Neutrino Physics

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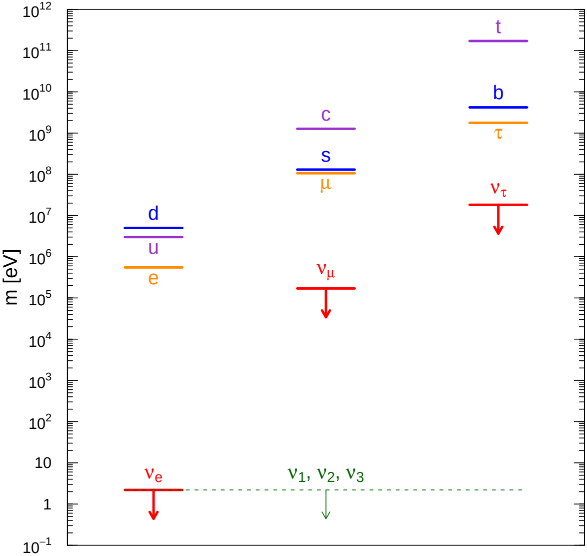
- **Part I: Theory of Neutrino Masses and Mixing**
- **Part II: Neutrino Oscillations**
- **Part III: Phenomenology**

Part I

Theory of Neutrino Masses and Mixing

- Dirac Masses and Mixing
- Dirac-Majorana Mass Term
- See-Saw Mechanism
- Effective Majorana Three-Neutrino Mixing

Fermion Mass Spectrum



Dirac Masses and Mixing

- Dirac Equation: $(i\partial\!\!\!/ - m)\,\nu(x) = 0 \qquad (\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu)$

- Dirac Lagrangian: $\mathscr{L}(x) = \overline{\nu}(x)\,(i\partial\!\!\!/ - m)\,\nu(x)$

- Chiral decomposition: $\nu_L \equiv P_L \nu\,, \qquad \nu_R \equiv P_R \nu\,, \qquad \nu = \nu_L + \nu_R$

$$P_L \equiv \frac{1-\gamma^5}{2}\,, \quad P_R \equiv \frac{1+\gamma^5}{2}$$

$$P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L + P_R = 1\,, \quad P_L P_R = P_R P_L = 0$$

$$\mathscr{L} = \overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_R} i\partial\!\!\!/ \nu_R - m\,(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

- In SM only $\nu_L \Longrightarrow$ no Dirac mass

- Oscillation experiments have shown that neutrinos are massive

- Simplest extension of the SM: add ν_R

Extended Standard Model

$(\alpha = e, \mu, \tau)$		I	I_3	Y	Q
lepton doublets	$L_{\alpha L} \equiv \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
charged-lepton singlets	α_R	0	0	-2	-1
neutrino singlets	$\nu_{\alpha R}$	0	0	0	0
Higgs doublet	$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

Lepton-Higgs Yukawa Lagrangian

$$\mathscr{L}_{H,\mathrm{L}} = - \sum_{\alpha,\beta=e,\mu,\tau} \left[Y^{\ell}_{\alpha\beta} \, \overline{L_{\alpha L}} \, \Phi \, \beta_R + Y^{\nu}_{\alpha\beta} \, \overline{L_{\alpha L}} \, \widetilde{\Phi} \, \nu_{\beta R} \right] + \mathrm{H.c.}$$

Electroweak Symmetry Breaking

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \qquad \widetilde{\Phi} = i\sigma_2 \, \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

Dirac Mass Term

$$\mathscr{L}_{\mathrm{D}} = -\sum_{\alpha,\beta=e,\mu,\tau} M^{\mathrm{D}}_{\alpha\beta}\,\overline{\nu_{\alpha L}}\,\nu_{\beta R} + \mathrm{H.c.} \qquad M^{\mathrm{D}}_{\alpha\beta} = \frac{v}{\sqrt{2}}\,Y_{\alpha\beta}$$

$v = \left(\sqrt{2}\,G_{\mathrm{F}}\right)^{-1/2} = 246\,\mathrm{GeV}$ complex 3×3 Dirac mass matrix

$$\mathscr{L}_{\mathrm{D}} = -\begin{pmatrix} \overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}} \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} + \mathrm{H.c.}$$

L_e, L_μ, L_τ are not conserved

L is conserved: $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Mixing

$$N_L^T = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}) \qquad N_R^T = (\nu_{eR}, \nu_{\mu R}, \nu_{\tau R})$$

$$\mathscr{L}_{\text{D}} = -\overline{N_L}\, M^{\text{D}}\, N_R + \text{H.c.}$$

Diagonalization: unitary transformation

$$N_L = U_L n_L \qquad N_R = U_R n_R$$

$$n_L^T = (\nu_{1L}, \nu_{2L}, \nu_{3L}) \qquad n_R^T = (\nu_{1R}, \nu_{2R}, \nu_{3R})$$

$$U_L^\dagger M^{\text{D}} U_R = M \qquad M_{kj}^{\text{D}} = m_k \delta_{kj}$$

$$\mathscr{L}_{\text{D}} = -\sum_k m_k\, \overline{\nu_{kL}}\, \nu_{kR} + \text{H.c.}$$

Mixing Matrix

- $U = U_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

- Unitary $N \times N$ matrix depends on N^2 independent real parameters

$$N = 3 \qquad \Longrightarrow \qquad \begin{array}{ll} \dfrac{N(N-1)}{2} = 3 & \quad \text{Mixing Angles} \\ \dfrac{N(N+1)}{2} = 6 & \quad \text{Phases} \end{array}$$

- Not all phases are physical observables

- Only physical effect of mixing matrix occurs through its presence in the Leptonic Weak Charged Current

- Weak Charged Current: $j_W^\rho = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, U_{\alpha k}^* \, \gamma^\rho \, \ell_{\alpha L}$
- Apart from the Weak Charged Current, the Lagrangian is invariant under the global phase transformations (6 arbitrary phases)
  $$\nu_k \to e^{i\varphi_k} \, \nu_k \quad (k = 1,2,3) \, , \qquad \ell_\alpha \to e^{i\varphi_\alpha} \, \ell_\alpha \quad (\alpha = e, \mu, \tau)$$
- Performing this transformation, the Charged Current becomes

$$j_W^\rho = 2 \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, e^{-i\varphi_k} \, U_{\alpha k}^* \, e^{i\varphi_\alpha} \, \gamma^\rho \, \ell_{\alpha L}$$

$$j_W^\rho = 2 \, \underbrace{e^{-i(\varphi_1 - \varphi_e)}}_{1} \sum_{k=1}^{3} \sum_{\alpha=e,\mu,\tau} \overline{\nu_{kL}} \, \underbrace{e^{-i(\varphi_k - \varphi_1)}}_{2} \, U_{\alpha k}^* \, \underbrace{e^{i(\varphi_\alpha - \varphi_e)}}_{2} \, \gamma^\rho \, \ell_{\alpha L}$$

- There are 5 arbitrary phases of the fields that can be chosen to eliminate 5 of the 6 phases of the mixing matrix
- 5 and not 6 phases of the mixing matrix can be eliminated because a common rephasing of all the fields leaves the Charged Current invariant $\Longleftrightarrow$ conservation of Total Lepton Number.

- The mixing matrix contains 1 Physical Phase.
- It is convenient to express the 3×3 unitary mixing matrix only in terms of the four physical parameters:

3 Mixing Angles and 1 Phase

Standard Parameterization of Mixing Matrix

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $s_{ab} \equiv \sin\vartheta_{ab}$ $0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $0 \leq \delta_{13} \leq 2\pi$

3 Mixing Angles ϑ_{12}, ϑ_{23}, ϑ_{13} and 1 Phase δ_{13}

CP Violation

- Parameterization invariants: $|U_{\alpha k}|^2 = U_{\alpha k} U_{\alpha k}^*\,, \quad U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}$

$$\mathfrak{Im}\big[U_{\alpha k} U_{\alpha j}^* U_{\beta k}^* U_{\beta j}\big] = \pm J$$

- Jarlskog invariant: $J = \mathfrak{Im}\big[U_{e2} U_{e3}^* U_{\mu 2}^* U_{\mu 3}\big]$
- In standard parameterization: $J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin\delta_{13}$
- Jarlskog invariant is useful for quantifying CP violation in a parameterization-independent way.
- All measurable CP-violation effects depend on J.

Dirac-Majorana Mass Term

- Dirac mass term

$$\mathscr{L}_{\mathrm{D}} = - \sum_{\alpha,\beta=e,\mu,\tau} M^{\mathrm{D}}_{\alpha\beta} \, \overline{\nu_{\alpha L}} \, \nu_{\beta R} + \text{H.c.}$$

- Singlet Majorana mass term allowed by $SU(2)_L \times U(1)_Y$

$$\mathscr{L}_R = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\alpha R} \, \mathcal{C}^\dagger \, M^R_{\alpha\beta} \, \nu_{\beta R} + \text{H.c.}$$

complex symmetric 3 × 3 Majorana mass matrix

- Forbidden Triplet Majorana mass term ($I_3 = 1$ and $Y = -2$)

$$\mathscr{L}_L = \frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu^T_{\alpha L} \, \mathcal{C}^\dagger \, M^L_{\alpha\beta} \, \nu_{\beta L} + \text{H.c.}$$

- Dirac-Majorana mass term

$$\mathscr{L}_{\mathrm{D+M}} = \mathscr{L}_{\mathrm{D}} + \mathscr{L}_R$$

- $N_L^T \equiv \left(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \nu_{eR}^C, \nu_{\mu R}^C, \nu_{\tau R}^C\right)$ $\qquad$ $\nu_{\alpha R}^C = \mathcal{C}\overline{\nu_{\alpha R}}^T$,

- Dirac-Majorana mass term
$$\mathscr{L}_{\text{D+M}} = \frac{1}{2} N_L^T \, \mathcal{C}^\dagger \, M^{\text{D+M}} \, N_L + \text{H.c.}$$

- Complex symmetric 6×6 Dirac-Majorana mass matrix
$$M^{\text{D+M}} = \begin{pmatrix} 0 & M^{\text{D}\,T} \\ M^{\text{D}} & M^R \end{pmatrix}$$

- Diagonalization: unitary transformation
$$N_L = V n_L \qquad\qquad n_L^T = (\nu_{1L}, \ldots, \nu_{6L})$$
$$V^T M^{\text{D+M}} V = M \qquad\qquad M_{kj} = m_k \delta_{kj}$$

- Dirac-Majorana mass term $n_L^T = (\nu_{1L}, \ldots, \nu_{6L})$

$$\begin{aligned}\mathscr{L}_{\text{D+M}} &= \frac{1}{2}\, n_L^T\, \mathcal{C}^\dagger\, M\, n_L + \text{H.c.} \\ &= \frac{1}{2}\sum_{k=1}^{6} m_k\, \nu_{kL}^T\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.} \\ &= \frac{1}{2}\sum_{k=1}^{6} m_k\, \nu_k^T\, \mathcal{C}^\dagger\, \nu_k\end{aligned}$$

- Massive Majorana neutrino fields

$$\nu_k = \nu_{kL} + \mathcal{C}\overline{\nu_{kL}}^T \qquad \nu_k = \mathcal{C}\overline{\nu_k}^T = \nu_k^C$$

- A Majorana field has half the degrees of freedom of a Dirac field

neutrino = antineutrino

- Total Lepton Number is not conserved: $\boxed{\Delta L = \pm 2}$

See-Saw Mechanism

[Minkowski, PLB 67 (1977) 42]

[Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

- Dirac-Majorana mass matrix: $M^{\text{D+M}} = \begin{pmatrix} 0 & {M^{\text{D}}}^T \\ M^{\text{D}} & M^R \end{pmatrix}$

- $M^{\text{D}} \lesssim 100\,\text{GeV}$ generated by electroweak symmetry breaking

