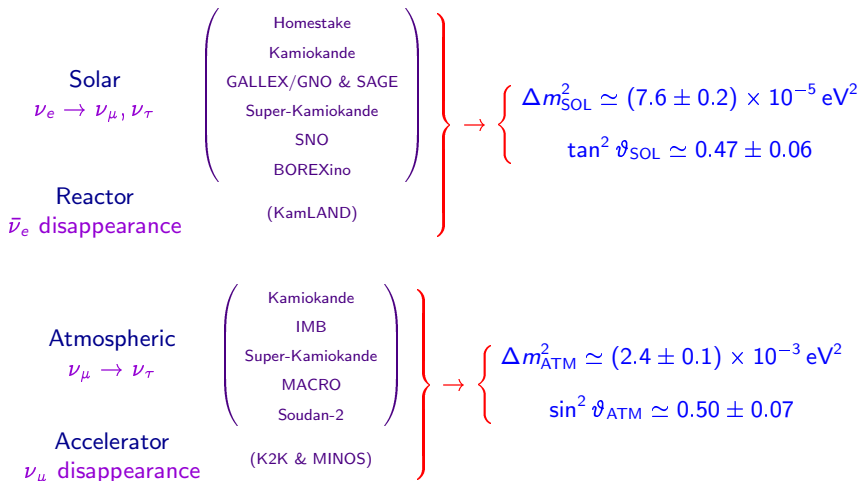
## FUTURE

MINOS:  $\nu_\mu \rightarrow \nu_\mu$ ,  $\nu_\mu \rightarrow \nu_e$ ,  $\nu_\mu \rightarrow \nu_{e,\mu,\tau}$  (NC)  
 CNGS: ICARUS:  $\nu_\mu \rightarrow \nu_e$ ,  $\nu_\mu \rightarrow \nu_\tau$  OPERA:  $\nu_\mu \rightarrow \nu_\tau$

# Phenomenology of Three-Neutrino Mixing

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
  - Experimental Evidences of Neutrino Oscillations
  - Three-Neutrino Mixing
  - Allowed Three-Neutrino Schemes
  - Mixing Matrix
  - Bilarge Mixing
  - Absolute Scale of Neutrino Masses
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay

# Experimental Evidences of Neutrino Oscillations



Two scales of  $\Delta m^2$ :  $\Delta m_{\text{ATM}}^2 \simeq 30 \Delta m_{\text{SOL}}^2$

Large mixings:  $\vartheta_{\text{ATM}} \simeq 45^\circ$ ,  $\vartheta_{\text{SOL}} \simeq 34^\circ$

## Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^3 U_{\alpha k} \nu_{kL} \quad (\alpha = e, \mu, \tau)$$

three flavor fields:  $\nu_e, \nu_\mu, \nu_\tau$

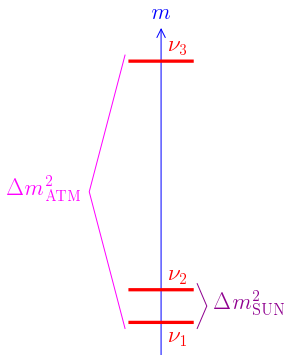
three massive fields:  $\nu_1, \nu_2, \nu_3$

$$\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = m_2^2 - m_1^2 + m_3^2 - m_2^2 + m_1^2 - m_3^2 = 0$$

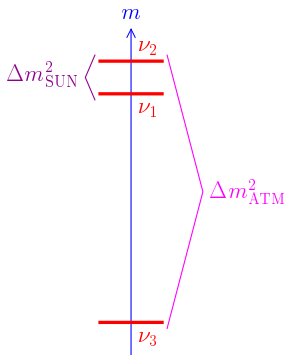
$$\Delta m_{\text{SOL}}^2 = \Delta m_{21}^2 \simeq (7.6 \pm 0.2) \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{\text{ATM}}^2 \simeq |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| \simeq (2.4 \pm 0.1) \times 10^{-3} \text{ eV}^2$$

# Allowed Three-Neutrino Schemes



"normal"



"inverted"

different signs of  $\Delta m_{31}^2 \simeq \Delta m_{32}^2$

absolute scale is not determined by neutrino oscillation data

# Mixing Matrix

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2|$$

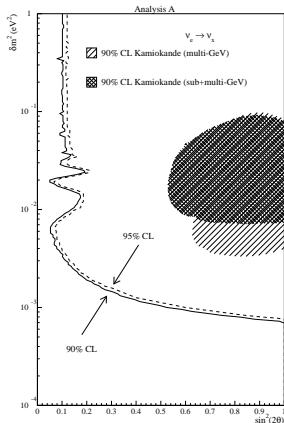
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

SOL →
↑
 ATM

$$\text{CHOOZ: } \begin{cases} \Delta m_{\text{CHOOZ}}^2 = \Delta m_{31}^2 = \Delta m_{\text{ATM}}^2 \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$$

$$|U_{e3}|^2 \lesssim 5 \times 10^{-2}$$

SOLAR AND ATMOSPHERIC  $\nu$  OSCILLATIONS  
ARE PRACTICALLY DECOUPLED!



