Theory and Phenomenology of Massive Neutrinos

Part II: Neutrino Oscillations

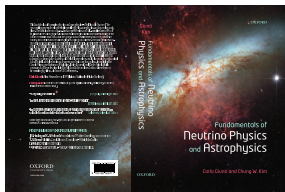### Carlo Giunti

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Neutrino Unbound: http://www.nu.to.infn.it

KIAS, Seoul, 30 November – 2 December 2015

http://www.nu.to.infn.it/slides/2015/giunti-151201-kias-2.pdf

C. Giunti and C.W. Kim
Fundamentals of Neutrino Physics and Astrophysics
Oxford University Press
15 March 2007 – 728 pages

Part II: Neutrino Oscillations

- Neutrino Oscillations

- Neutrino Oscillations in Vacuum

- Two-Neutrino Oscillations

- Neutrino Oscillations in Matter

- Wave-Packet Theory of NuOsc

- Common Question: Do Charged Leptons Oscillate?

- Mistake: Oscillation Phase Larger by a Factor of 2

Neutrino Oscillations

▸ 1957: Pontecorvo proposed Neutrino Oscillations in analogy with
  $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\implies$ $\nu \leftrightarrows \bar{\nu}$

▸ In 1957 only one neutrino $\nu = \nu_e$ was known!

▸ 1958: Goldhaber, Grodzins and Sunyar measure neutrino helicity:
  $\nu(h = -1)$

▸ Then, in weak interactions $\nu(h = -1)$ and $\bar{\nu}(h = +1)$

▸ Helicity conservation $\implies$ $\nu(h = -1) \leftrightarrows \bar{\nu}(h = -1)$

▸ $\bar{\nu}(h = -1)$ is a sterile neutrino (Pontecorvo, 1967)

▸ 1962: Lederman, Schwartz and Steinberger discover ν_μ

▸ 1962: Maki, Nakagava, Sakata proposed a model with neutrino mixing:

$$\nu_e = \cos\vartheta\, \nu_1 + \sin\vartheta\, \nu_2$$
$$\nu_\mu = -\sin\vartheta\, \nu_1 + \cos\vartheta\, \nu_2$$

"weak neutrinos are not stable due to the occurrence of a virtual
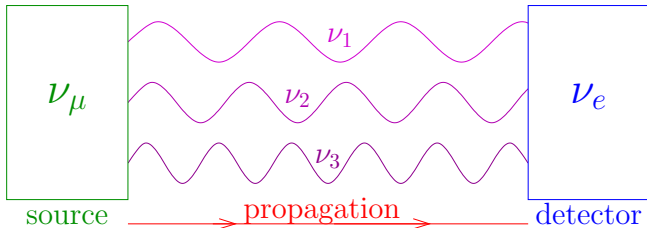transmutation $\nu_e \leftrightarrows \nu_\mu$"

▸ 1967: Pontecorvo: $\nu_e \leftrightarrows \nu_\mu$ oscillations and applications (solar
neutrinos)

- Flavor Neutrinos: ν_e, ν_μ, ν_τ produced in Weak Interactions

- Massive Neutrinos: ν_1, ν_2, ν_3 propagate from Source to Detector

- A Flavor Neutrino is a superposition of Massive Neutrinos

$$|\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$
$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$$
$$|\nu_\tau\rangle = U_{\tau 1}|\nu_1\rangle + U_{\tau 2}|\nu_2\rangle + U_{\tau 3}|\nu_3\rangle$$

- U is the 3×3 Neutrino Mixing Matrix

$$|\nu(t=0)\rangle = |\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$$

$$|\nu(t>0)\rangle = U_{\mu 1}\,e^{-iE_1 t}|\nu_1\rangle + U_{\mu 2}\,e^{-iE_2 t}|\nu_2\rangle + U_{\mu 3}\,e^{-iE_3 t}|\nu_3\rangle \neq |\nu_\mu\rangle$$
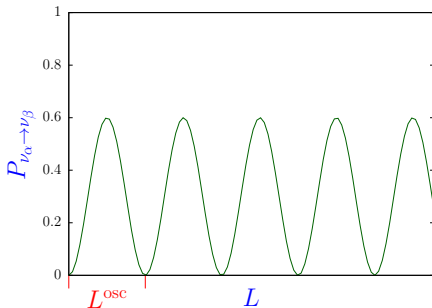
$$E_k^2 = p^2 + m_k^2 \qquad t \simeq L$$

