Neutrino Physics

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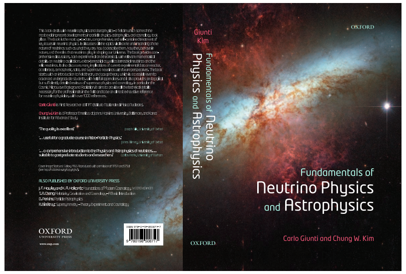


C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics
and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part I: Theory of Neutrino Masses and Mixing

- Dirac Neutrino Masses and Mixing
- Majorana Neutrino Masses and Mixing
- Dirac-Majorana Mass Term
- Number of Flavor and Massive Neutrinos?
- Sterile Neutrinos

Part II: Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

Part III: Phenomenology

- Solar Neutrinos and KamLAND
- Atmospheric and LBL Oscillation Experiments
- Phenomenology of Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

Part II

Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

Neutrino Oscillations in Vacuum

- Neutrino Oscillations in Vacuum
  - Ultrarelativistic Approximation
  - Easy Example of Neutrino Production
  - Neutrino Oscillations
  - Neutrinos and Antineutrinos
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1\,MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{array}{c} \nu + A \to B + C \\ \Downarrow \\ s = 2 E m_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \dfrac{(m_B + m_C)^2}{2 m_A} - \dfrac{m_A}{2} \end{array}}$$

$$\begin{array}{ll} \nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\ \nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\ \bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\ \nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\ \nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2 m_e} = 10.9\,\text{GeV} \end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO) , $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Easy Example of Neutrino Production

$$\underline{\pi^+ \to \mu^+ + \nu_\mu} \qquad \boxed{\nu_\mu = \sum_k U_{\mu k} \nu_k}$$

two-body decay $\Longrightarrow$ fixed kinematics $\qquad \boxed{E_k^2 = p_k^2 + m_k^2}$

$$\pi \text{ at rest: } \begin{cases} p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\, m_\pi^2} \\ E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\, m_\pi^2} \end{cases}$$

$$0^{\text{th}} \text{ order: } m_k = 0 \Rightarrow p_k = E_k = E = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$$

$$1^{\text{st}} \text{ order: } \boxed{E_k \simeq E + \xi\, \frac{m_k^2}{2E}} \qquad \boxed{p_k \simeq E - (1 - \xi)\, \frac{m_k^2}{2E}}$$

$$\xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

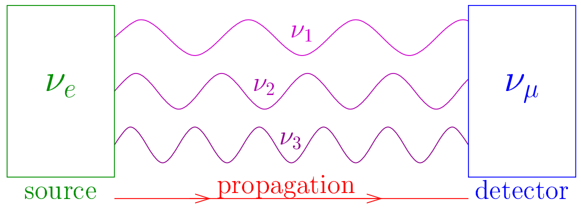
Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955)
- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector
- A Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{aligned} |\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\ |\nu_\mu\rangle &= U_{\mu 1}\,|\nu_1\rangle + U_{\mu 2}\,|\nu_2\rangle + U_{\mu 3}\,|\nu_3\rangle \\ |\nu_\tau\rangle &= U_{\tau 1}\,|\nu_1\rangle + U_{\tau 2}\,|\nu_2\rangle + U_{\tau 3}\,|\nu_3\rangle \end{aligned}$$

- U is the 3×3 Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

at the detector there is a probability > 0 to see the neutrino as a ν_μ

Neutrino Oscillations are Flavor Transitions

$$\nu_e \to \nu_\mu \qquad \nu_e \to \nu_\tau \qquad \nu_\mu \to \nu_e \qquad \nu_\mu \to \nu_\tau$$

$$\bar\nu_e \to \bar\nu_\mu \qquad \bar\nu_e \to \bar\nu_\tau \qquad \bar\nu_\mu \to \bar\nu_e \qquad \bar\nu_\mu \to \bar\nu_\tau$$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{CC} \sim W_\rho \left(\overline{\nu_{eL}}\gamma^\rho e_L + \overline{\nu_{\mu L}}\gamma^\rho \mu_L + \overline{\nu_{\tau L}}\gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k}\nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k}|\nu_k\rangle \quad$ States