[CHOOZ, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

$$|U_{e1}|^2 \simeq \cos^2 \vartheta_{\text{SOL}} \quad |U_{e2}|^2 \simeq \sin^2 \vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2 \vartheta_{\text{ATM}} \quad |U_{\tau 3}|^2 \simeq \cos^2 \vartheta_{\text{ATM}}$$



$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\sin^2 \vartheta_{12} = 0.304_{-0.016}^{+0.022}$$

$$\sin^2 \vartheta_{23} = 0.50_{-0.06}^{+0.07}$$

$$\sin^2 \vartheta_{13} < 0.035 \quad (90\% \text{ C.L.})$$

[Schwetz, Tortola, Valle, New J. Phys. 10 (2008) 113011]

Hint of  $\vartheta_{13} > 0$

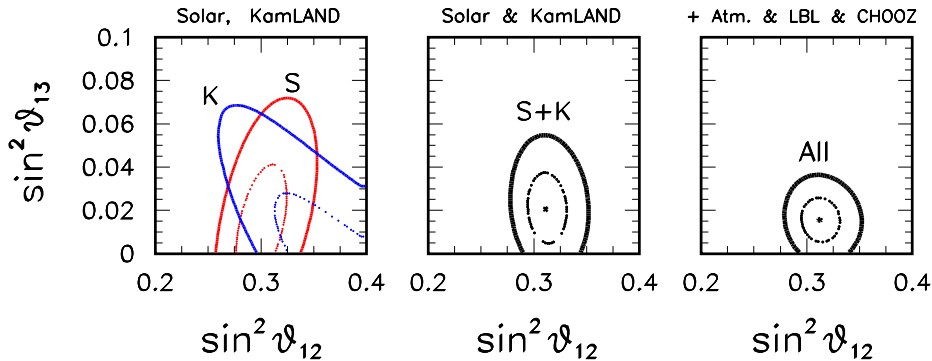
[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$$\sin^2 \vartheta_{13} = 0.016 \pm 0.010 \quad [\text{Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801}]$$

future: measure  $\vartheta_{13} \neq 0 \implies$  CP violation, matter effects, mass hierarchy

## Hint of $\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]



$$\sin^2 \vartheta_{13} = 0.016 \pm 0.010 \quad [\text{Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801}]$$

$$P_{\nu_e \rightarrow \nu_e}^{(-)} \simeq \begin{cases} (1 - \sin^2 \vartheta_{13})^2 (1 - 0.5 \sin^2 \vartheta_{12}) & \text{SOL low-energy \& KamLAND} \\ (1 - \sin^2 \vartheta_{13})^2 \sin^2 \vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

# Bilarge Mixing

$$|U_{e3}|^2 \ll 1$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \implies \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \phantom{\nu_a^{(S)}} = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \implies \vartheta_A \simeq \frac{\pi}{4} \implies U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\text{Solar } \nu_e \rightarrow \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$$

$$\frac{\Phi_{CC}^{\text{SNO}}}{\Phi_{\nu_e}^{\text{SSM}}} \simeq \frac{1}{3} \implies \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6 \text{ MeV}$$

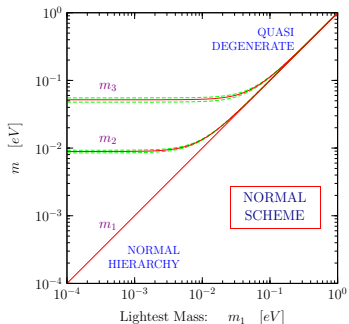
$$\sin^2 \vartheta_S \simeq \frac{1}{3} \implies U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

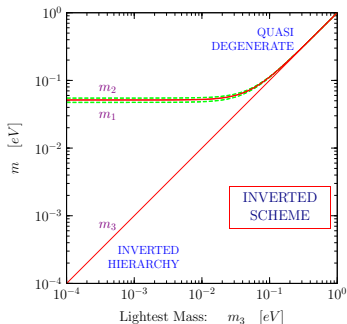
[Harrison, Perkins, Scott, hep-ph/0202074]

# Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$

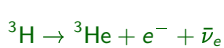
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$

$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for  $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2} \text{ eV}$

# Tritium Beta-Decay

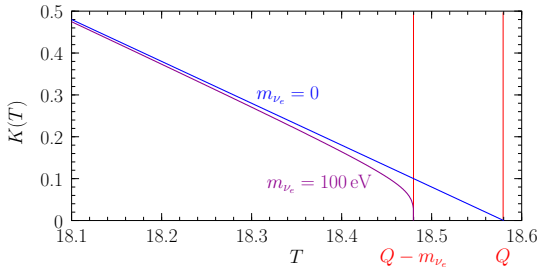


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) p E (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^3\text{H}} - M_{{}^3\text{He}} - m_e = 18.58 \text{ keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{\frac{d\Gamma/dT}{(\cos\vartheta_C G_F)^2} |\mathcal{M}|^2 F(E) p E}{2\pi^3}} = \left[ (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2} \right]^{1/2}$$