- M^R can be arbitrarily large (not protected by SM symmetries): scale of new physics beyond Standard Model

$$M^R \gg M^{\text{D}}$$

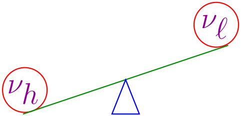
- Approximate diagonalization by blocks

$$W^T \, M^{\text{D+M}} \, W = \begin{pmatrix} M^\ell & 0 \\ 0 & M^h \end{pmatrix} + \text{O}\Big[(M^R)^{-1} M^{\text{D}}\Big]$$

$$M^{\ell} \simeq {M^{\mathrm{D}}}^{T} \, (M^{R})^{-1} \, M^{\mathrm{D}} \qquad\qquad M^{h} \simeq M^{R}$$

- Three-generation generalization of the well-known see-saw formula

$$m_{\ell} \simeq \frac{m_{\mathrm{D}}^{2}}{m_{R}}$$



- Natural explanation of smallness of light neutrino masses

- Effective low-energy Majorana three-neutrino mixing

$$\begin{pmatrix} \left(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}\right)^{T} \\ \left(\nu_{eR}^{C}, \nu_{\mu R}^{C}, \nu_{\tau R}^{C}\right)^{T} \end{pmatrix} \simeq \begin{pmatrix} 1 & [(M^{R})^{-1} M^{\mathrm{D}}]^{\dagger} \\ -(M^{R})^{-1} M^{\mathrm{D}} & 1 \end{pmatrix} \begin{pmatrix} \left(\nu_{eL}^{\ell}, \nu_{\mu L}^{\ell}, \nu_{\tau L}^{\ell}\right)^{T} \\ \left({\nu_{eR}^{h}}^{C}, {\nu_{\mu R}^{h}}^{C}, {\nu_{\tau R}^{h}}^{C}\right)^{T} \end{pmatrix}$$

Effective Majorana Three-Neutrino Mixing

- Light Majorana mass term $\quad {N^{\ell}_{L}}^{T} = \left(\nu^{\ell}_{eL}, \nu^{\ell}_{\mu L}, \nu^{\ell}_{\tau L}\right)$

$$\mathscr{L}_{\ell} = \frac{1}{2} \, {N^{\ell}_{L}}^{T} \, \mathcal{C}^{\dagger} \, M^{\ell} \, N^{\ell}_{L} + \text{H.c.}$$

- Diagonalization: unitary transformation

$$N^{\ell}_{L} = U n_{L} \qquad n^{T}_{L} = (\nu_{1L}, \nu_{2L}, \nu_{3L})$$

$$U^{T} M^{\ell} U = M \qquad M_{kj} = m_{k} \delta_{kj}$$

- Massive neutrinos are Majorana!

- Mixing: $\quad \nu_{\alpha L} \simeq \nu^{\ell}_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$

Mixing Matrix

- Weak Charged Current: $j_W^{\rho\,\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} \overline{\ell_{\alpha L}}\, \gamma^\rho\, U_{\alpha k}\, \nu_{kL}$
- U depends on 6 phases
- Majorana Mass Term $\quad \mathscr{L}_{\text{mass}}^{\text{M}} = \frac{1}{2} \sum_{k=1}^{3} m_k\, \nu_{kL}^{T}\, \mathcal{C}^\dagger\, \nu_{kL} + \text{H.c.}$

  is not invariant under global U(1) transformations

$$\nu_{kL} \to e^{i\varphi_k}\, \nu_{kL} \qquad (k = 1, 2, 3)$$

- Left-handed massive neutrino fields cannot be rephased
- Only 3 phases can be eliminated by rephasing the charged-lepton fields on the left of U
- Two Majorana phases factorized on the right of mixing matrix:

$$\boxed{U = U^{\text{D}}\, D^{\text{M}}} \qquad D^{\text{M}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- U^{D} is analogous to a Dirac mixing matrix, with one Dirac phase

- Standard parameterization:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

- Jarlskog invariant depends only on Dirac phase:

$$J = c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13}\sin\delta_{13}$$

Part II

Neutrino Oscillations

- Neutrino Oscillations in Vacuum
- Neutrino Oscillations in Matter

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{array}{c} \nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \dfrac{(m_B + m_C)^2}{2m_A} - \dfrac{m_A}{2} \end{array}}$$

$$\begin{array}{ll} \nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^- & E_{\text{th}} = 0.233\,\text{MeV} \\ \nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^- & E_{\text{th}} = 0.81\,\text{MeV} \\ \bar{\nu}_e + p \to n + e^+ & E_{\text{th}} = 1.8\,\text{MeV} \\ \nu_\mu + n \to p + \mu^- & E_{\text{th}} = 110\,\text{MeV} \\ \nu_\mu + e^- \to \nu_e + \mu^- & E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV} \end{array}$$

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO) , $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

Neutrino Oscillations

- 1957: Bruno Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955)

- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions

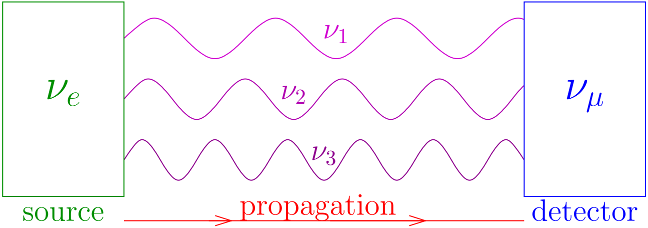
- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector

- A Flavor Neutrino is a superposition of Massive Neutrinos

$$\begin{aligned}
|\nu_e\rangle &= U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle \\
|\nu_\mu\rangle &= U_{\mu1}\,|\nu_1\rangle + U_{\mu2}\,|\nu_2\rangle + U_{\mu3}\,|\nu_3\rangle \\
|\nu_\tau\rangle &= U_{\tau1}\,|\nu_1\rangle + U_{\tau2}\,|\nu_2\rangle + U_{\tau3}\,|\nu_3\rangle
\end{aligned}$$

- U is the 3×3 Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U_{e1}\,e^{-iE_1 t}\,|\nu_1\rangle + U_{e2}\,e^{-iE_2 t}\,|\nu_2\rangle + U_{e3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_e\rangle$$

at the detector there is a probability > 0 to see the neutrino as a ν_μ

Neutrino Oscillations are Flavor Transitions $\boxed{\Delta L = 0}$

$\nu_e \to \nu_\mu$ $\nu_e \to \nu_\tau$ $\nu_\mu \to \nu_e$ $\nu_\mu \to \nu_\tau$

$\bar{\nu}_e \to \bar{\nu}_\mu$ $\bar{\nu}_e \to \bar{\nu}_\tau$ $\bar{\nu}_\mu \to \bar{\nu}_e$ $\bar{\nu}_\mu \to \bar{\nu}_\tau$

Neutrino Oscillations in Vacuum

[Eliezer, Swift, NPB 105 (1976) 45] [Fritzsch, Minkowski, PLB 62 (1976) 72] [Bilenky, Pontecorvo, SJNP 24 (1976) 316]

$$\mathcal{L}_{\text{CC}} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\quad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad \Longrightarrow \quad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \quad$ States

initial flavor: $\alpha \quad = \quad e$ or μ or τ

$$|\nu_k(t,x)\rangle = e^{-iE_k t + ip_k x} \, |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_k U^*_{\alpha k} \, e^{-iE_k t + ip_k x} \, |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t,x)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + ip_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0,0) = \sum_k U^*_{\alpha k} U_{\beta k} = \delta_{\alpha\beta} \qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t > 0, x > 0) \neq \delta_{\alpha\beta}$$

$$P_{\nu_\alpha \to \nu_\beta}(t,x) = \left|\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t,x)\right|^2 = \left|\sum_k U_{\alpha k}^* e^{-iE_k t + i p_k x} U_{\beta k}\right|^2$$

ultra-relativistic neutrinos ⟹ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L,E) &= \left|\sum_k U_{\alpha k}^* \, e^{-i m_k^2 L / 2E} \, U_{\beta k}\right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \end{aligned}$$

$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu}^T = -\mathcal{C} \, \nu^*$$

C $\Longrightarrow$ Particle ⇆ Antiparticle
P $\Longrightarrow$ Left-Handed ⇆ Right-Handed

Fields: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \xrightarrow{\mathsf{CP}} \quad \nu_{\alpha L}^{\mathsf{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\mathsf{CP}}$

States: $|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\mathsf{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS U ⇆ U^* ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\Longrightarrow$ $A_{\alpha\beta}^{\text{CPT}} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$ $\boxed{\text{CPT} \quad \Rightarrow \quad A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

violation of CP in neutrino oscillations is proportional to

$|U_{e3}| = \sin\vartheta_{13}$ and $\sin\delta_{13}$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\mathsf{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $A^{\mathsf{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

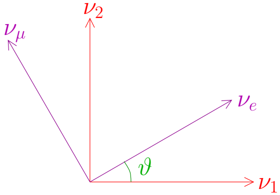
$$\begin{aligned} \text{CPT} \quad \Longrightarrow \quad 0 &= A^{\mathsf{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} + P_{\nu_\beta \to \nu_\alpha} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\ &= A^{\mathsf{T}}_{\alpha\beta} + A^{\mathsf{CP}}_{\beta\alpha} = A^{\mathsf{T}}_{\alpha\beta} - A^{\mathsf{CP}}_{\alpha\beta} \quad \Longrightarrow \quad \boxed{A^{\mathsf{T}}_{\alpha\beta} = A^{\mathsf{CP}}_{\alpha\beta}} \end{aligned}$$

$$\boxed{A^{\mathsf{T}}_{\alpha\beta}(L,E) = 4 \sum_{k>j} \mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\mathrm{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k} \, |\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{array}{l} |\nu_e\rangle = \cos\vartheta \, |\nu_1\rangle + \sin\vartheta \, |\nu_2\rangle \\ |\nu_\mu\rangle = -\sin\vartheta \, |\nu_1\rangle + \cos\vartheta \, |\nu_2\rangle \end{array}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $P_{\nu_e \to \nu_\mu} = P_{\nu_\mu \to \nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

Survival Probabilities: $P_{\nu_e \to \nu_e} = P_{\nu_\mu \to \nu_\mu} = 1 - P_{\nu_e \to \nu_\mu}$

two-neutrino mixing transition probability

$$\alpha \neq \beta \qquad \alpha, \beta = e, \mu, \tau$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L, E) &= \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \\ &= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{m}]}{E[\text{MeV}]}\right) \\ &= \sin^2 2\vartheta \, \sin^2\left(1.27 \, \frac{\Delta m^2[\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}\right) \end{aligned}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \, \frac{E \, [\text{MeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{m} = 2.47 \, \frac{E \, [\text{GeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{km}$$

Types of Experiments

Two-Neutrino Mixing

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

observable if $\frac{\Delta m^2 L}{4E} \gtrsim 1$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}\,,\ E \sim 1\,\text{MeV}$

Accelerator: $L \sim 1\,\text{km}\,,\ E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$

$\Downarrow$

$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Reactor: $L \sim 1\,\text{km}\,,\ E \sim 1\,\text{MeV}$ CHOOZ, PALO VERDE

Accelerator: $L \sim 10^3\,\text{km}\,,\ E \gtrsim 1\,\text{GeV}$ K2K, MINOS, CNGS

Atmospheric: $L \sim 10^2 - 10^4\,\text{km}\,,\ E \sim 0.1 - 10^2\,\text{GeV}$

Kamiokande, IMB, Super-Kamiokande, Soudan, MACRO, MINOS

SUN

$\frac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}\,,\quad E \sim 0.1 - 10\,\text{MeV}$

Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1\,,\ 10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$

VLBL

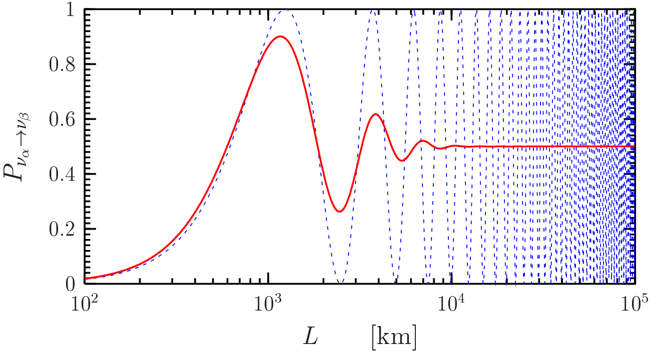
$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}\,,\ E \sim 1\,\text{MeV}$