$$P_{\nu_\mu \to \nu_e}(t>0) = |\langle \nu_e|\nu(t>0)\rangle|^2 \sim \sum_{k>j} \mathrm{Re}\big[U_{ek}U_{\mu k}^* U_{ej}^* U_{\mu j}\big]\sin^2\!\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$

transition probabilities depend on U and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$

$$\begin{array}{cccc}
\nu_e \to \nu_\mu & \nu_e \to \nu_\tau & \nu_\mu \to \nu_e & \nu_\mu \to \nu_\tau \\
\bar\nu_e \to \bar\nu_\mu & \bar\nu_e \to \bar\nu_\tau & \bar\nu_\mu \to \bar\nu_e & \bar\nu_\mu \to \bar\nu_\tau
\end{array}$$

- ▶ Neutrino Oscillations are due to interference of different phases of massive neutrinos: pure quantum-mechanical effect!

- ▶ Phases of massive neutrinos depend on distance $\Longrightarrow$ Oscillations depend on distance L

Ultrarelativistic Approximation

Only neutrinos with energy $\gtrsim 0.1\,MeV$ are detectable!

Charged-Current Processes: Threshold

$$\nu + A \to B + C$$
$$\Downarrow$$
$$s = 2Em_A + m_A^2 \geq (m_B + m_C)^2$$
$$\Downarrow$$
$$E_{th} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2}$$

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$	$E_{th} = 0.233$ MeV
$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$	$E_{th} = 0.81$ MeV
$\bar{\nu}_e + p \to n + e^+$	$E_{th} = 1.8$ MeV
$\nu_\mu + n \to p + \mu^-$	$E_{th} = 110$ MeV
$\nu_\mu + e^- \to \nu_e + \mu^-$	$E_{th} \simeq \frac{m_\mu^2}{2m_e} = 10.9$ GeV

Elastic Scattering Processes: Cross Section $\propto$ Energy

$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow E_{th} \simeq 5\,\text{MeV}\,(\text{SK, SNO})\,,\ 0.25\,\text{MeV}\,(\text{Borexino})$

Laboratory and Astrophysical Limits $\Longrightarrow \boxed{m_\nu \lesssim 1\,\text{eV}}$

Simple Example of Neutrino Production

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \boxed{\nu_\mu = \sum_k U_{\mu k} \nu_k}$$

two-body decay $\Longrightarrow$ fixed kinematics $\qquad \boxed{E_k^2 = p_k^2 + m_k^2}$

π at rest:
$$
\begin{cases}
p_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 - \dfrac{m_k^2}{2}\left(1 + \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2} \\[4mm]
E_k^2 = \dfrac{m_\pi^2}{4}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right)^2 + \dfrac{m_k^2}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) + \dfrac{m_k^4}{4\,m_\pi^2}
\end{cases}
$$

0th order: $m_k = 0 \Rightarrow p_k = E_k = E = \dfrac{m_\pi}{2}\left(1 - \dfrac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\text{MeV}$

1st order: $\qquad \boxed{E_k \simeq E + \xi\,\dfrac{m_k^2}{2E}} \qquad \boxed{p_k \simeq E - (1-\xi)\,\dfrac{m_k^2}{2E}}$

$$\xi = \frac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.2$$

Neutrino Oscillations in Vacuum

$$\mathcal{L}_{CC} \sim W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right)$$

Fields $\qquad \nu_\alpha = \sum_k U_{\alpha k} \nu_k \qquad \Longrightarrow \qquad |\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \qquad$ States

$$\text{initial flavor:} \quad \alpha \quad = \quad e \quad \text{or} \quad \mu \quad \text{or} \quad \tau$$

$$|\nu_k(t, x)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_k U^*_{\alpha k} e^{-iE_k t + i p_k x} |\nu_k\rangle$$

$$|\nu_k\rangle = \sum_{\beta = e, \mu, \tau} U_{\beta k} |\nu_\beta\rangle \quad \Rightarrow \quad |\nu_\alpha(t, x)\rangle = \sum_{\beta = e, \mu, \tau} \underbrace{\left(\sum_k U^*_{\alpha k} e^{-iE_k t + i p_k x} U_{\beta k} \right)}_{\mathcal{A}_{\nu_\alpha \to \nu_\beta}(t, x)} |\nu_\beta\rangle$$