initial flavor: $\alpha = e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + ip_k x}\,|\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U^*_{\alpha k}\,e^{-iE_k t + ip_k x}\,|\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k}|\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha\to\nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha\to\nu_\beta}(0,0) = \sum_k U^*_{\alpha k} U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha\to\nu_\beta}(t>0, x>0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha\to\nu_\beta}(t,x) = \left| \mathcal{A}_{\nu_\alpha\to\nu_\beta}(t,x) \right|^2 = \left| \sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\Longrightarrow$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k)\,L = \frac{E_k^2 - p_k^2}{E_k + p_k}\,L = \frac{m_k^2}{E_k + p_k}\,L \simeq \frac{m_k^2}{2E}\,L$$

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta}(L,E) &= \left| \sum_k U^*_{\alpha k}\,e^{-im_k^2 L/2E}\,U_{\beta k} \right|^2 \\ &= \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{kj} L}{2E} \right) \end{aligned}$$

$$\Delta m^2_{kj} \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathrm{CP}} = \gamma^0 \mathcal{C} \overline{\nu}^T = -\mathcal{C} \nu^*$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\mathrm{CP}} \nu_{\alpha L}^{\mathrm{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathrm{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathrm{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $U \leftrightarrows U^*$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT, CP and T Symmetries

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\mathrm{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A_{\alpha\beta}^{\mathrm{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\Longrightarrow$ $A_{\alpha\beta}^{\mathrm{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $U \leftrightarrows U^*$ $\alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\mathrm{CP}} P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A_{\alpha\beta}^{\mathrm{CP}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$ $\boxed{\mathrm{CPT} \Rightarrow A_{\alpha\beta}^{\mathrm{CP}} = -A_{\beta\alpha}^{\mathrm{CP}}}$

$$\boxed{A_{\alpha\beta}^{\mathrm{CP}}(L, E) = 4 \sum_{k>j} \mathrm{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)}$$

Jarlskog rephasing invariant: $\mathrm{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^*\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$$

violation of CP in neutrino oscillations is proportional to

$|U_{e3}| = \sin\vartheta_{13}$ and $\sin\delta_{13}$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\mathsf{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A^{\mathrm{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\begin{aligned}
\mathrm{CPT} \quad \Longrightarrow \quad 0 = A^{\mathrm{CPT}}_{\alpha\beta} &= P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\
&= A^{\mathrm{T}}_{\alpha\beta} + A^{\mathrm{CP}}_{\beta\alpha} = A^{\mathrm{T}}_{\alpha\beta} - A^{\mathrm{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\mathrm{T}}_{\alpha\beta} = A^{\mathrm{CP}}_{\alpha\beta}}
\end{aligned}$$

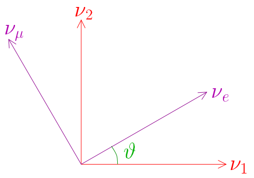
$$\boxed{A^{\mathrm{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\quad \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

Two-Neutrino Oscillations

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k} |\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m^2_2 - m^2_1$$

Transition Probability: $\quad P_{\nu_e \to \nu_\mu} = P_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

Survival Probabilities: $\quad P_{\nu_e \to \nu_e} = P_{\nu_\mu \to \nu_\mu} = 1 - P_{\nu_e \to \nu_\mu}$

two-neutrino mixing transition probability

$$\alpha \neq \beta \qquad \alpha, \beta = e, \mu, \tau$$

$$\begin{aligned}
P_{\nu_\alpha \to \nu_\beta}(L,E) &= \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2 [\mathrm{eV}^2] \, L[\mathrm{m}]}{E[\mathrm{MeV}]}\right) \\
&= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2 [\mathrm{eV}^2] \, L[\mathrm{km}]}{E[\mathrm{GeV}]}\right)
\end{aligned}$$

oscillation length

$$L^{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \, \frac{E\,[\mathrm{MeV}]}{\Delta m^2\,[\mathrm{eV}^2]} \, \mathrm{m} = 2.47 \, \frac{E\,[\mathrm{GeV}]}{\Delta m^2\,[\mathrm{eV}^2]} \, \mathrm{km}$$