KamLAND

Average over Energy Resolution of the Detector

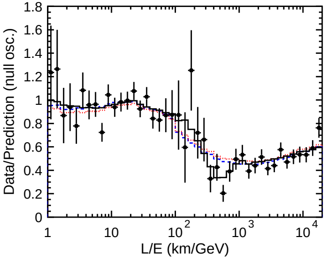
$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2} \sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$



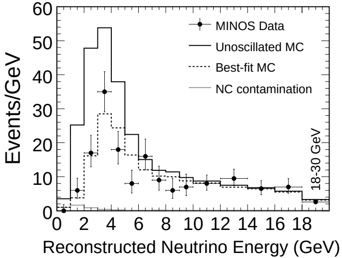
$\Delta m^2 = 10^{-3}\,\text{eV}$ $\sin^2 2\vartheta = 1$ $\langle E \rangle = 1\,\text{GeV}$ $\Delta E = 0.2\,\text{GeV}$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\, \mathrm{d}E\right] \qquad (\alpha \neq \beta)$$

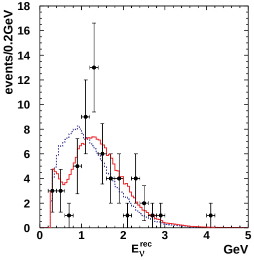
Observations of Neutrino Oscillations



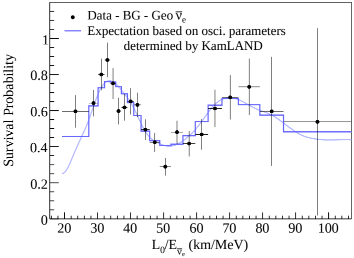
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]



[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]



[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

Neutrino Oscillations in Matter

a flavor neutrino ν_α with momentum p is described by

$$|\nu_\alpha(p)\rangle = \sum_k U_{\alpha k}^* \, |\nu_k(p)\rangle$$

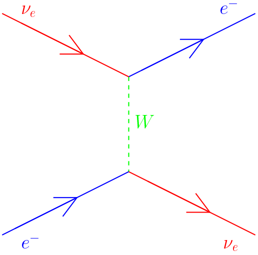
$$\mathcal{H}_0 \, |\nu_k(p)\rangle = E_k \, |\nu_k(p)\rangle \qquad\qquad E_k = \sqrt{p^2 + m_k^2}$$

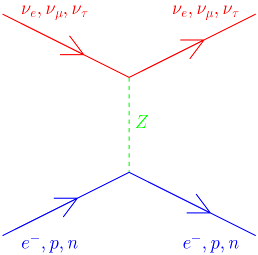
in matter $\qquad \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad\qquad \mathcal{H}_I \, |\nu_\alpha(p)\rangle = V_\alpha \, |\nu_\alpha(p)\rangle$

V_α = effective potential due to coherent interactions with the medium

forward elastic CC and NC scattering

Effective Potentials in Matter





$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

only $V_{\text{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}}$ $\overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Neutrino Flavors in Matter

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\,\Psi_\alpha = \frac{1}{2E}\left(U\,\mathbb{M}^2\,U^\dagger + \mathbb{A}\right)\Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2}EG_{\mathrm{F}}N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U\,\mathbb{M}^2\,U^\dagger \xrightarrow{\text{matter}} U\,\mathbb{M}^2\,U^\dagger + 2\,E\,\underset{\uparrow}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\ \Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix}$

$$\boxed{\begin{aligned}&P_{\nu_e\to\nu_\mu}(x)=|\psi_\mu(x)|^2\\ &P_{\nu_e\to\nu_e}(x)=|\psi_e(x)|^2=1-P_{\nu_e\to\nu_\mu}(x)\end{aligned}}$$

Constant Matter Density

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin2\vartheta\\\Delta m^2\sin2\vartheta & \Delta m^2\cos2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x}=0$$

Diagonalization of Effective Hamiltonian

$$\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\-\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}=\left[\underset{\uparrow}{\frac{A_{\mathrm{CC}}}{4E}}+\frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}$$

irrelevant common phase

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\text{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\text{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\text{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\text{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\text{M}} = \pi/4$)

$$A^{\text{R}}_{\text{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\text{R}}_{e} = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\text{F}}}$$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

$$\nu_e \to \nu_\mu \quad \Longrightarrow \quad \begin{pmatrix}\psi_e(0)\\\psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix}\psi_1(0)\\\psi_2(0)\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\\sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

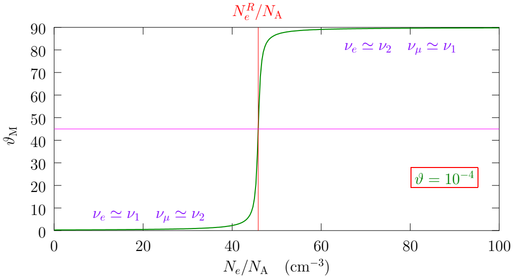
$$\psi_1(x) = \cos\vartheta_{\mathrm{M}} \exp\left(i\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)$$

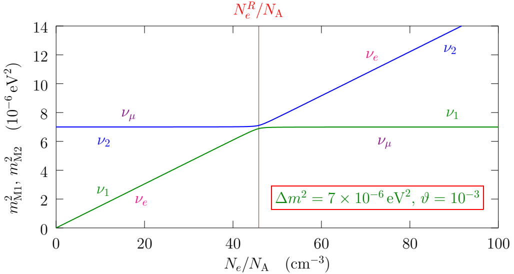
$$\psi_2(x) = \sin\vartheta_{\mathrm{M}} \exp\left(-i\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 = |-\sin\vartheta_{\mathrm{M}}\psi_1(x) + \cos\vartheta_{\mathrm{M}}\psi_2(x)|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x) = \sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m^2_{\mathrm{M}} x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)





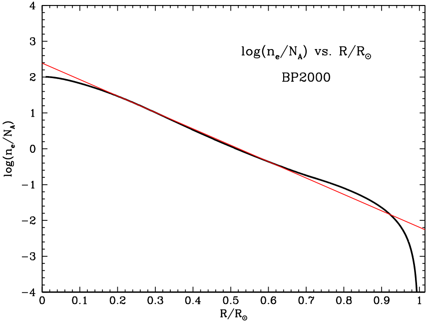
$$\boxed{\begin{aligned}\nu_e &= \cos\vartheta_{\rm M}\,\nu_1 + \sin\vartheta_{\rm M}\,\nu_2\\ \nu_\mu &= -\sin\vartheta_{\rm M}\,\nu_1 + \cos\vartheta_{\rm M}\,\nu_2\end{aligned}}$$

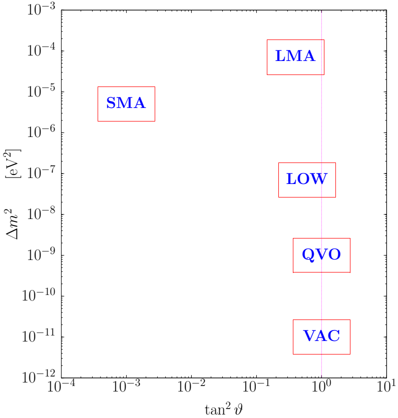
$$\boxed{\tan 2\vartheta_{\rm M} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\rm CC}}{\Delta m^2 \cos 2\vartheta}}}$$

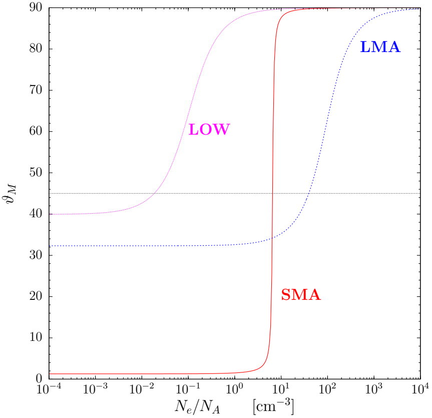
$$\boxed{\Delta m^2_{\rm M} = \left[\left(\Delta m^2 \cos 2\vartheta - A_{\rm CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2\right]^{1/2}}$$

Phenomenology of Solar Neutrinos

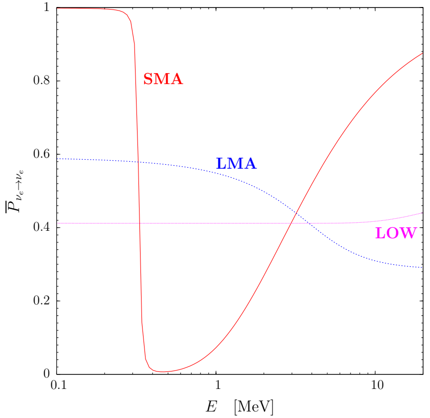
LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}\,\text{eV}^2$,	$\tan^2\vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}\,\text{eV}^2$,	$\tan^2\vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}\,\text{eV}^2$,	$\tan^2\vartheta \sim 1$







ϑ_M
0
10
20
30
40
50
60
70
80
90
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
N_e/N_A [cm^{-3}]
LOW
LMA
SMA



$\overline{P}_{\nu_e \to \nu_e}$
0
0.2
0.4
0.6
0.8
1
0.1
1
10
E [MeV]
SMA
LMA
LOW

In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes: $$i\,\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}x} = \frac{1}{2E}\sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})} D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2 D \quad \Longrightarrow \quad DM^2 D^\dagger = M^2$$

$$U^{(\mathrm{M})} M^2 (U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})} D M^2 D^\dagger (U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})} M^2 (U^{(\mathrm{D})})^\dagger$$

Part III

Phenomenology

- Three-Neutrino Mixing
- Absolute Scale of Neutrino Masses
- Tritium Beta-Decay
- Neutrinoless Double-Beta Decay
- Anomalies Beyond Three-Neutrino Mixing
- Conclusions

Experimental Evidences of Neutrino Oscillations

Solar
$\nu_e \to \nu_\mu, \nu_\tau$

Reactor
$\bar{\nu}_e$ disappearance

Homestake
Kamiokande
GALLEX/GNO & SAGE
Super-Kamiokande
SNO
BOREXino

(KamLAND)

$$\to \begin{cases} \Delta m^2_{\text{SOL}} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2 \\ \tan^2\vartheta_{\text{SOL}} \simeq 0.47 \pm 0.06 \end{cases}$$

Atmospheric
$\nu_\mu \to \nu_\tau$

Accelerator
ν_μ disappearance

Kamiokande
IMB
Super-Kamiokande
MACRO
Soudan-2

(K2K & MINOS)

$$\to \begin{cases} \Delta m^2_{\text{ATM}} \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2 \\ \sin^2\vartheta_{\text{ATM}} \simeq 0.50 \pm 0.07 \end{cases}$$

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ$, $\vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

three flavor fields: ν_e, ν_μ, ν_τ

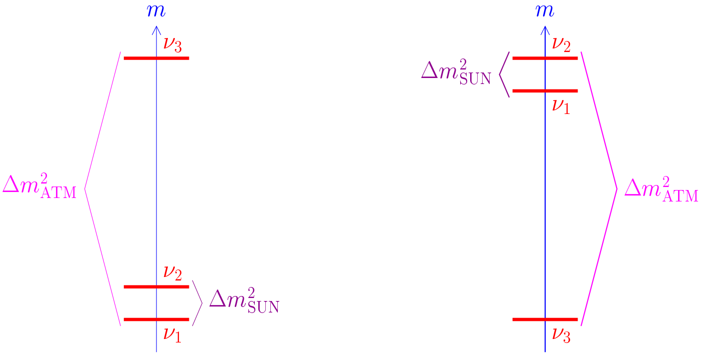
three massive fields: ν_1, ν_2, ν_3

$$\Delta m^2_{21} + \Delta m^2_{32} + \Delta m^2_{31} = m^2_2 - m^2_1 + m^2_3 - m^2_2 + m^2_1 - m^2_3 = 0$$

$$\Delta m^2_{\text{SOL}} = \Delta m^2_{21} \simeq (7.6 \pm 0.2) \times 10^{-5}\,\text{eV}^2$$

$$\Delta m^2_{\text{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq (2.4 \pm 0.1) \times 10^{-3}\,\text{eV}^2$$