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta}(0, 0) = \sum_k U^*_{\alpha k} U_{\beta k} = \delta_{\alpha \beta} \qquad\qquad \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t > 0, x > 0) \neq \delta_{\alpha \beta}$$

$$P_{\nu_\alpha \to \nu_\beta}(t, x) = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta}(t, x) \right|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t + ip_k x} U_{\beta k} \right|^2$$

ultra-relativistic neutrinos $\implies$ $t \simeq x = L$ source-detector distance

$$E_k t - p_k x \simeq (E_k - p_k) L = \frac{E_k^2 - p_k^2}{E_k + p_k} L = \frac{m_k^2}{E_k + p_k} L \simeq \frac{m_k^2}{2E} L$$

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L, E) &= \left| \sum_k U_{\alpha k}^* \, e^{-im_k^2 L/2E} \, U_{\beta k} \right|^2 \\ &= \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right) \end{aligned}$$
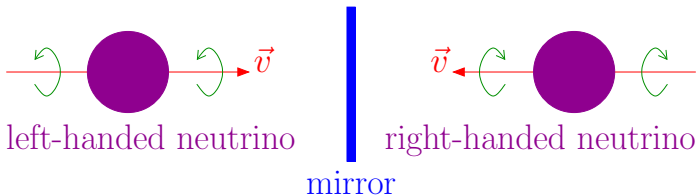
$$\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Neutrinos and Antineutrinos

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\mathsf{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu_{\alpha L}}^T$$

C	$\Longrightarrow$	Particle $\leftrightarrows$ Antiparticle
P	$\Longrightarrow$	Left-Handed $\leftrightarrows$ Right-Handed

left-handed neutrino   right-handed neutrino

mirror

Fields: $\nu_{\alpha L} = \displaystyle\sum_k U_{\alpha k} \nu_{kL} \xrightarrow{\text{CP}} \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$

States: $|\nu_\alpha\rangle = \displaystyle\sum_k U_{\alpha k}^* |\nu_k\rangle \xrightarrow{\text{CP}} |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$

NEUTRINOS $\quad U \;\leftrightarrows\; U^*\quad$ ANTINEUTRINOS

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L,E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\text{CPT}} P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $\quad A_{\alpha\beta}^{\text{CPT}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\quad\Longrightarrow\quad A_{\alpha\beta}^{\text{CPT}} = 0 \quad$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i\frac{\Delta m_{kj}^2 L}{2E}\right)$$

is invariant under CPT: $\qquad U \;\leftrightarrows\; U^* \qquad \alpha \;\leftrightarrows\; \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}} \qquad \text{(solar } \nu_e, \text{ reactor } \bar{\nu}_e, \text{ accelerator } \nu_\mu\text{)}$$

CP Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \xrightarrow{\ \text{CP}\ } P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A_{\alpha\beta}^{\text{CP}} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$A_{\alpha\beta}^{\text{CP}}(L, E) = 4 \sum_{k>j} \text{Im}\big[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \big] \sin\!\left(\frac{\Delta m_{kj}^2 L}{2E} \right)$$

Jarlskog rephasing invariant: $\quad \text{Im}\big[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \big] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c_{13}^2 s_{13} \sin \delta_{13}$$

$$J \neq 0 \iff \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

$$\text{CPT} \implies 0 = A_{\alpha\beta}^{\text{CPT}}$$

$$= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

$$= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \leftarrow A_{\alpha\beta}^{\text{CP}}$$

$$+ P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow -A_{\beta\alpha}^{\text{CPT}} = 0$$

$$+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A_{\beta\alpha}^{\text{CP}}$$

$$= A_{\alpha\beta}^{\text{CP}} + A_{\beta\alpha}^{\text{CP}} \qquad \implies \boxed{A_{\alpha\beta}^{\text{CP}} = -A_{\beta\alpha}^{\text{CP}}}$$

T Symmetry

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\mathsf{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $\quad A_{\alpha\beta}^{\mathsf{T}} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\mathsf{CPT} \quad \Longrightarrow \quad 0 = A_{\alpha\beta}^{\mathsf{CPT}}$$

$$= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

$$= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow A_{\alpha\beta}^{\mathsf{T}}$$