Types of Experiments

Two-Neutrino Mixing $\boxed{P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)}$ observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL
$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$
Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$
Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL
$L/E \lesssim 10^4\,\text{eV}^{-2}$
$\Downarrow$
$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$
Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE
Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS
Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$
Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

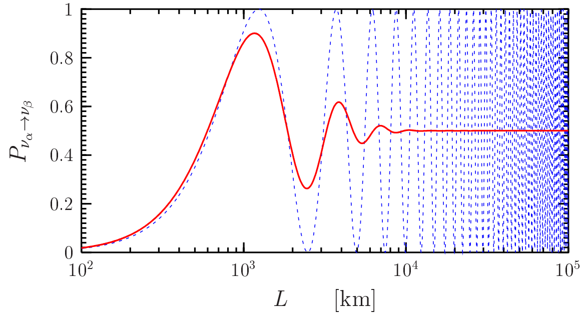
SUN
$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$
$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$
Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$
Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$
KamLAND

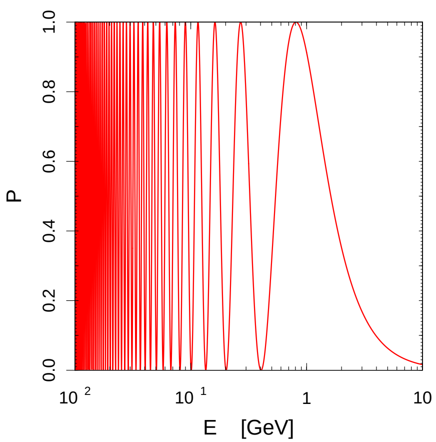
Average over Energy Resolution of the Detector

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\sin^2 2\vartheta\left[1-\cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$



$\Delta m^2 = 10^{-3}\,\text{eV}$ $\quad \sin^2 2\vartheta = 1$ $\quad \langle E\rangle = 1\,\text{GeV}$ $\quad \Delta E = 0.2\,\text{GeV}$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha\neq\beta)$$

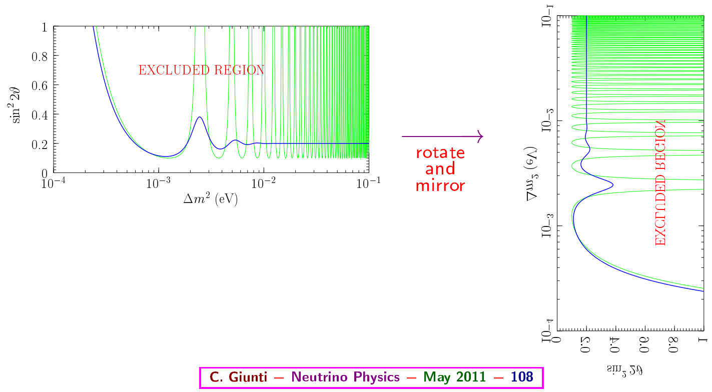


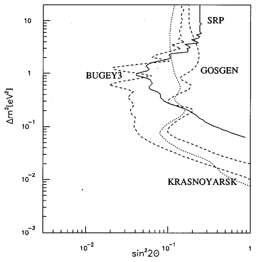
$\Delta m^2 = 1\,\text{eV}$ $\quad \sin^2 2\vartheta = 1$ $\quad L = 1\,\text{km}$

Exclusion Curves

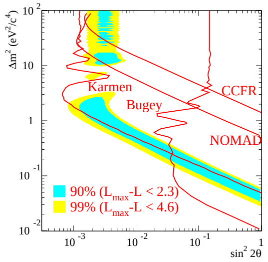
$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle = \frac{1}{2}\sin^2 2\vartheta\left[1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E\right] \qquad (\alpha\neq\beta)$$

$$\langle P_{\nu_\alpha\to\nu_\beta}(L,E)\rangle \leq P^{\text{max}}_{\nu_\alpha\to\nu_\beta} \quad\Longrightarrow\quad \sin^2 2\vartheta \leq \frac{2\,P^{\text{max}}_{\nu_\alpha\to\nu_\beta}}{1-\int\cos\left(\frac{\Delta m^2 L}{2E}\right)\phi(E)\,\mathrm{d}E}$$