Allowed Three-Neutrino Schemes



"normal" "inverted"

different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

ATM and LBL Oscillation Probabilities

$$P_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-iE_k t} \right|^2 * \left| e^{iE_1 t} \right|^2$$

$$= \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} e^{-i(E_k - E_1)t} \right|^2 \to \left| \sum_{k=1}^{3} U_{\alpha k}^{*} U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E} \right) \right|^2$$

$$E_k \simeq E + \frac{m_k^2}{2E} \qquad \frac{\Delta m_{21}^2 L}{2E} \ll 1$$

$$P_{\nu_\alpha \to \nu_\beta} = \left| U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} + U_{\alpha 3}^{*} U_{\beta 3} \exp\left(-i \frac{\Delta m_{31}^2 L}{2E} \right) \right|^2$$

$$U_{\alpha 1}^{*} U_{\beta 1} + U_{\alpha 2}^{*} U_{\beta 2} = \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3}$$

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\beta} = {} & \left| \delta_{\alpha\beta} - U_{\alpha 3}^{*} U_{\beta 3} \left[1 - \exp\left(-i \frac{\Delta m^2_{31} L}{2E} \right) \right] \right|^2 \\
= {} & \delta_{\alpha\beta} + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \left(2 - 2 \cos \frac{\Delta m^2_{31} L}{2E} \right) \\
& - 2 \delta_{\alpha\beta} |U_{\alpha 3}|^2 \left(1 - \cos \frac{\Delta m^2_{31} L}{2E} \right) \\
= {} & \delta_{\alpha\beta} - 2 |U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \left(1 - \cos \frac{\Delta m^2_{31} L}{2E} \right) \\
= {} & \delta_{\alpha\beta} - 4 |U_{\alpha 3}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 3}|^2 \right) \sin^2 \frac{\Delta m^2_{31} L}{4E}
\end{aligned}
$$

$$\alpha \neq \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\beta} = 4 |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

$$\alpha = \beta \quad \Longrightarrow \quad P_{\nu_\alpha \to \nu_\alpha} = 1 - 4 |U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2 \right) \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right)$$

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \qquad (\alpha \neq \beta)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 3}|^2 \left(1 - |U_{\alpha 3}|^2\right)$$

$$U = \left(\begin{array}{cc|c|} \cline{3-3} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \\ \cline{3-3} \end{array}\right)$$

$\uparrow$

ATM & LBL

$$\sin^2 2\vartheta_{ee} \ll 1$$

$$\Downarrow$$

$$|U_{e3}|^2 \simeq \frac{\sin^2 2\vartheta_{ee}}{4}$$

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e3}|^2 \left(1 - |U_{e3}|^2\right) \simeq 4|U_{e3}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right)$$

$$|U_{\mu 3}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - \sin^2 2\vartheta_{\mu\mu}}\right)$$

- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{\mu e} = 4|U_{e3}|^2 |U_{\mu 3}|^2$$

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

SOL →

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

↑ ATM & LBL

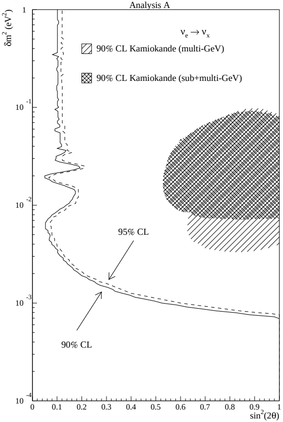
Chooz: $\begin{cases} \Delta m^2_{\text{Chooz}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{Chooz}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2}}$$

[Bilenky, Giunti, PLB 444 (1998) 379]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS
ARE PRACTICALLY DECOUPLED!



[Chooz, PLB 466 (1999) 415]

[Palo Verde, PRD 64 (2001) 112001]

$$|U_{e1}|^2 \simeq \cos^2\vartheta_{\text{SOL}} \qquad |U_{e2}|^2 \simeq \sin^2\vartheta_{\text{SOL}}$$

$$|U_{\mu 3}|^2 \simeq \sin^2\vartheta_{\text{ATM}} \qquad |U_{\tau 3}|^2 \simeq \cos^2\vartheta_{\text{ATM}}$$

$$U^{\mathrm{D}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\mathrm{ATM}}} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\mathrm{SOL}}}$$

$$\Delta m^2_{21} = \left(7.65^{+0.23}_{-0.20}\right) \times 10^{-5}\,\mathrm{eV}^2 \qquad |\Delta m^2_{31}| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\mathrm{eV}^2$$

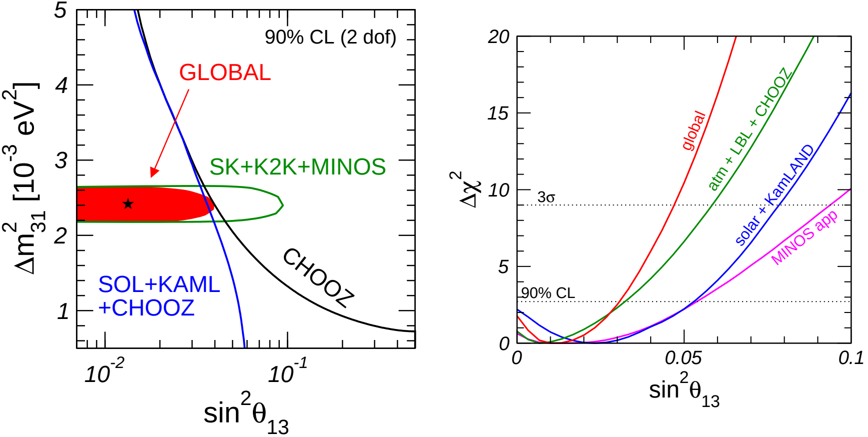
$$\sin^2\vartheta_{12} = 0.304^{+0.022}_{-0.016} \qquad \sin^2\vartheta_{23} = 0.50^{+0.07}_{-0.06}$$

$$\sin^2\vartheta_{13} < 0.035 \quad (90\%\ \mathrm{C.L.})$$

[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]

Current Research

measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

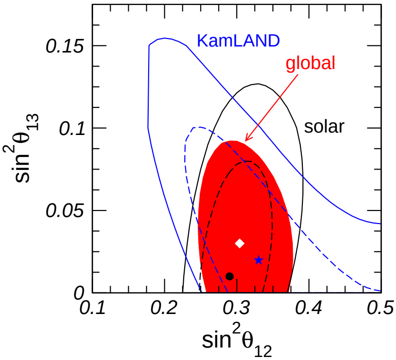


[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

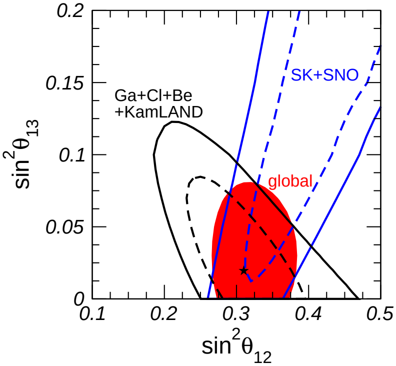
Hint of $\vartheta_{13} > 0$

[Fogli, Lisi, Marrone, Palazzo, Rotunno, NO-VE, April 2008] [Balantekin, Yilmaz, JPG 35 (2008) 075007]

$\sin^2\vartheta_{13} = 0.016 \pm 0.010$ [Fogli, Lisi, Marrone, Palazzo, Rotunno, PRL 101 (2008) 141801]



[Schwetz, Tortola, Valle, arXiv:0808.2016v3, 11 Feb 2010]



[Mezzetto, Schwetz, arXiv:1003.5800, 10 Aug 2010]

$$P_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \simeq \begin{cases} \left(1 - \sin^2\vartheta_{13}\right)^2 \left(1 - 0.5 \sin^2\vartheta_{12}\right) & \text{SOL low-energy \& KamLAND} \\ \left(1 - \sin^2\vartheta_{13}\right)^2 \sin^2\vartheta_{12} & \text{SOL high-energy (matter effect)} \end{cases}$$

LBL Oscillation Probabilities

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad \alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \qquad A = \frac{2EV}{\Delta m^2_{31} L} \qquad V = \sqrt{2} G_{\text{F}} N_e$$

$$\sin\theta_{13} \ll 1 \qquad \alpha \ll 1$$

$$P^{\text{LBL}}_{\nu_e\to\nu_e} \simeq 1 - \sin^2 2\vartheta_{13} \sin^2\Delta - \alpha^2 \Delta^2 \sin^2 2\vartheta_{12}$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq\ & \sin^2 2\vartheta_{13} \sin^2\vartheta_{23} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & + \alpha \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \delta_{13}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & + \alpha^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

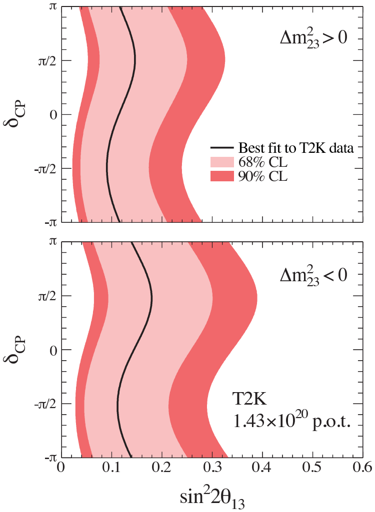
[Mezzetto, Schwetz, arXiv:1003.5800]

T2K

[PRL 107 (2011) 041801, arXiv:1106.2822]

ND at 280 m FD at 295 km

2.5° off-axis ⇒ NBB with $\langle E \rangle \simeq 0.6\,\text{GeV} \simeq |\Delta m^2_{31}|L/2\pi$



$\nu_\mu \to \nu_e$

6 ν_e events in FD

background: 1.5 ± 0.3

2.5σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.11^{+0.17}_{-0.08} & \text{(NH)} \\ 0.14^{+0.20}_{-0.10} & \text{(IH)} \end{cases}$$

90% C.L. $\delta_{13} = 0$

Assumptions

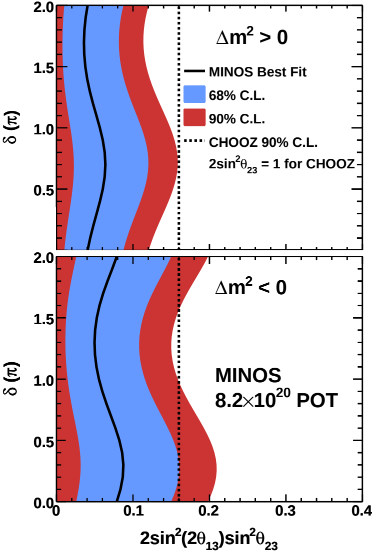
$$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\text{eV}\,,\ \sin^2 2\vartheta_{12} = 0.87$$

$$|\Delta m^2_{31}| = 2.4 \times 10^{-3}\,\text{eV}\,,\ \sin^2 2\vartheta_{23} = 1$$

MINOS

[PRL 107 (2011) 181802, arXiv:1108.0015]

ND at 1.04 km FD at 735 km $\langle E \rangle \simeq 3\,\mathrm{GeV}$



$$\nu_\mu \to \nu_e$$

62 ν_e events in FD

background: 49.6 ± 7.5

1.6σ effect

$$\sin^2 2\vartheta_{13} = \begin{cases} 0.041^{+0.047}_{-0.031} & \text{(NH)} \\ 0.079^{+0.071}_{-0.053} & \text{(IH)} \end{cases}$$