$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

Mainz & Troitsk

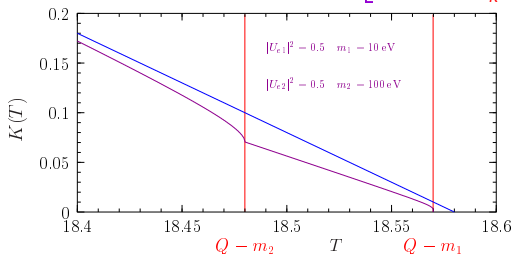
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity:  $m_{\nu_e} \simeq 0.2 \text{ eV}$

$$\text{Neutrino Mixing} \implies K(T) = \left[ (Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}$$



analysis of data is  
different from the  
no-mixing case:

$2N - 1$  parameters

$$\left( \sum_k |U_{ek}|^2 = 1 \right)$$

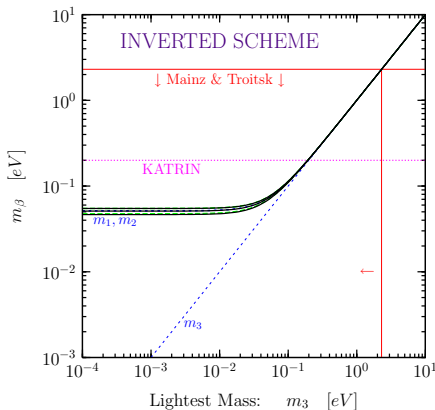
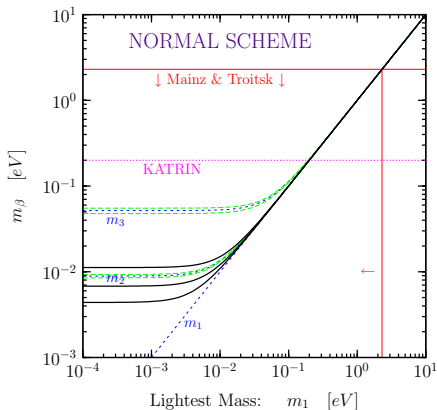
if experiment is not sensitive to masses ( $m_k \ll Q - T$ )

effective mass:

$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$\begin{aligned} K^2 &= (Q - T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q - T)^2}} \simeq (Q - T)^2 \sum_k |U_{ek}|^2 \left[ 1 - \frac{1}{2} \frac{m_k^2}{(Q - T)^2} \right] \\ &= (Q - T)^2 \left[ 1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T)^2} \right] \simeq (Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \end{aligned}$$

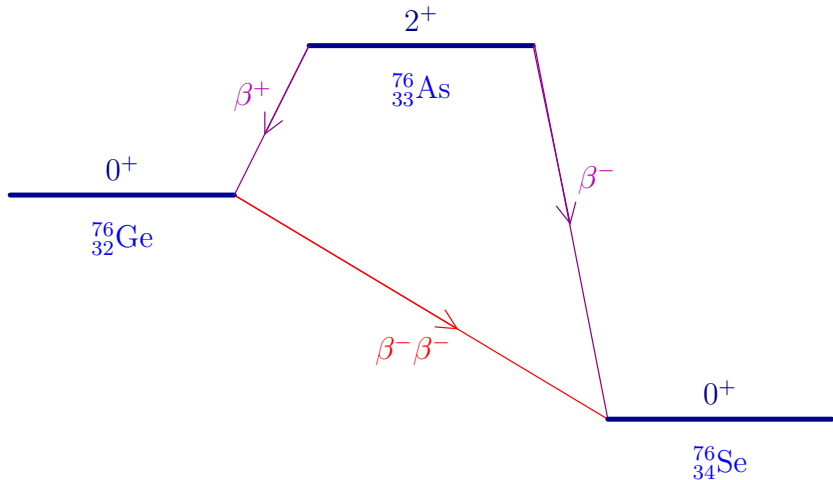
$$m_\beta^2 = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$



Quasi-Degenerate:  $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF  $m_\beta \lesssim 4 \times 10^{-2} \text{ eV} \implies$  NORMAL HIERARCHY

# Neutrinoless Double-Beta Decay



Effective Majorana Neutrino Mass:

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

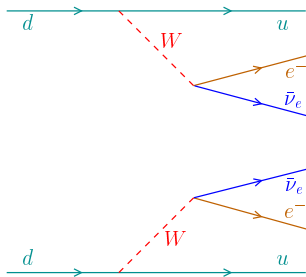


## Two-Neutrino Double- $\beta$ Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process  
in the Standard Model



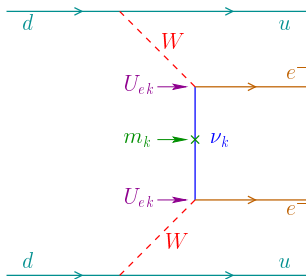
## Neutrinoless Double- $\beta$ Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \rightarrow \mathcal{N}(A, Z + 2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 |m_{\beta\beta}|^2$$

effective  
Majorana  
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

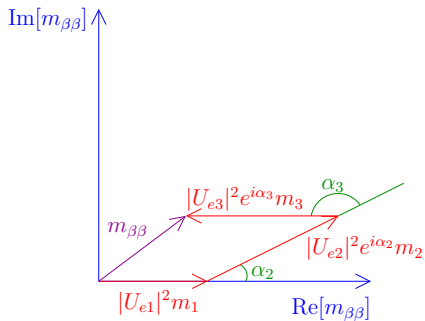
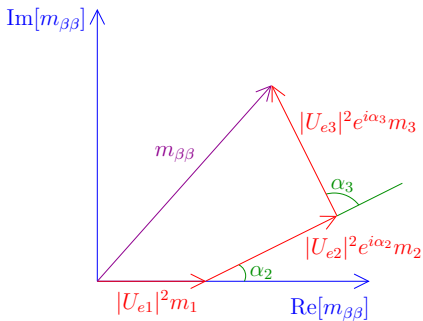


## Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k \quad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$

$$\alpha_2 = 2\lambda_2 \quad \alpha_3 = 2(\lambda_3 - \delta_{13})$$



## Experimental Bounds

CUORICINO ( $^{130}\text{Te}$ ) [PRC 78 (2008) 035502]

$$T_{1/2}^{0\nu} > 3 \times 10^{24} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.19 - 0.68 \text{ eV}$$

Heidelberg-Moscow ( $^{76}\text{Ge}$ ) [EPJA 12 (2001) 147]

$$T_{1/2}^{0\nu} > 1.9 \times 10^{25} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.32 - 1.0 \text{ eV}$$

IGEX ( $^{76}\text{Ge}$ ) [PRD 65 (2002) 092007]

$$T_{1/2}^{0\nu} > 1.57 \times 10^{25} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.33 - 1.35 \text{ eV}$$

NEMO 3 ( $^{100}\text{Mo}$ ) [PRL 95 (2005) 182302]

$$T_{1/2}^{0\nu} > 4.6 \times 10^{23} \text{ y (90\% C.L.)} \implies |m_{\beta\beta}| \lesssim 0.7 - 2.8 \text{ eV}$$

## FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

$$|m_{\beta\beta}| \sim \text{few } 10^{-1} \text{ eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few } 10^{-2} \text{ eV}$$

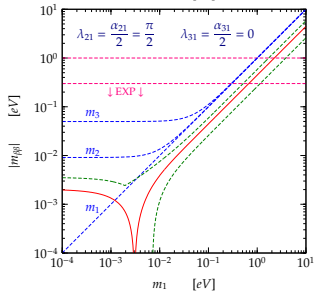
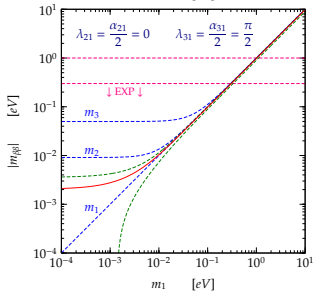
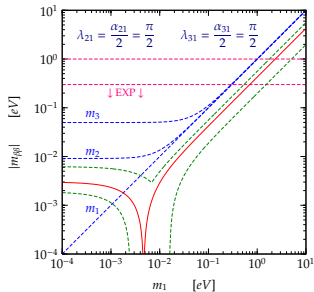
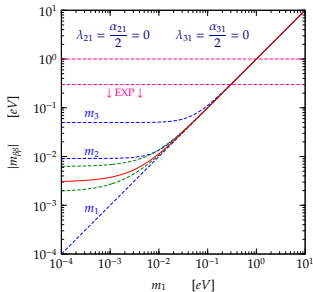
# Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3$$

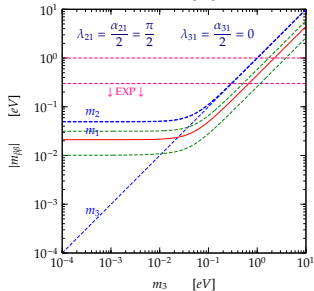
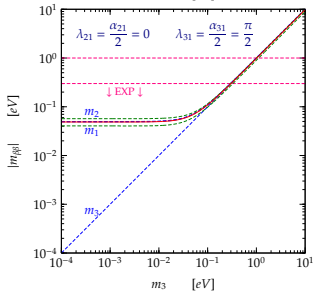
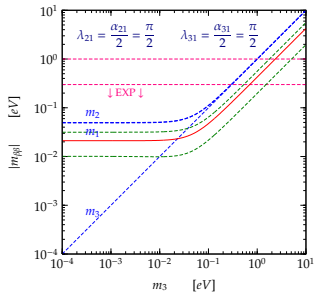
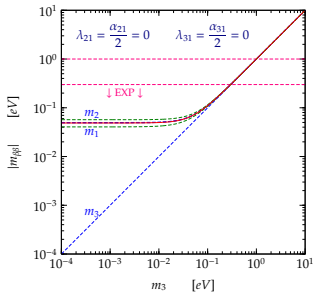
CP conservation