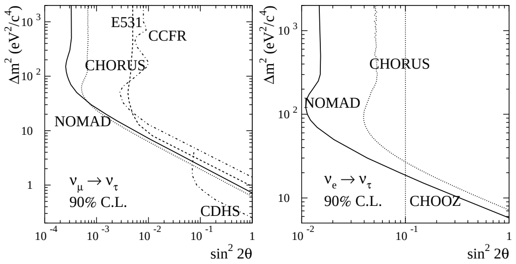




Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$

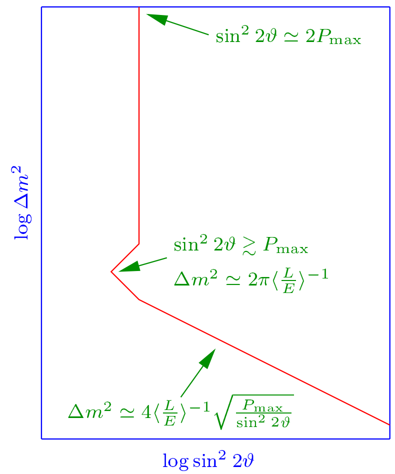


Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$



Accelerator SBL Experiments: $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\tau$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau$

Anatomy of Exclusion Plots



$\frac{1}{2}\sin^2 2\vartheta \left[1 - \left\langle \cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle\right] = P_{\max}$

- $\Delta m^2 \gg \langle L/E \rangle^{-1}$

  $P_{\max} \simeq \frac{1}{2}\sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P_{\max}$

- $\text{Min}\left\langle \cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle \geq -1$

  $\sin^2 2\vartheta = \frac{2\,P_{\max}}{1-\text{Min}\left\langle\cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle} \geq P_{\max}$

  $\Delta m^2 \simeq 2\pi\langle L/E\rangle^{-1}$

- $\Delta m^2 \ll 2\pi\langle L/E\rangle^{-1}$

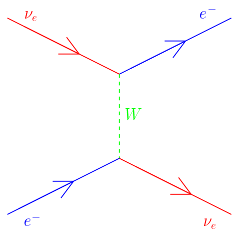
  $\cos\left(\frac{\Delta m^2 L}{2E}\right) \simeq 1 - \frac{1}{2}\left(\frac{\Delta m^2 L}{2E}\right)^2$

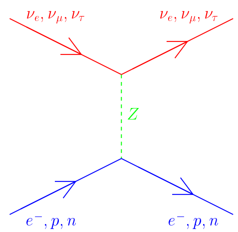
  $\Delta m^2 \simeq 4\left\langle\frac{L}{E}\right\rangle^{-1}\sqrt{\frac{P_{\max}}{\sin^2 2\vartheta}}$

Neutrino Oscillations in Matter

- Neutrino Oscillations in Vacuum
- CPT, CP and T Symmetries
- Two-Neutrino Oscillations
- Neutrino Oscillations in Matter
  - Effective Potentials in Matter
  - Evolution of Neutrino Flavors in Matter
  - MSW Effect (Resonant Transitions in Matter)
  - Solar Neutrinos
  - In Neutrino Oscillations Dirac = Majorana

Effective Potentials in Matter





$\boxed{V_{CC} = \sqrt{2}G_F N_e}$ $\quad V_{NC}^{(e^-)} = -V_{NC}^{(p)} \Rightarrow$ $\boxed{V_{NC} = V_{NC}^{(n)} = -\frac{\sqrt{2}}{2}G_F N_n}$

$V_e = V_{CC} + V_{NC} \qquad V_\mu = V_\tau = V_{NC}$

only $V_{CC} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\quad \overline{V}_{CC} = -V_{CC} \qquad \overline{V}_{NC} = -V_{NC}$