68% C.L. $\delta_{13} = 0$

Assumptions

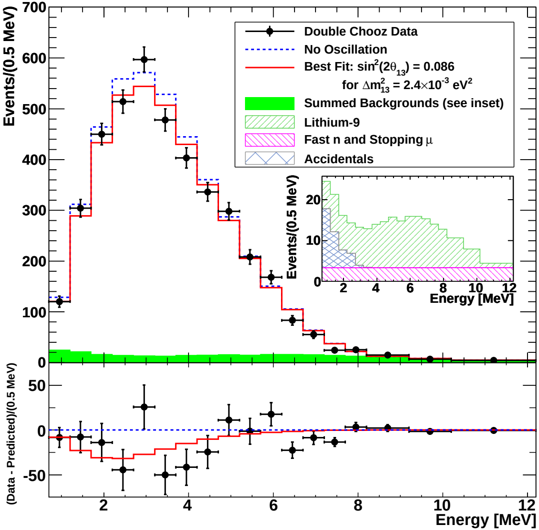
$$\Delta m^2_{21} = 7.6 \times 10^{-5}\,\mathrm{eV}\,, \ \sin^2 2\vartheta_{12} = 0.87$$

$$|\Delta m^2_{31}| = 2.3 \times 10^{-3}\,\mathrm{eV}\,, \ \sin^2 2\vartheta_{23} = 1$$

Double Chooz

[arXiv:1112.6353]

$L = 1050\,\mathrm{m} \qquad \langle E \rangle \simeq 3.6\,\mathrm{GeV}$ Reactor $\bar{\nu}_e$



$\bar{\nu}_e$ disappearance

$R = 0.944 \pm 0.043$

1.3σ effect

$\sin^2 2\vartheta_{13} = 0.086 \pm 0.051$

2013: ND at 400 m

~ 0.02 1σ precision

Other Active Experiments

RENO	Reactor $\bar{\nu}_e$. ND at 290 m, FD at 1380 m. First results expected in summer 2012. Expected sensitivity: ~ 0.016 at 90% C.L. in 3 years.
Daya Bay	Reactor $\bar{\nu}_e$. ND at 360 m, ND at 500 m, FD at 1600 m. First results expected in 2013. Expected sensitivity: ~ 0.01 at 90% C.L. in 3 years.

Under Construction

NOvA	Accelerator $\nu_\mu \to \nu_e$. ND at 1 km, FD at 810 km. 0.8° off-axis $\Rightarrow$ NBB with $\langle E \rangle \simeq 2\,\text{GeV} \simeq \lvert\Delta m^2_{31}\rvert L/2\pi$. Data taking will start in 2013. Expected sensitivity: ~ 0.01 at 90% C.L. in 3 years. Sensitive to CP violation and mass hierarchy.

CP Violation

$$P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = -16 J_{\alpha\beta} \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \sin\left(\frac{\Delta m^2_{31} L}{4E}\right) \sin\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c^2_{13} \sin\delta_{13}$$

Necessary conditions for observation of CP violation:

- Sensitivity to small ϑ_{13}
- Sensitivity to oscillations due to Δm^2_{21} and Δm^2_{31}

Mass Hierarchy

- $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $\cos 2\vartheta_{13} = \frac{2EV}{\Delta m^2_{31}}$

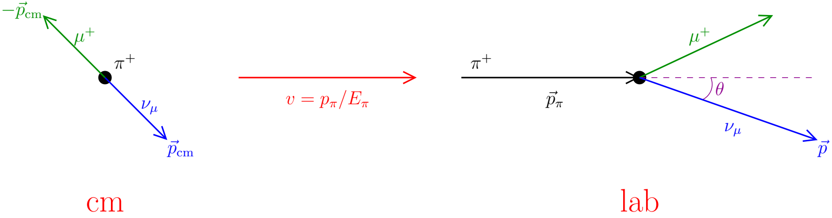
  Requires $\Delta m^2_{31} > 0$ Normal Hierarchy

- $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $\cos 2\vartheta_{13} = -\frac{2EV}{\Delta m^2_{31}}$

  Requires $\Delta m^2_{31} < 0$ Inverted Hierarchy

Off-Axis Experiments

high-intensity WB beam
detector shifted by a small angle from axis of beam
almost monochromatic neutrino energy



$$E_{\rm cm} = p_{\rm cm} = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 29.79\,\text{MeV}$$

$$\gamma = \left(1 - v^2\right)^{-1/2} = E_\pi / m_\pi \gg 1 \qquad \begin{cases} E = \gamma\left(E_{\rm cm} + v\, p^z_{\rm cm}\right) \\ p^z = \gamma\left(v\, E_{\rm cm} + p^z_{\rm cm}\right) \end{cases}$$

$$p^z = p\,\cos\theta \qquad \Longrightarrow \qquad E = \frac{E_{\rm cm}}{\gamma\left(1 - v\,\cos\theta\right)}$$

$$\cos\theta \simeq 1 - \theta^2/2 \qquad \text{and} \qquad v \simeq 1$$

$$E = \frac{E_{\text{cm}}}{\gamma\left(1 - v\cos\theta\right)} \simeq \frac{\gamma\left(1+v\right)}{1+\gamma^2\theta^2 v\left(1+v\right)/2}\,E_{\text{cm}} \simeq \frac{2\gamma}{1+\gamma^2\theta^2}\,E_{\text{cm}}$$

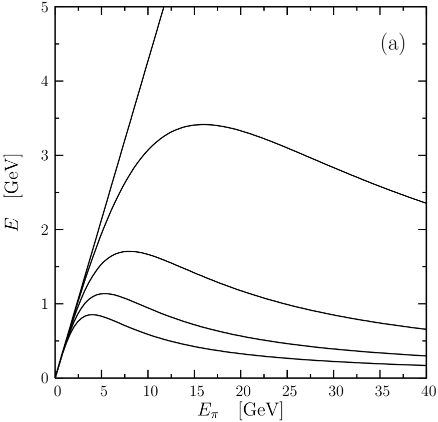
$$E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi}{1+\gamma^2\,\theta^2} = \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi\, m_\pi^2}{m_\pi^2 + E_\pi^2\,\theta^2}$$

- $\theta = 0 \quad \Longrightarrow \quad E \propto E_\pi$ WB beam
- $E_\pi\theta \gg m_\pi \quad \Longrightarrow \quad E \propto \dfrac{m_\pi^2}{E_\pi\,\theta^2}$ high-energy π^+ give low-energy ν_μ

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{1-\gamma^2\,\theta^2}{\left(1+\gamma^2\,\theta^2\right)^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}E_\pi} \simeq 0 \quad \text{for} \quad \theta = \gamma^{-1} = \frac{m_\pi}{E_\pi} \quad \Longrightarrow \quad E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)\frac{m_\pi}{2\theta} \simeq \frac{29.79\,\text{MeV}}{\theta}$$

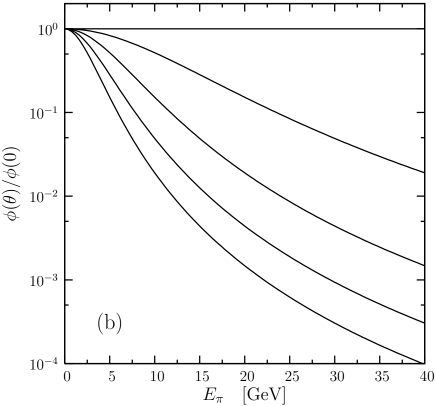
$$\text{off-axis angle} \quad \theta \simeq m_\pi / \langle E_\pi \rangle \quad \Longrightarrow \quad E \simeq \frac{29.79\,\text{MeV}}{\theta}$$



$$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$$

- E can be tuned on oscillation peak $E_{\text{peak}} = \Delta m^2 L / 2\pi$
- small $E \Longrightarrow$ short $L_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2} \Longrightarrow$ sensitivity to small values of Δm^2

$$\frac{\phi(\theta)}{\phi(0)} = \frac{1}{4} \left(\frac{2}{1 + \gamma^2 \, \theta^2} \right)^2$$

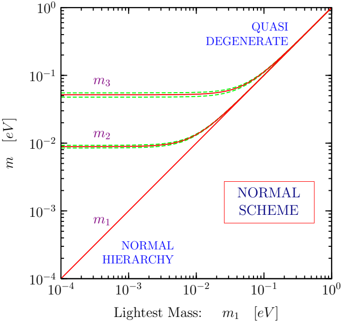


$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

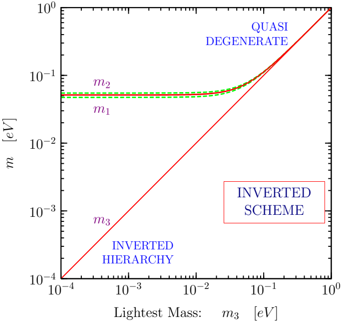
flux suppression requires superbeam

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

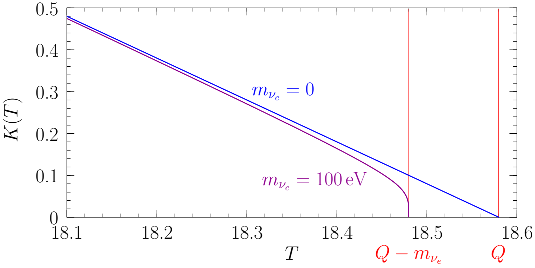
$^{3}\mathrm{H} \to {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$

Mainz & Troitsk

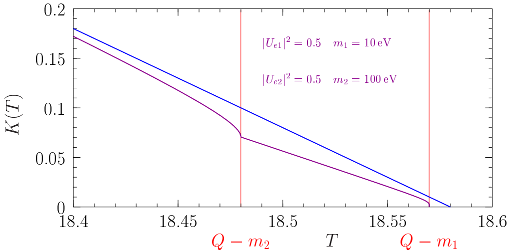
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2012)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case:
$2N - 1$ parameters

$$\left(\sum_k |U_{ek}|^2 = 1 \right)$$

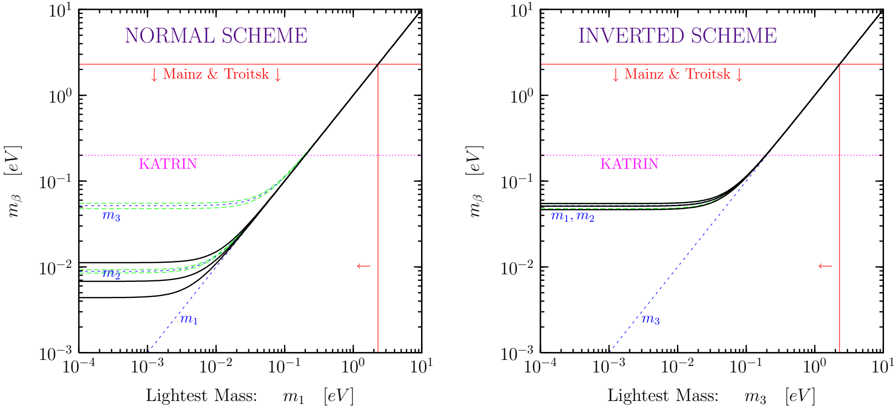
if experiment is not sensitive to masses $(m_k \ll Q - T)$

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

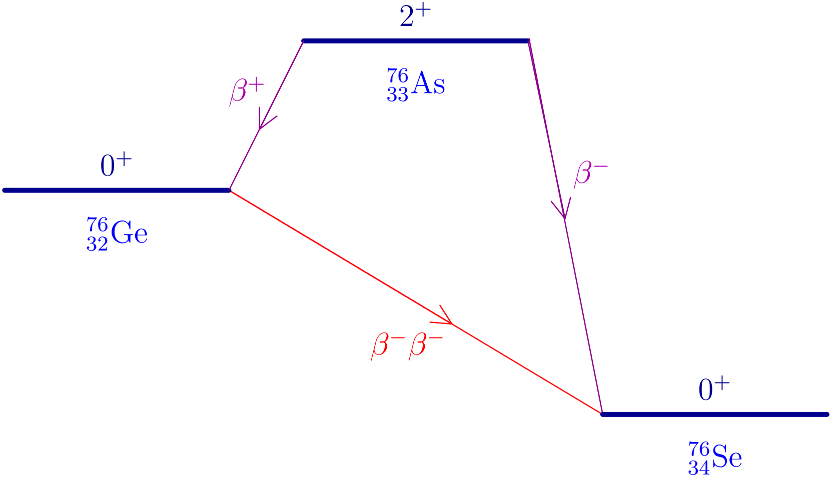
$$m_\beta^2 = |U_{e1}|^2\, m_1^2 + |U_{e2}|^2\, m_2^2 + |U_{e3}|^2\, m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Neutrinoless Double-Beta Decay