$$\alpha_{21} = 0, \pi \quad \alpha_{31} = 0, \pi$$

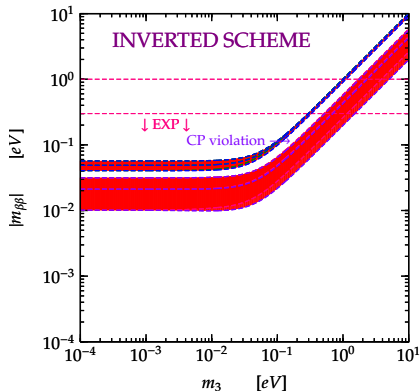
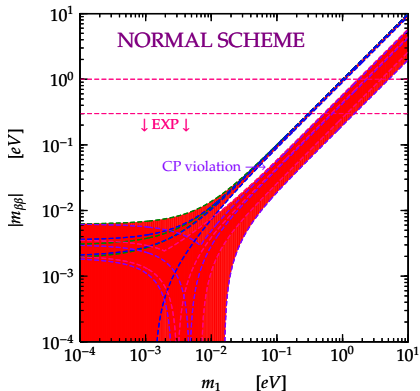
# CP Conservation: Normal Scheme



## CP Conservation: Inverted Scheme



$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_{21}} m_2 + |U_{e3}|^2 e^{i\alpha_{31}} m_3$$

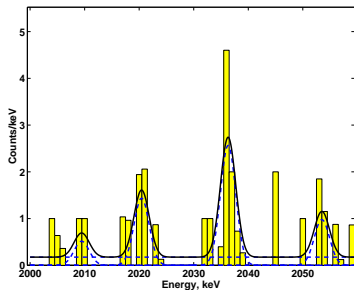


FUTURE: IF  $|m_{\beta\beta}| \lesssim 10^{-2} \text{ eV} \Rightarrow$  NORMAL HIERARCHY

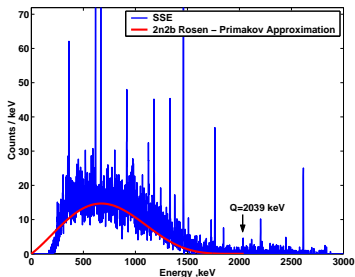
# Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$$T_{1/2}^{0\nu \text{bf}} = 1.19 \times 10^{25} \text{ y} \quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25} \text{ y} (3\sigma) \quad 4.2\sigma \text{ evidence}$$



pulse-shape selected spectrum



3.8 $\sigma$  evidence

[PLB 586 (2004) 198]

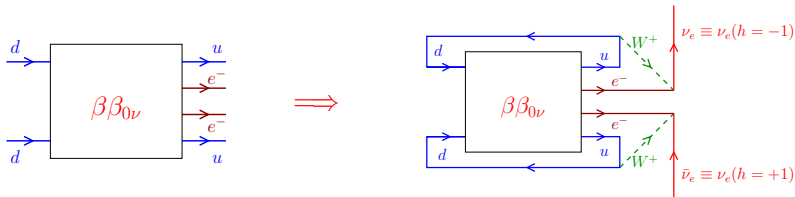
the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Rightarrow \quad 0.22 \text{ eV} \lesssim |m_{\beta\beta}| \lesssim 1.6 \text{ eV}$$

if confirmed, very exciting (Majorana  $\nu$  and large mass scale)



# $\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

## Majorana Mass Term

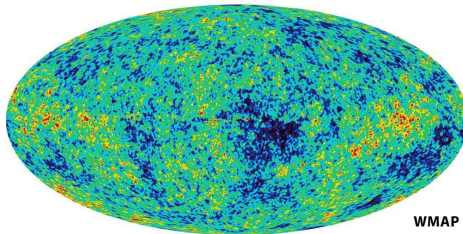
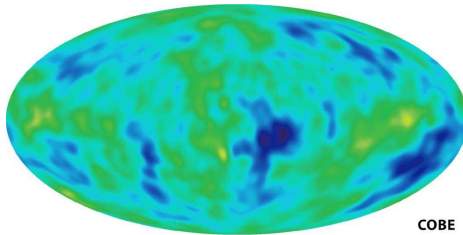
$$\mathcal{L}_{eL}^M = -\frac{1}{2} m_{ee} \left( \overline{\nu_{eL}^c} \nu_{eL} + \overline{\nu_{eL}} \nu_{eL}^c \right)$$

# Cosmological Bound on Neutrino Masses

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay
- **Cosmological Bound on Neutrino Masses**
  - WMAP (Wilkinson Microwave Anisotropy Probe)
  - Galaxy Redshift Surveys
  - Lyman-alpha Forest
  - Relic Neutrinos
  - Power Spectrum of Density Fluctuations

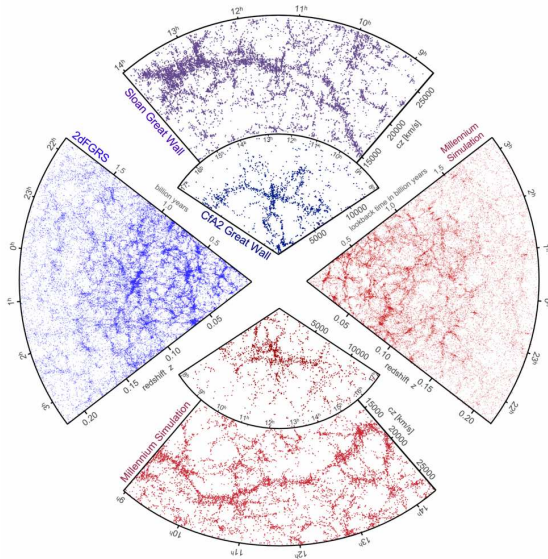
# WMAP (Wilkinson Microwave Anisotropy Probe)