Matter Effects

a flavor neutrino ν_α with momentum p is described by

$$|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* \, |\nu_k(p)\rangle$$

$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

in matter $\quad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$

V_α = effective potential due to coherent interactions with the medium

forward elastic CC and NC scattering

Evolution of Neutrino Flavors in Matter

Schrödinger picture: $\quad i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle \,, \qquad |\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$

flavor transition amplitudes: $\quad \varphi_\beta(p,t) = \langle \nu_\beta(p) | \nu(p,t) \rangle \,, \qquad \varphi_\beta(p,0) = \delta_{\alpha\beta}$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu(p,t) \rangle = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle$$

$$\begin{aligned} \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle &= \sum_\rho \langle \nu_\beta(p) | \mathcal{H}_0 | \nu_\rho(p) \rangle \underbrace{\langle \nu_\rho(p) | \nu(p,t) \rangle}_{\varphi_\rho(p,t)} \\ &= \sum_\rho \sum_{k,j} U_{\beta k} \underbrace{\langle \nu_k(p) | \mathcal{H}_0 | \nu_j(p) \rangle}_{\delta_{kj} E_k} U_{\rho j}^* \, \varphi_\rho(p,t) \end{aligned}$$

$$\langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle = \sum_\rho \underbrace{\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\rho(p) \rangle}_{\delta_{\beta\rho} V_\beta} \varphi_\rho(p,t) = V_\beta \, \varphi_\beta(p,t)$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \, E_k \, U_{\rho k}^* + \delta_{\beta\rho} \, V_\beta \right) \varphi_\rho}$$

ultrarelativistic neutrinos: $\quad E_k = p + \frac{m_k^2}{2E} \qquad E = p \qquad t = x$

$$V_e = V_{\mathrm{CC}} + V_{\mathrm{NC}} \qquad V_\mu = V_\tau = V_{\mathrm{NC}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \varphi_\beta(p,x) = (p + V_{\mathrm{NC}}) \, \varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \, \delta_{\rho e} \, V_{\mathrm{CC}} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x) \, e^{ipx + i \int_0^x V_{\mathrm{NC}}(x') \, \mathrm{d}x'}$$

$$\Downarrow$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = e^{ipx + i \int_0^x V_{\mathrm{NC}}(x') \, \mathrm{d}x'} \left(-p - V_{\mathrm{NC}} + i \frac{\mathrm{d}}{\mathrm{d}x} \right) \varphi_\beta$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e} \, \delta_{\rho e} \, V_{\mathrm{CC}} \right) \psi_\rho}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2}$$

evolution of flavor transition amplitudes in matrix form

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2} E G_{\mathrm{F}} N_e$$

effective mass-squared matrix in vacuum $\quad \mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \underset{\substack{\uparrow \\ \text{potential due to coherent} \\ \text{forward elastic scattering}}}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}} \quad$ effective mass-squared matrix in matter

Two-Neutrino Mixing

$\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta\, m_1^2 + \sin^2\vartheta\, m_2^2 & \cos\vartheta \sin\vartheta\, (m_2^2 - m_1^2) \\ \cos\vartheta \sin\vartheta\, (m_2^2 - m_1^2) & \sin^2\vartheta\, m_1^2 + \cos^2\vartheta\, m_2^2 \end{pmatrix}$$

$$= \underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\frac{1}{2}\Sigma m^2} + \frac{1}{2}\begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}}$$

initial $\nu_e \quad \Longrightarrow \quad \begin{pmatrix} \psi_e(0) \\ \psi_\mu(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\boxed{\begin{aligned} &P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 \\ &P_{\nu_e\to\nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x) \end{aligned}}$$

Constant Matter Density

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x} = 0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}} \\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}} \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\underset{\substack{\uparrow \\ \text{irrelevant common phase}}}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E}\begin{pmatrix} -\Delta m_{\mathrm{M}}^2 & 0 \\ 0 & \Delta m_{\mathrm{M}}^2 \end{pmatrix}\right]\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m_{\mathrm{M}}^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A_{\mathrm{CC}}^{\mathrm{R}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N_e^{\mathrm{R}} = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2}EG_{\mathrm{F}}}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\text{M}} & 0\\ 0 & \Delta m^2_{\text{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\text{M}} & \sin\vartheta_{\text{M}}\\ -\sin\vartheta_{\text{M}} & \cos\vartheta_{\text{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \Rightarrow \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\text{M}} & -\sin\vartheta_{\text{M}}\\ \sin\vartheta_{\text{M}} & \cos\vartheta_{\text{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1(0)\\ \psi_2(0)\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\text{M}}\\ \sin\vartheta_{\text{M}}\end{pmatrix}$$