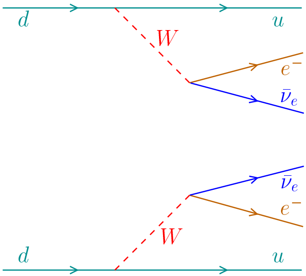
Effective Majorana Neutrino Mass: $m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu} \, |\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



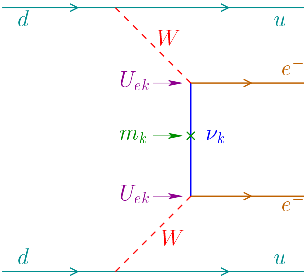
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A, Z) \to \mathcal{N}(A, Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, |m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$

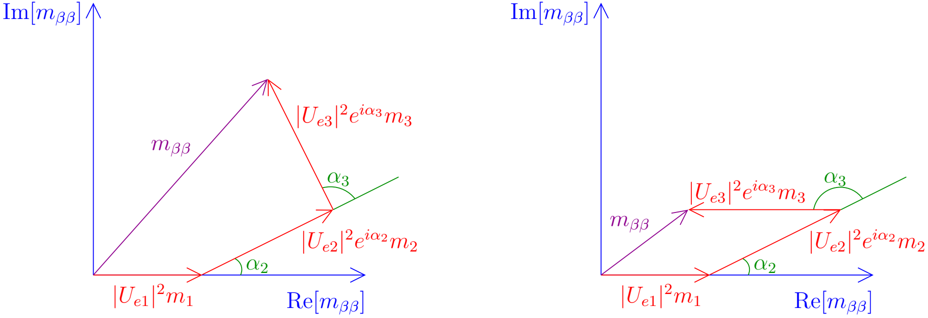


Effective Majorana Neutrino Mass

$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$



Experimental Bounds

CUORICINO (^{130}Te) [arXiv:1012.3266]

$$\boxed{T^{0\nu}_{1/2} > 2.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.3 - 0.7\,\text{eV}}$$

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \Longrightarrow \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

COBRA, XMASS, CAMEO, CANDLES

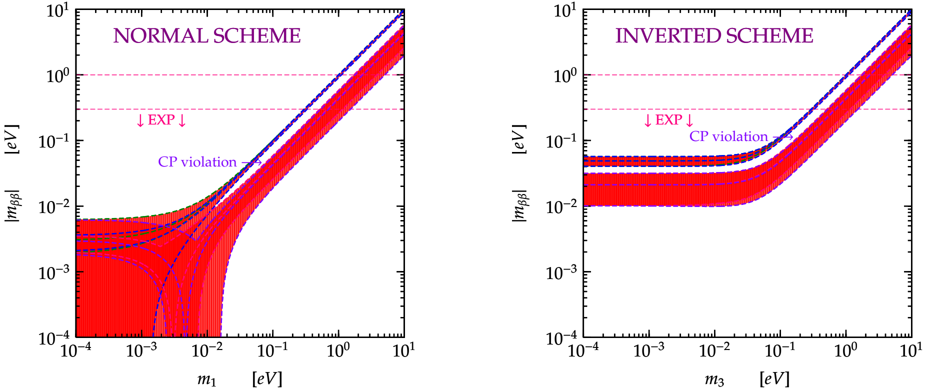
$|m_{\beta\beta}| \sim \text{few}\,10^{-1}\,\text{eV}$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$|m_{\beta\beta}| \sim \text{few}\,10^{-2}\,\text{eV}$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_{21}}\, m_2 + |U_{e3}|^2\, e^{i\alpha_{31}}\, m_3$$

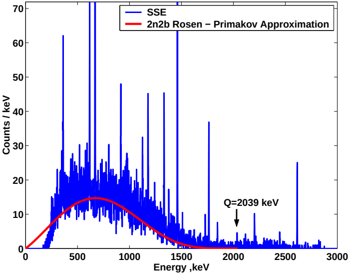


FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}\,\mathrm{eV} \Longrightarrow$ NORMAL HIERARCHY

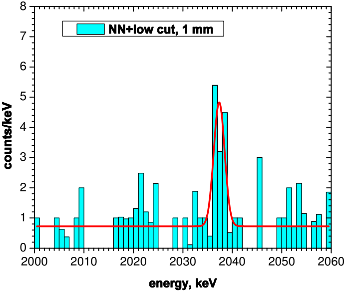
Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409]

$$T^{0\nu}_{1/2} = (2.23^{+0.44}_{-0.31}) \times 10^{25}\,\text{y}$$ 6.5σ evidence [MPLA 21 (2006) 1547]



[PLB 586 (2004) 198]



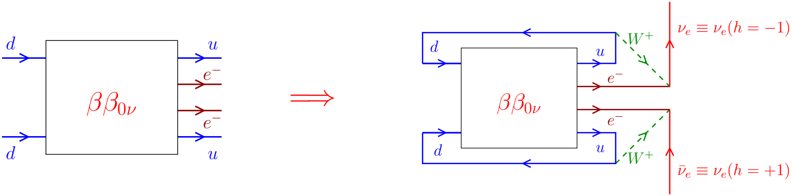
[MPLA 21 (2006) 1547]

the indication must be checked by other experiments

$$|m_{\beta\beta}| = 0.32 \pm 0.03\,\text{eV}$$ [MPLA 21 (2006) 1547]

if confirmed, very exciting (Majorana ν and large mass scale)

$\beta\beta_{0\nu}$ Decay ⇔ Majorana Neutrino Mass



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

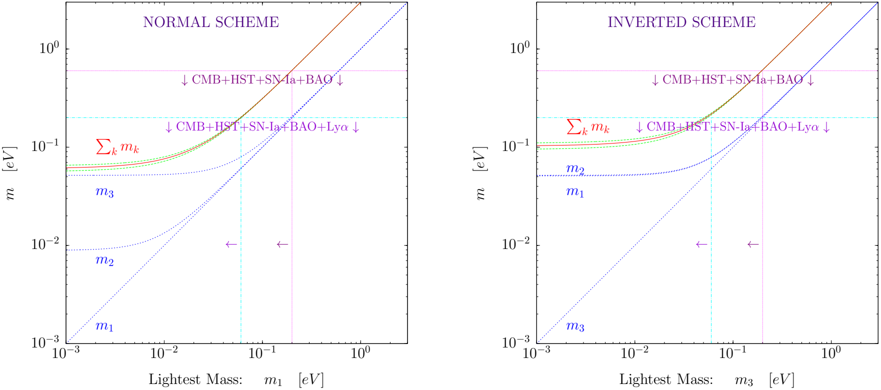
Majorana Mass Term

$$\boxed{\mathcal{L}^{\mathrm{M}}_{eL} = -\frac{1}{2}\, m_{ee} \left(\overline{\nu_{eL}^{c}}\, \nu_{eL} + \overline{\nu_{eL}}\, \nu_{eL}^{c} \right)}$$

Cosmological Bound on Neutrino Masses

$$\sum_{k=1}^{3} m_k \lesssim 0.6\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO}$$

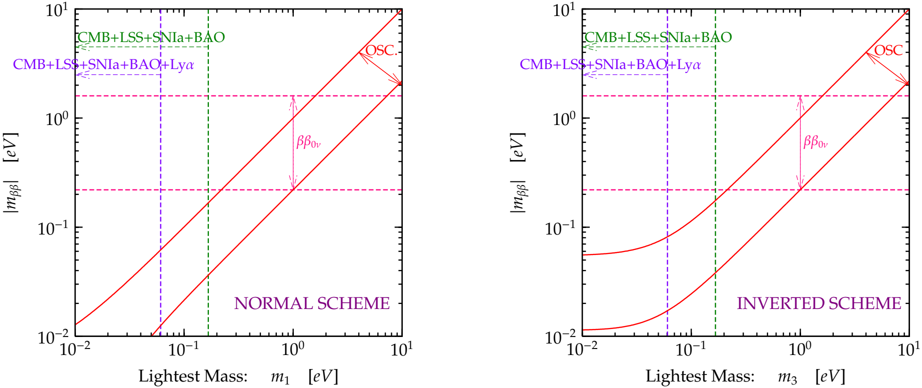
$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\text{eV} \qquad (\sim 2\sigma) \qquad \text{CMB + HST + SN-Ia + BAO + Ly}\alpha$$



FUTURE: IF $\sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\text{eV} \Longrightarrow$ NORMAL HIERARCHY

Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ ($\sim 3\sigma$ range)

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]



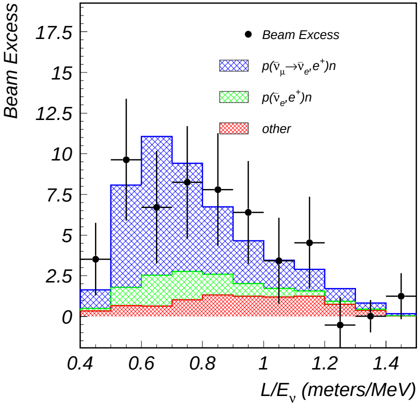
tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

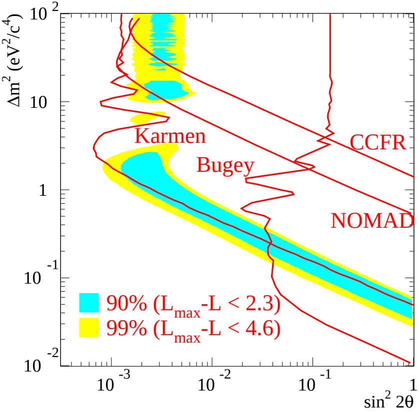
Anomalies Beyond Three-Neutrino Mixing

LSND

[LSND, PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\quad L \simeq 30\,\mathrm{m}$ $\quad 20\,\mathrm{MeV} \leq E \leq 200\,\mathrm{MeV}$



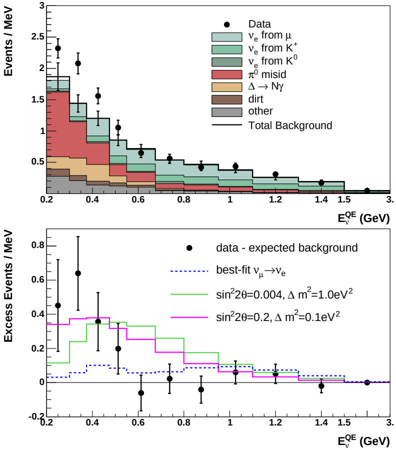


$$\Delta m^2_{\mathrm{LSND}} \gtrsim 0.2\,\mathrm{eV}^2 \qquad (\gg \Delta m^2_{\mathrm{ATM}} \gg \Delta m^2_{\mathrm{SOL}})$$

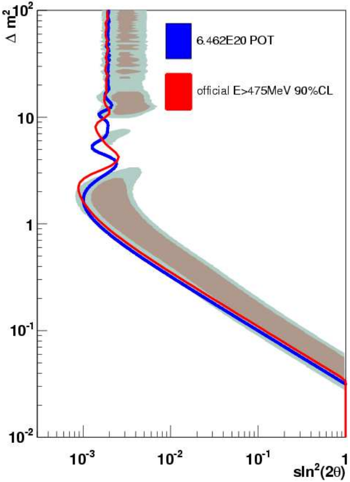
MiniBooNE Neutrinos

[PRL 98 (2007) 231801; PRL 102 (2009) 101802]

$\nu_\mu \to \nu_e$ $\quad L \simeq 541\,\mathrm{m}$ $\quad 475\,\mathrm{MeV} \le E \lesssim 3\,\mathrm{GeV}$



[MiniBooNE, PRL 102 (2009) 101802, arXiv:0812.2243]



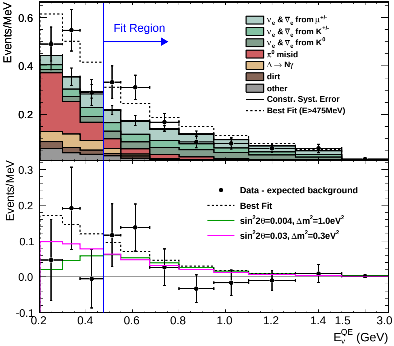
[Djurcic, arXiv:0901.1648]

Low-Energy Anomaly!