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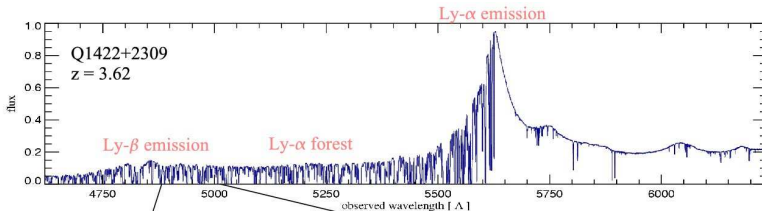
[WMAP, <http://map.gsfc.nasa.gov>]

# Galaxy Redshift Surveys

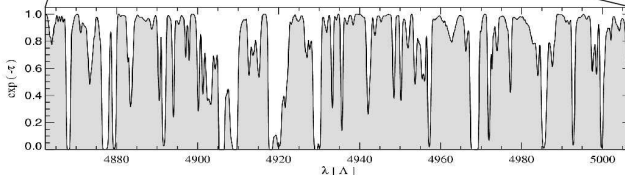
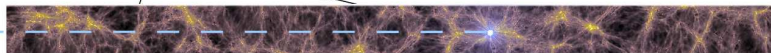


[Springel, Frenk, White, astro-ph/0604561]

# Lyman-alpha Forest



to Earth



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman  $\alpha$ ,  $\beta$ ,  $\gamma$  wavelengths:  $\lambda_{\alpha}^0 = 1215.67 \text{ \AA}$ ,  $\lambda_{\beta}^0 = 1025.72 \text{ \AA}$ ,  $\lambda_{\gamma}^0 = 972.54 \text{ \AA}$

Lyman- $\alpha$  forest: The region in which only Ly $\alpha$  photons can be absorbed:  $[(1+z_q)\lambda_{\beta}^0, (1+z_q)\lambda_{\alpha}^0]$

# Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions



weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \implies T_{\text{dec}} \sim 1 \text{ MeV}$$

neutrino decoupling

$$\text{Relic Neutrinos: } T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945 \text{ K} \implies k T_\nu \simeq 1.676 \times 10^{-4} \text{ eV}$$

( $T_\gamma = 2.725 \pm 0.001 \text{ K}$ )

$$\text{number density: } n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \implies n_{\nu_k, \bar{\nu}_k} \simeq 0.1827 T_\nu^3 \simeq 112 \text{ cm}^{-3}$$

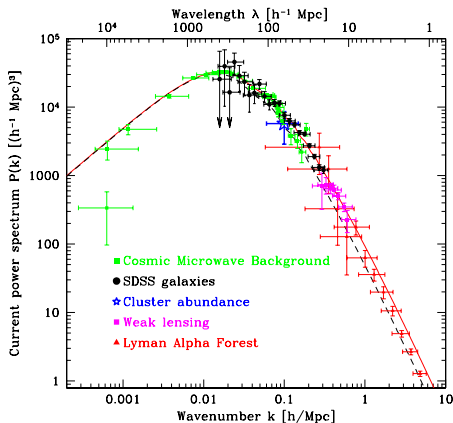
$$\text{density contribution: } \Omega_k = \frac{n_{\nu_k, \bar{\nu}_k} m_k}{\rho_c} \simeq \frac{1}{h^2} \frac{m_k}{94.14 \text{ eV}} \implies \Omega_\nu h^2 = \frac{\sum_k m_k}{94.14 \text{ eV}}$$

( $\rho_c = \frac{3H^2}{8\pi G_N}$ )

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \quad \implies \quad \sum_k m_k \lesssim 14 \text{ eV}$$

# Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat  $\Lambda$ CDM model

$$(\Omega_M^0 = 0.28, h = 0.72, \Omega_B^0/\Omega_M^0 = 0.16)$$

Dashed Curve:  $\sum_{k=1}^3 m_k = 1 \text{ eV}$

hot dark matter  
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1) \delta(\vec{x}_2) \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} P(\vec{k})$$

small scale suppression

$$\begin{aligned} \frac{\Delta P(k)}{P(k)} &\approx -8 \frac{\Omega_\nu}{\Omega_m} \\ &\approx -0.8 \left( \frac{\sum_k m_k}{1 \text{ eV}} \right) \left( \frac{0.1}{\Omega_m h^2} \right) \end{aligned}$$

for

$$k \gtrsim k_{\text{nr}} \approx 0.026 \sqrt{\frac{m_\nu}{1 \text{ eV}}} \sqrt{\Omega_m} h \text{ Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SN-Ia  $\implies$  Flat  $\Lambda$ CDM

$$T_0 = 13.7 \pm 0.2 \text{ Gyr} \quad h = 0.71_{-0.03}^{+0.04}$$
$$\Omega_0 = 1.02 \pm 0.02 \quad \Omega_b = 0.044 \pm 0.004 \quad \Omega_m = 0.27 \pm 0.04$$

$$\Omega_\nu h^2 < 0.0076 \quad (95\% \text{ conf.}) \implies \sum_{k=1}^3 m_k < 0.71 \text{ eV}$$