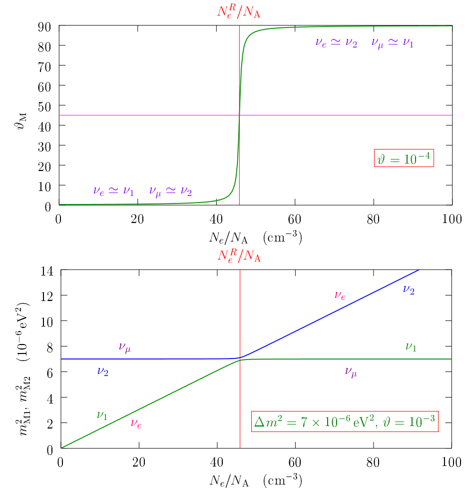
$$\psi_1(x) = \cos\vartheta_{\text{M}} \exp\left(i\frac{\Delta m^2_{\text{M}} x}{4E}\right)$$

$$\psi_2(x) = \sin\vartheta_{\text{M}} \exp\left(-i\frac{\Delta m^2_{\text{M}} x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 = |-\sin\vartheta_{\text{M}}\psi_1(x) + \cos\vartheta_{\text{M}}\psi_2(x)|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\vartheta_{\text{M}} \, \sin^2\left(\frac{\Delta m^2_{\text{M}} x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)



$$\boxed{\begin{aligned}\nu_e &= \cos\vartheta_{\text{M}}\,\nu_1 + \sin\vartheta_{\text{M}}\,\nu_2\\ \nu_\mu &= -\sin\vartheta_{\text{M}}\,\nu_1 + \cos\vartheta_{\text{M}}\,\nu_2\end{aligned}}$$

$$\boxed{\tan 2\vartheta_{\text{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\text{CC}}}{\Delta m^2 \cos 2\vartheta}}}$$

$$\boxed{\Delta m^2_{\text{M}} = \left[\left(\Delta m^2\cos 2\vartheta - A_{\text{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2\right]^{1/2}}$$

$$\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\text{M}} & \sin\vartheta_{\text{M}}\\ -\sin\vartheta_{\text{M}} & \cos\vartheta_{\text{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\underbrace{\frac{A_{\text{CC}}}{4E}}_{\text{irrelevant common phase}} + \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\text{M}} & 0\\ 0 & \Delta m^2_{\text{M}}\end{pmatrix} + \underbrace{\begin{pmatrix}0 & -i\dfrac{d\vartheta_{\text{M}}}{dx}\\ i\dfrac{d\vartheta_{\text{M}}}{dx} & 0\end{pmatrix}}_{\text{maximum near resonance}}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_1(0)\\ \psi_2(0)\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\text{M}} & -\sin\vartheta^0_{\text{M}}\\ \sin\vartheta^0_{\text{M}} & \cos\vartheta^0_{\text{M}}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\text{M}}\\ \sin\vartheta^0_{\text{M}}\end{pmatrix}$$

$$\begin{aligned}\psi_1(x) \simeq{}& \left[\cos\vartheta^0_{\text{M}} \exp\left(i\int_0^{x_R}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\mathcal{A}^{\text{R}}_{11} + \sin\vartheta^0_{\text{M}} \exp\left(-i\int_0^{x_R}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\mathcal{A}^{\text{R}}_{21}\right]\\ &\times \exp\left(i\int_{x_R}^{x}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\\ \psi_2(x) \simeq{}& \left[\cos\vartheta^0_{\text{M}} \exp\left(i\int_0^{x_R}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\mathcal{A}^{\text{R}}_{12} + \sin\vartheta^0_{\text{M}} \exp\left(-i\int_0^{x_R}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\mathcal{A}^{\text{R}}_{22}\right]\\ &\times \exp\left(-i\int_{x_R}^{x}\frac{\Delta m^2_{\text{M}}(x')}{4E}\,dx'\right)\end{aligned}$$