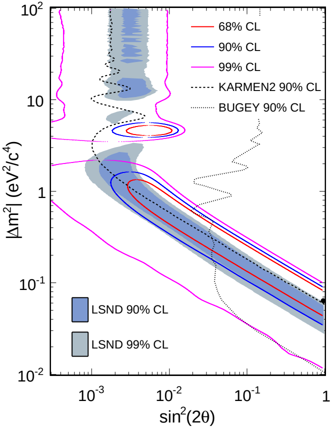
MiniBooNE Antineutrinos

[PRL 103 (2009) 111801; PRL 105 (2010) 181801]

$\bar{\nu}_\mu \to \bar{\nu}_e$ $L \simeq 541\,\text{m}$ $475\,\text{MeV} \leq E \lesssim 3\,\text{GeV}$



[MiniBooNE, PRL 105 (2010) 181801, arXiv:1007.1150]



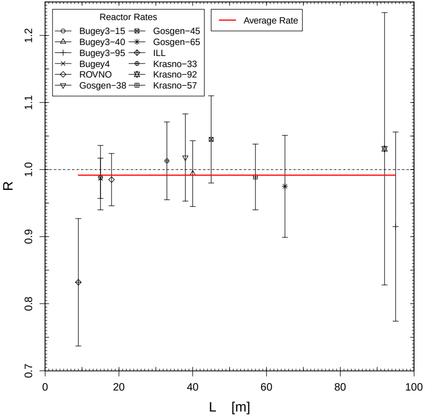
Agreement with LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ signal!

Similar L/E but different L and E $\Longrightarrow$ Oscillations!

Reactor Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006, arXiv:1101.2755]

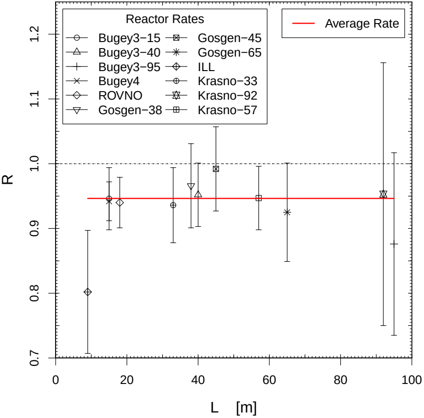
Old Reactor $\bar{\nu}_e$ Fluxes



$$\overline{R} = 0.992 \pm 0.024$$

New Reactor $\bar{\nu}_e$ Fluxes

[Mueller et al, PRC 83 (2011) 054615, arXiv:1101.2663]



$$\overline{R} = 0.946 \pm 0.024$$

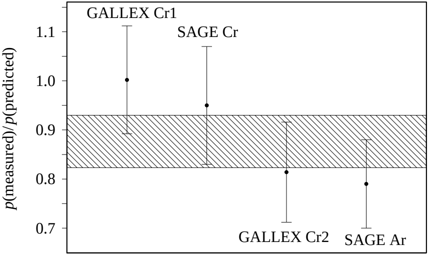
Gallium Anomaly

Gallium Radioactive Source Experiments

Tests of the solar neutrino detectors GALLEX (Cr1, Cr2) and SAGE (Cr, Ar)

ν_e Sources: $e^- + {}^{51}\mathrm{Cr} \to {}^{51}\mathrm{V} + \nu_e$ $e^- + {}^{37}\mathrm{Ar} \to {}^{37}\mathrm{Cl} + \nu_e$

Detection Process: $\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^-$



[SAGE, PRC 73 (2006) 045805, nucl-ex/0512041]

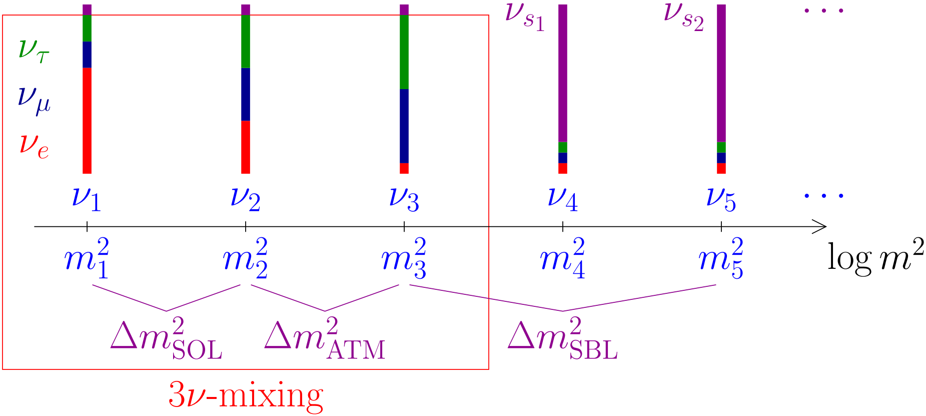
$$\langle L \rangle_{\mathrm{GALLEX}} = 1.9\,\mathrm{m}$$

$$\langle L \rangle_{\mathrm{SAGE}} = 0.6\,\mathrm{m}$$

$$\boxed{R^{\mathrm{Ga}} = 0.76^{+0.09}_{-0.08}}$$

[Giunti, Laveder, PRC 83 (2011) 065504, arXiv:1006.3244]

Beyond Three-Neutrino Mixing



Sterile Neutrinos

- Light anti-ν_R are called sterile neutrinos

$$\nu_R^c \to \nu_{sL} \qquad \text{(left-handed)}$$

- Sterile means no standard model interactions
- Active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ can oscillate into sterile neutrinos (ν_s)
- Observables:
  - Disappearance of active neutrinos (neutral current deficit)
  - Indirect evidence through combined fit of data (current indication)
- Powerful window on new physics beyond the Standard Model

Cosmology

- $N_s =$ number of thermalized sterile neutrinos (not necessarily integer)
- CMB and LSS in ΛCDM: $N_s = 1.3 \pm 0.9 \quad m_s < 0.66\,\text{eV} \quad (95\%\ \text{C.L.})$

[Hamann, Hannestad, Raffelt, Tamborra, Wong, PRL 105 (2010) 181301, arXiv:1006.5276]

$$N_s = 1.61 \pm 0.92 \qquad m_s < 0.70\,\text{eV} \quad (95\%\ \text{C.L.})$$

[Giusarma, Corsi, Archidiacono, de Putter, Melchiorri, Mena, Pandolfi, PRD 83 (2011) 115023, arXiv:1102.4774]

$N_s = 1.12^{+0.86}_{-0.74} \quad (95\%\ \text{C.L.})$ [Archidiacono, Calabrese, Melchiorri, PRD 84 (2011) 123008, arXiv:1109.2767]

- BBN: $\begin{cases} N_s = 0.22 \pm 0.59 \\ N_s = 0.64^{+0.40}_{-0.35} \\ N_s \leq 1 \text{ at } 95\%\ \text{C.L.} \end{cases}$
  [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]
  [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]
  [Mangano, Serpico, PLB 701 (2011) 296, arXiv:1103.1261]
- CMB+LSS+BBN: $N_s = 0.85^{+0.39}_{-0.56} \quad (95\%\ \text{C.L.})$

[Hamann, Hannestad, Raffelt, Wong, JCAP 1109 (2011) 034, arXiv:1108.4136]

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \qquad \sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$

Perturbation of 3ν Mixing

$$|U_{e4}|^2 \ll 1\,, \quad |U_{\mu 4}|^2 \ll 1\,, \quad |U_{\tau 4}|^2 \ll 1\,, \quad |U_{s4}|^2 \simeq 1$$

$$U = \left(\begin{array}{ccc|c|} \hline U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \\ \hline \end{array}\right)$$

$\uparrow$

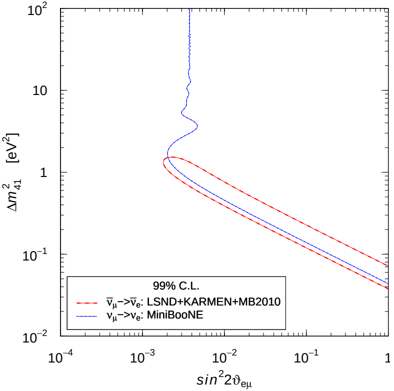
SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

$$\Downarrow$$

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

MiniBooNE



GoF = 17%

PGoF = 0.15%

- Tension between $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ data is reduced with MiniBooNE 2011 antineutrino data.
- 3+2 $\Longrightarrow$ CP Violation [Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004, hep-ph/0305255], [Maltoni, Schwetz, PRD 76, 093005 (2007), arXiv:0705.0107], [Karagiorgi et al, PRD 80 (2009) 073001, arXiv:0906.1997], [Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801, arXiv:1103.4570], [Giunti, Laveder, PRD 84 (2011) 073008, arXiv:1107.1452]
- 3+1+NSI $\Longrightarrow$ CP Violation [Akhmedov, Schwetz, JHEP 10 (2010) 115, arXiv:1007.4171]

Goodness of Fit

- Assumption or approximation: Gaussian uncertainties and linear model
- χ^2_{min} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF} = N_{\text{D}} - N_{\text{P}}$$

$$N_{\text{D}} = \text{Number of Data} \qquad N_{\text{P}} = \text{Number of Fitted Parameters}$$

- $\langle \chi^2_{\text{min}} \rangle = \text{NDF}$ $\qquad$ $\text{Var}(\chi^2_{\text{min}}) = 2\text{NDF}$
- $\text{GoF} = \int_{\chi^2_{\text{min}}}^{\infty} p_{\chi^2}(z, \text{NDF})\, \text{d}z$ $\qquad$ $p_{\chi^2}(z, n) = \dfrac{z^{n/2-1} e^{-z/2}}{2^{n/2}\Gamma(n/2)}$

Parameter Goodness of Fit

Maltoni, Schwetz, PRD 68 (2003) 033020, arXiv:hep-ph/0304176

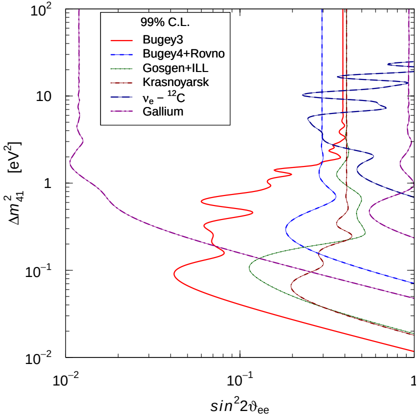
- Measure compatibility of two (or more) sets of data points A and B under fitting model
- $\chi^2_{\text{PGoF}} = (\chi^2_{\text{min}})_{\text{A+B}} - [(\chi^2_{\text{min}})_{\text{A}} + (\chi^2_{\text{min}})_{\text{B}}]$
- χ^2_{PGoF} has χ^2 distribution with Number of Degrees of Freedom

$$\text{NDF}_{\text{PGoF}} = N_{\text{P}}^{\text{A}} + N_{\text{P}}^{\text{B}} - N_{\text{P}}^{\text{A+B}}$$

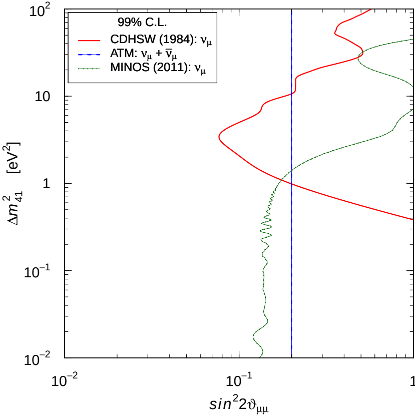
- $\text{PGoF} = \int_{\chi^2_{\text{PGoF}}}^{\infty} p_{\chi^2}(z, \text{NDF}_{\text{PGoF}})\, \text{d}z$