CMB + HST + SN-Ia + BAO

$$T_0 = 13.72 \pm 0.12 \text{ Gyr} \quad h = 0.705 \pm 0.013$$

$$-0.0179 < \Omega_0 - 1 < 0.0081 \quad (95\% \text{ C.L.})$$

$$\Omega_b = 0.0456 \pm 0.0015 \quad \Omega_m = 0.274 \pm 0.013$$

$$\sum_{k=1}^3 m_k < 0.67 \text{ eV} \quad (95\% \text{ C.L.}) \quad N_{\text{eff}} = 4.4 \pm 1.5$$



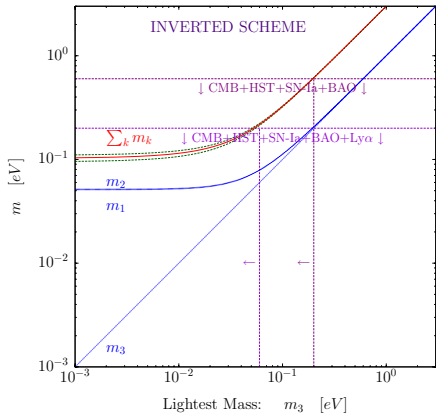
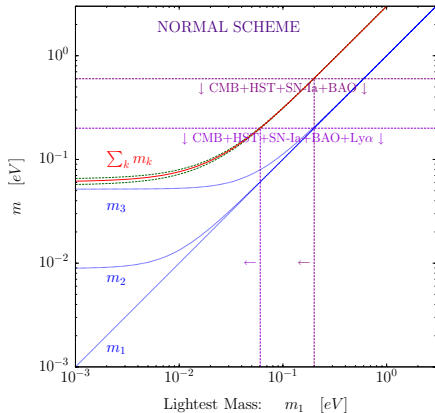
## Flat $\Lambda$ CDM

Case	Cosmological data set	$\Sigma$ (at $2\sigma$ )
1	CMB	$< 1.19$ eV
2	CMB + LSS	$< 0.71$ eV
3	CMB + HST + SN-Ia	$< 0.75$ eV
4	CMB + HST + SN-Ia + BAO	$< 0.60$ eV
5	CMB + HST + SN-Ia + BAO + $\text{Ly}\alpha$	$< 0.19$ eV

$2\sigma$  (95% C.L.) constraints on the sum of  $\nu$  masses  $\Sigma$ .

$$\sum_{k=1}^3 m_k \lesssim 0.6 \text{ eV} \quad (\sim 2\sigma) \quad \text{CMB + HST + SN-Ia + BAO}$$

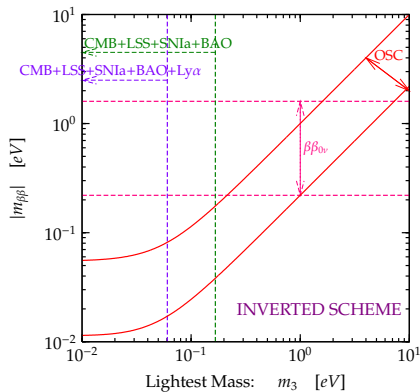
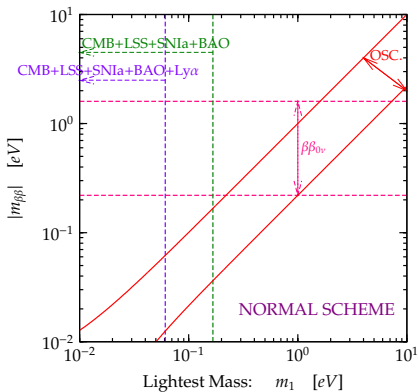
$$\sum_{k=1}^3 m_k \lesssim 0.2 \text{ eV} \quad (\sim 2\sigma) \quad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$



FUTURE: IF  $\sum_{k=1}^3 m_k \lesssim 9 \times 10^{-2} \text{ eV} \implies$  NORMAL HIERARCHY

Indication of  $\beta\beta_{0\nu}$  Decay:  $0.22 \text{ eV} \lesssim |m_{\beta\beta}| \lesssim 1.6 \text{ eV}$  ( $\sim 3\sigma$  range)

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]



tension among oscillation data, CMB+LSS+BAO(+Ly $\alpha$ ) and  $\beta\beta_{0\nu}$  signal

# Experimental Neutrino Anomalies

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay
- Cosmological Bound on Neutrino Masses
- Experimental Neutrino Anomalies
  - LSND
  - MiniBooNE
  - Gallium Radioactive Source Experiments