Averaged Survival Probability

$$\psi_e(x) = \cos\vartheta^x_{\text{M}}\,\psi_1(x) + \sin\vartheta^x_{\text{M}}\,\psi_2(x)$$

neglect interference (averaged over energy spectrum)

$$\begin{aligned}\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 ={}& \cos^2\vartheta^x_{\text{M}}\,\cos^2\vartheta^0_{\text{M}}\,|\mathcal{A}^{\text{R}}_{11}|^2 + \cos^2\vartheta^x_{\text{M}}\,\sin^2\vartheta^0_{\text{M}}\,|\mathcal{A}^{\text{R}}_{21}|^2\\ &+ \sin^2\vartheta^x_{\text{M}}\,\cos^2\vartheta^0_{\text{M}}\,|\mathcal{A}^{\text{R}}_{12}|^2 + \sin^2\vartheta^x_{\text{M}}\,\sin^2\vartheta^0_{\text{M}}\,|\mathcal{A}^{\text{R}}_{22}|^2\end{aligned}$$

conservation of probability (unitarity)

$$|\mathcal{A}^{\text{R}}_{12}|^2 = |\mathcal{A}^{\text{R}}_{21}|^2 = P_{\text{c}} \qquad |\mathcal{A}^{\text{R}}_{11}|^2 = |\mathcal{A}^{\text{R}}_{22}|^2 = 1 - P_{\text{c}}$$

$P_{\text{c}} \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{c}}\right)\cos 2\vartheta^0_{\text{M}}\,\cos 2\vartheta^x_{\text{M}}}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $\gamma = \left.\frac{\Delta m^2_M/2E}{2|d\vartheta_M/dx|}\right|_R = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{CC}}{dx}\right|_R}$

$A \propto x$ — $F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

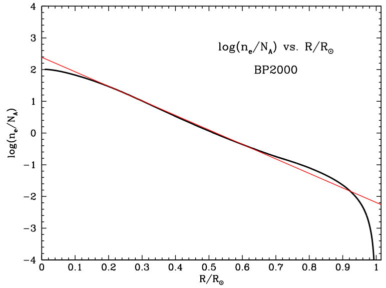
$A \propto 1/x$ — $F = \left(1-\tan^2\vartheta\right)^2 / \left(1+\tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ — $F = 1 - \tan^2\vartheta$ [Pizzochero, PRD 36 (1987) 2293] [Toshev, PLB 196 (1987) 170] [Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $N_e(x) \simeq N_e^c \exp\left(-\frac{x}{x_0}\right)$ $\quad N_e^c = 245\, N_A/\mathrm{cm}^3$ $\quad x_0 = \frac{R_\odot}{10.54}$



$$\overline{P}^{\text{sun}}_{\nu_e\to\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta^0_M \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{CC}}{dx}\right|_R}$$

$$F = 1 - \tan^2\vartheta$$

$$A_{CC} = 2\sqrt{2} E G_F N_e$$

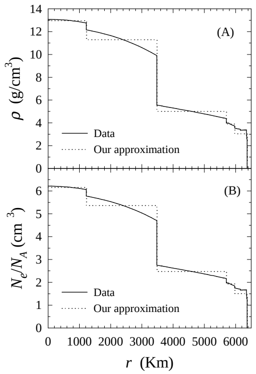
practical prescription: [Lisi et al., PRD 63 (2001) 093002]

$$\begin{cases} \text{numerical } |d\ln A_{CC}/dx|_R & \text{for } x \leq 0.904 R_\odot \\ |d\ln A_{CC}/dx|_R \to \frac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{cases}$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e\to\nu_e} = \overline{P}^{\text{sun}}_{\nu_e\to\nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e\to\nu_e}\right)\left(P^{\text{earth}}_{\nu_2\to\nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]



[Giunti, Kim, Monteno, NP B 521 (1998) 3]

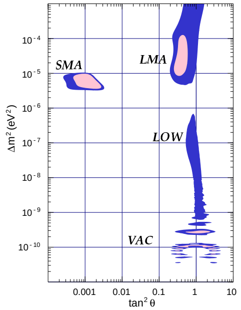
$P^{\text{earth}}_{\nu_2\to\nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

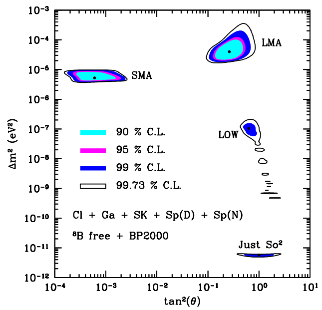
Wave functions of flavor neutrinos are joined at the boundaries of steps.