Disappearance Constraints

$\bar{\nu}_e$ Disappearance



ν_μ and $\bar{\nu}_\mu$ Disappearance



- New Reactor $\bar{\nu}_e$ Fluxes
  [Mueller et al., arXiv:1101.2663]
  [Mention et al., arXiv:1101.2755]
- KARMEN+LSND $\nu_e + {}^{12}\text{C} \rightarrow {}^{12}\text{N}_{\text{g.s.}} + e^-$
  [Conrad, Shaevitz, arXiv:1106.5552]
  [Giunti, Laveder, arXiv:1111.1069]
- ATM constraint on $|U_{\mu 4}|^2$
  [Maltoni, Schwetz, arXiv:0705.0107]
- MINOS constraint on $|U_{\mu 4}|^2$
  [Giunti, Laveder, arXiv:1109.4033]

3+1

- ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 \left(1 - |U_{e4}|^2\right) \simeq 4|U_{e4}|^2$$

- ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 \left(1 - |U_{\mu4}|^2\right) \simeq 4|U_{\mu4}|^2$$

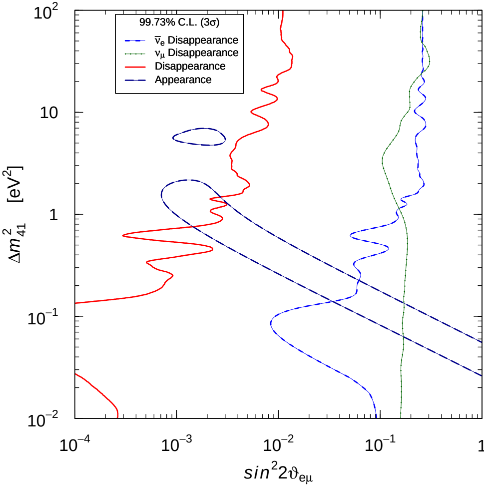
- $\nu_\mu \to \nu_e$ experiments:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

- Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu}$ $\Longrightarrow$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694, arXiv:hep-ph/9606411]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247, arXiv:hep-ph/9607372]



3+1

GoF = 50%

PGoF = 0.3%

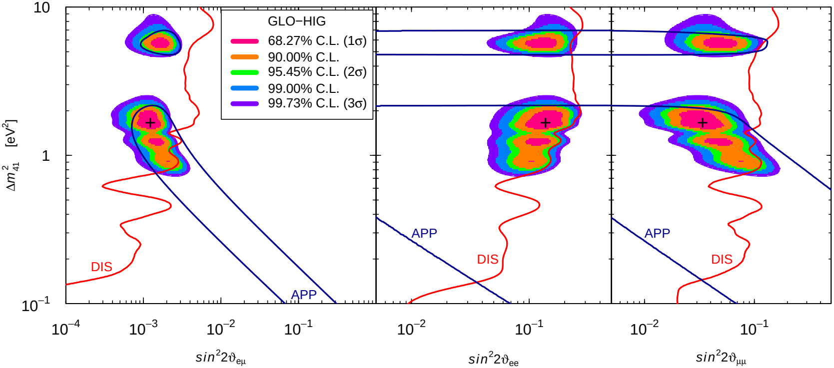
[Giunti, Laveder, arXiv:1111.1069]

- 3+1: Appearance-Disappearance tension
- 3+2: same tension [Kopp, Maltoni, Schwetz, arXiv:1103.4570], [Giunti, Laveder, arXiv:1107.1452]
- Tension reduced in 3+1+NSI [Akhmedov, Schwetz, JHEP 10 (2010) 115, arXiv:1007.4171]
- No tension in 3+1+CPTV [Barger, Marfatia, Whisnant, PLB 576 (2003) 303]
[Giunti, Laveder, PRD 82 (2010) 093016, PRD 83 (2011) 053006]

Global 3+1 Fit of SBL Data

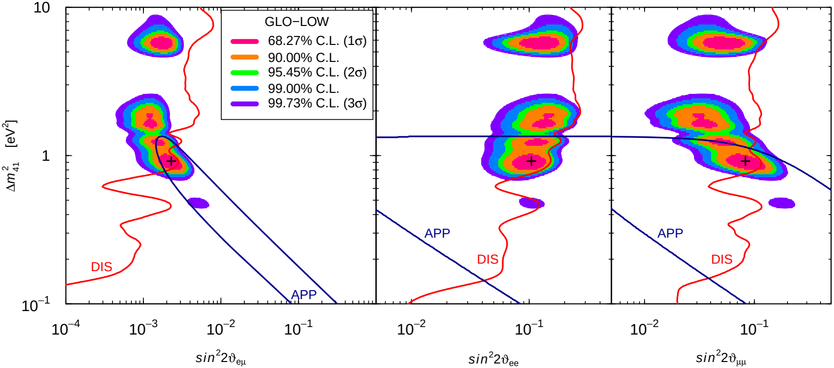
[Giunti, Laveder, arXiv:1109.4033, arXiv:1111.1069]

- Simplest scheme beyond standard three-neutrino mixing which can partially explain the data.
- It corresponds to the natural addition of one new entity (a sterile neutrino) to explain a new effect (short-baseline oscillations).



Best fit: $\Delta m^2_{41} \approx 1.62\,\mathrm{eV}^2 \quad \Longrightarrow \quad m_4 \approx 1.3\,\mathrm{eV}$

MiniBooNE Low-Energy Anomaly



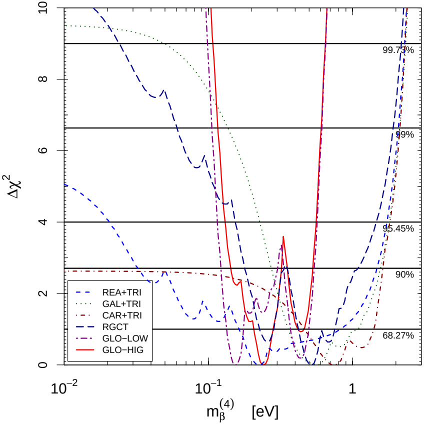
GoF = 30% PGoF = 0.008%

- Best fit at $\Delta m^2_{41} \approx 0.89\,\mathrm{eV}^2 \Longrightarrow m_4 \approx 0.94\,\mathrm{eV} \Longrightarrow$ Better compatibility with cosmological bound on m_4.
- APP-DIS tension indicates that MiniBooNE low-energy anomaly may have an explanation different from $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ oscillations.

Testable Implications

[Giunti, Laveder, PLB 706 (2011) 200, arXiv:1111.1069]

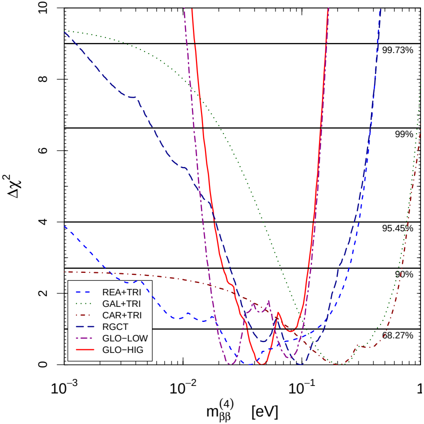
β Decay



$$m_\beta = \sqrt{\sum_k |U_{ek}|^2 m_k^2}$$

$$m_\beta^{(4)} = |U_{e4}| \sqrt{\Delta m_{41}^2}$$

$(\beta\beta)_{0\nu}$ Decay



$$m_{\beta\beta} = \left|\sum_k U_{ek}^2 \, m_k\right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

Future

LSND and MiniBooNE

- MiniBooNE is continuing to take antineutrino data.

$$\bar{\nu}_\mu \to \bar{\nu}_e + \bar{\nu}_\mu \to \bar{\nu}_\mu$$

- ICARUS@CERN-PS: $L \sim 1\text{km}$ $E \sim 1\text{GeV}$ [C. Rubbia et al, CERN-SPSC-2011-012]

$$\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e + \overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu + \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e$$

- MicroBooNE will test the MiniBooNE low-energy anomaly by measuring $\pi^0 \to 2\gamma$ background.

- LSND reloaded? Super-Kamiokande doped with Gadolinium

[Agarwalla, Huber, PLB 696 (2011) 359, arXiv:1007.3228.]

Reactor Anomaly

- Nucifer (France), small liquid scintillator detector at $L = 7\text{m}$

  [Lasserre, talk at EPS-HEP 2011]

- DANSS (Russia), movable solid scintillator at $L \simeq 10\text{m}$

  [Egorov, talk at LowNu11]

- SONGS (USA) [Bowden, talk at LowNu11]

- South Korea [Y.D. Kim, talk at LowNu11]

- Possible experiment at a small-size fast neutron reactor

  [Yasuda, JHEP 1109 (2011) 036, arXiv:1107.4766]

Gallium Anomaly

- New SAGE Gallium source experiments with 2 spherical shells

  [Gavrin et al, arXiv:1006.2103]

- ν_e and $\bar{\nu}_e$ radioactive source experiments with low-threshold detectors.

- Borexino: $\overset{(-)}{\nu}_e + e^- \to \overset{(-)}{\nu}_e + e^-$

  [Cribier et al, PRL 107 (2011) 201801, arXiv:1107.2335]

  [Pallavicini, talk at BEYOND3NU]

  [Ianni, Montanino, Scioscia, EPJC 8 (1999) 609, arXiv:hep-ex/9901012]

- LENS (Low Energy Neutrino Spectroscopy): [Agarwalla, Raghavan, arXiv:1011.4509]

$$\nu_e + {}^{115}\text{In} \to {}^{115}\text{Sn} + e^- + 2\gamma \qquad E_{\text{th}} = 0.1\,\text{MeV}$$

$$\bar{\nu}_e + p \to n + e^+ \qquad E_{\text{th}} = 1.8\,\text{MeV}$$

- Spherical Gaseous TPC: [Vergados, Giomataris, Novikov, NPB 854 (2012) 54, arXiv:1103.5307]

  Targets: ^{131}Xe, ^{40}Ar, ^{20}Ne, ^{4}He. Sources: ^{37}Ar, ^{51}Cr, ^{65}Zn, ^{32}P.

- Coherent neutrino-nucleon scattering in low-temperature bolometers

  [Formaggio, Figueroa-Feliciano, Anderson, PRD 85 (2012) 013009, arXiv:1107.3512]

- Near-Detector Beta-Beam experiments: [Agarwalla, Huber, Link, JHEP 01 (2010) 071]

$$N(A,Z) \to N(A,Z+1) + e^- + \bar{\nu}_e \qquad (\beta^-)$$

$$N(A,Z) \to N(A,Z-1) + e^+ + \nu_e \qquad (\beta^+)$$

- Near-Detector Neutrino Factory experiments: [Giunti, Laveder, Winter, PRD 80 (2009) 073005]

$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$$

- ND+FD at Low Energy Neutrino Factory (LENF) [Pascoli and Wong, poster at NUFACT 11]

- CPT tests: $A_{\alpha\beta}^{\mathrm{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8 \times 10^{-5}\,\text{eV}^2$ (SOL, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2 \times 10^{-3}\,\text{eV}^2$ (ATM, K2K, MINOS)

$\sin^2\vartheta_{12} \simeq 0.3$ $\sin^2\vartheta_{23} \simeq 0.5$ $\sin^2\vartheta_{13} \lesssim 0.05$ (Chooz)

$\sin^2\vartheta_{13} \simeq 0.02$? (T2K, MINOS, Double Chooz)

β & $\beta\beta_{0\nu}$ Decay and Cosmology $\Longrightarrow$ $m_\nu \lesssim 1\,\text{eV}$

To Do

Exp.: Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy.
Find absolute mass scale.
Check anomalies beyond three-neutrino mixing.

Theory: Why lepton mixing $\neq$ quark mixing?
(Due to Majorana nature of ν's?)
Why ϑ_{23} is maximal, ϑ_{12} is large and ϑ_{23} is small?
Explain anomalies beyond three-neutrino mixing.