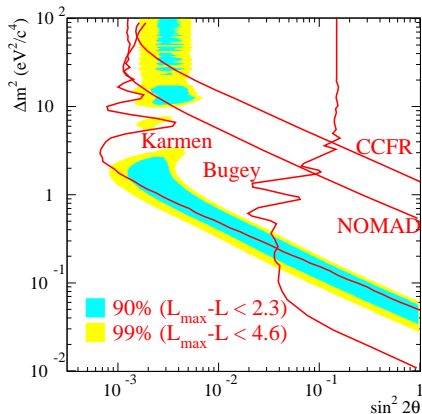
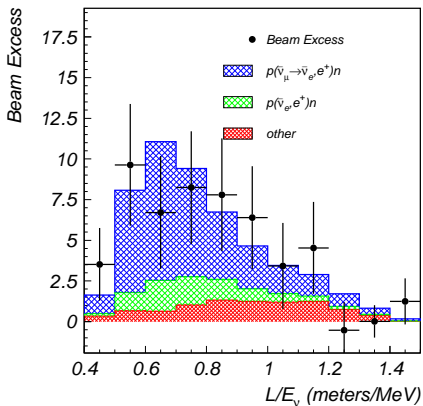
# LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

$$L \simeq 30 \text{ m}$$

$$20 \text{ MeV} \leq E \leq 200 \text{ MeV}$$



$$\Delta m_{\text{LSND}}^2 \gtrsim 0.2 \text{ eV}^2 \quad (\gg \Delta m_{\text{ATM}}^2 \gg \Delta m_{\text{SOL}}^2)$$

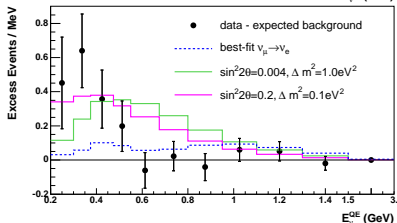
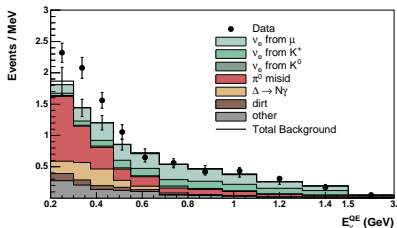
# MiniBooNE

[PRL 98 (2007) 231801]

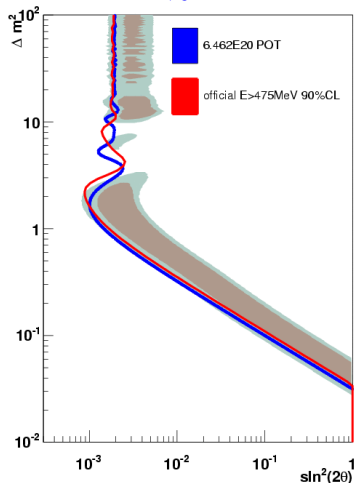
$\nu_\mu \rightarrow \nu_e$

$L \simeq 541 \text{ m}$

$475 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$



[arXiv:0812.2243]



[arXiv:0901.1648]

Low-Energy Anomaly!

# Gallium Radioactive Source Experiments

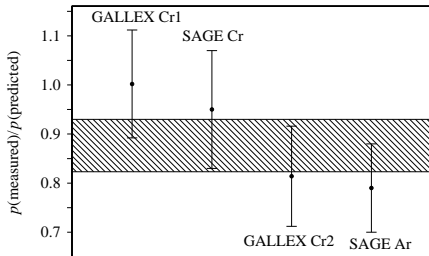
tests of solar neutrino detectors

GALLEX [PLB 342 (1995) 440; PLB 420 (1998) 114]

SAGE [PRL 77 (1996) 4708; PRC 59 (1999) 2246; PRC 73 (2006) 045805; arXiv:0901.2200]

Sources:  $e^- + {}^{51}\text{Cr} \rightarrow {}^{51}\text{V} + \nu_e$        $e^- + {}^{37}\text{Ar} \rightarrow {}^{37}\text{Cl} + \nu_e$

Detector:  $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$



$$R_{\text{Ga}} = 0.87 \pm 0.05$$

[SAGE, arXiv:0901.2200]

[SAGE, PRC 73 (2006) 045805]

## Conclusions

$\nu_e \rightarrow \nu_\mu, \nu_\tau$  with  $\Delta m_{\text{SOL}}^2 \simeq 8.3 \times 10^{-5} \text{ eV}^2$  (SOL, KamLAND)

$\nu_\mu \rightarrow \nu_\tau$  with  $\Delta m_{\text{ATM}}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$  (ATM, K2K, MINOS)



Bilarge  $3\nu$ -Mixing with  $|U_{e3}|^2 \ll 1$  (CHOOZ)

$\beta$  &  $\beta\beta_{0\nu}$  Decay and Cosmology  $\implies m_\nu \lesssim 1 \text{ eV}$

## FUTURE

**Theory:** Why lepton mixing  $\neq$  quark mixing?

(Due to Majorana nature of  $\nu$ 's?)

Why only  $|U_{e3}|^2 \ll 1$ ?

Explain experimental neutrino anomalies (sterile  $\nu$ 's?).

**Exp.:** Measure  $|U_{e3}| > 0 \implies$  CP viol., matter effects, mass hierarchy.

Check experimental neutrino anomalies.

Check  $\beta\beta_{0\nu}$  signal at Quasi-Degenerate mass scale.

Improve  $\beta$  &  $\beta\beta_{0\nu}$  Decay and Cosmology measurements.