Phenomenology of Solar Neutrinos

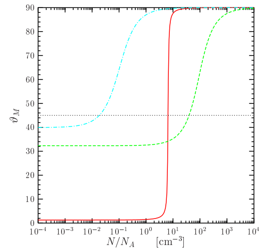
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5\times10^{-5}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7\times10^{-8}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5\times10^{-6}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5\times10^{-10}\,\mathrm{eV}^2$,	$\tan^2\vartheta \sim 1$



[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]



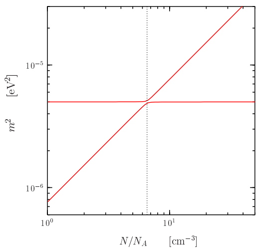
[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]



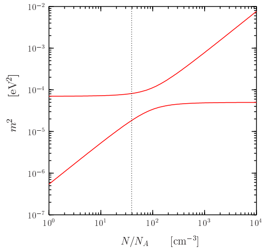
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\mathrm{eV}^2$
(typical SMA) $\tan^2\vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\mathrm{eV}^2$
(typical LMA) $\tan^2\vartheta = 0.4$

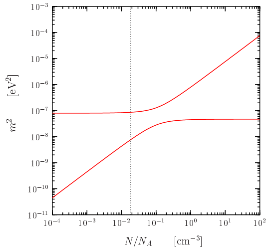
dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\mathrm{eV}^2$
(typical LOW) $\tan^2\vartheta = 0.7$



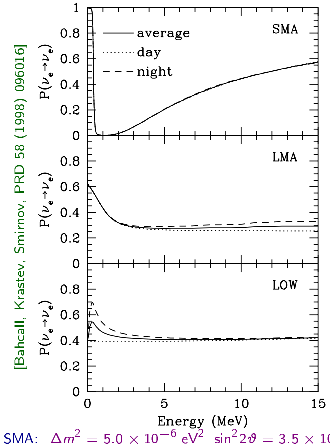
typical SMA



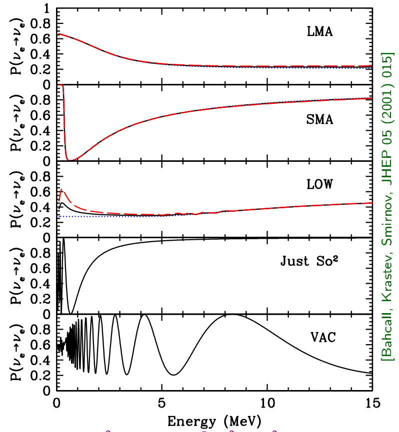
typical LMA



typical LOW



SMA: $\Delta m^2 = 5.0 \times 10^{-6}\,\mathrm{eV}^2$ $\sin^2 2\vartheta = 3.5 \times 10^{-3}$
LMA: $\Delta m^2 = 1.6 \times 10^{-5}\,\mathrm{eV}^2$ $\sin^2 2\vartheta = 0.57$
LOW: $\Delta m^2 = 7.9 \times 10^{-8}\,\mathrm{eV}^2$ $\sin^2 2\vartheta = 0.95$



LMA: $\Delta m^2 = 4.2 \times 10^{-5}\,\mathrm{eV}^2$ $\tan^2\vartheta = 0.26$
SMA: $\Delta m^2 = 5.2 \times 10^{-6}\,\mathrm{eV}^2$ $\tan^2\vartheta = 5.5 \times 10^{-4}$
LOW: $\Delta m^2 = 7.6 \times 10^{-8}\,\mathrm{eV}^2$ $\tan^2\vartheta = 0.72$
Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}\,\mathrm{eV}^2$ $\tan^2\vartheta = 1.0$
VAC: $\Delta m^2 = 1.4 \times 10^{-10}\,\mathrm{eV}^2$ $\tan^2\vartheta = 0.38$

In Neutrino Oscillations Dirac = Majorana

Evolution of Amplitudes: $$i\,\frac{d\psi_\alpha}{dx} = \frac{1}{2E}\sum_\beta \left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\psi_\beta$$

difference: $\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2 D \quad \Longrightarrow \quad DM^2D^\dagger = M^2$$

$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$