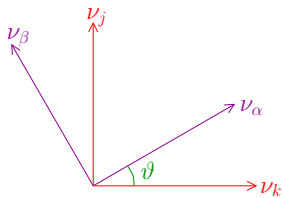$$+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A_{\beta\alpha}^{\mathsf{CP}}$$

$$= A_{\alpha\beta}^{\mathsf{T}} + A_{\beta\alpha}^{\mathsf{CP}}$$

$$= A_{\alpha\beta}^{\mathsf{T}} - A_{\alpha\beta}^{\mathsf{CP}} \qquad \Longrightarrow \quad \boxed{A_{\alpha\beta}^{\mathsf{T}} = A_{\alpha\beta}^{\mathsf{CP}}}$$

Two-Neutrino Oscillations

$$|\nu_\alpha\rangle = \cos\vartheta \, |\nu_k\rangle + \sin\vartheta \, |\nu_j\rangle$$
$$|\nu_\beta\rangle = -\sin\vartheta \, |\nu_k\rangle + \cos\vartheta \, |\nu_j\rangle$$

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Transition Probability:
$$P_{\nu_\alpha \to \nu_\beta} = P_{\nu_\beta \to \nu_\alpha} = \sin^2 2\vartheta \, \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$
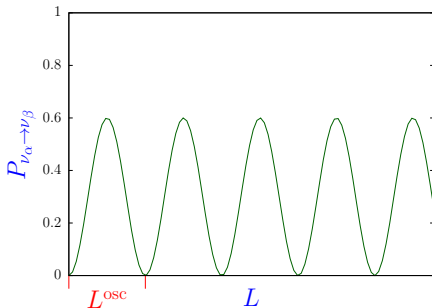
Survival Probabilities:
$$P_{\nu_\alpha \to \nu_\alpha} = P_{\nu_\beta \to \nu_\beta} = 1 - P_{\nu_\alpha \to \nu_\beta}$$

oscillation phase

$$\frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m^2 [\text{eV}^2] \, L[\text{m}]}{E[\text{MeV}]} = 1.27 \frac{\Delta m^2 [\text{eV}^2] \, L[\text{km}]}{E[\text{GeV}]}$$

oscillation length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.47 \frac{E \, [\text{MeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{m} = 2.47 \frac{E \, [\text{GeV}]}{\Delta m^2 \, [\text{eV}^2]} \, \text{km}$$

Types of Experiments

transitions due to Δm^2 observable only if $\dfrac{\Delta m^2 L}{E} \gtrsim 1 \Leftrightarrow \Delta m^2 \gtrsim \left(\dfrac{L}{E}\right)^{-1}$

SBL

$L/E \lesssim 10\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 0.1\,\text{eV}^2$

Reactor: $L \sim 10\,\text{m}$, $E \sim 1\,\text{MeV}$
Accelerator: $L \sim 1\,\text{km}$, $E \gtrsim 0.1\,\text{GeV}$

ATM & LBL

$L/E \lesssim 10^4\,\text{eV}^{-2}$
$\Downarrow$
$\Delta m^2 \gtrsim 10^{-4}\,\text{eV}^2$

Rea.: $L \sim 1\,\text{km}$, $E \sim 1\,\text{MeV}$ Daya Bay, RENO, Double Chooz
Acc.: $L \sim 10^3\,\text{km}$, $E \gtrsim 1\,\text{GeV}$ K2K, MINOS, OPERA, T2K,
NOνA
Atmospheric: $L \sim 10^2 - 10^4\,\text{km}$, $E \sim 0.1 - 10^2\,\text{GeV}$
Kamiokande, IMB, Super-Kamiokande, Soudan-2, MACRO

Solar

$\dfrac{L}{E} \sim 10^{11}\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-11}\,\text{eV}^2$

$L \sim 10^8\,\text{km}$, $E \sim 0.1 - 10\,\text{MeV}$
Homestake, Kamiokande, GALLEX, SAGE,
Super-Kamiokande, GNO, SNO, Borexino

Matter Effect (MSW) $\Rightarrow 10^{-4} \lesssim \sin^2 2\vartheta \lesssim 1$, $10^{-8}\,\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^{-4}\,\text{eV}^2$
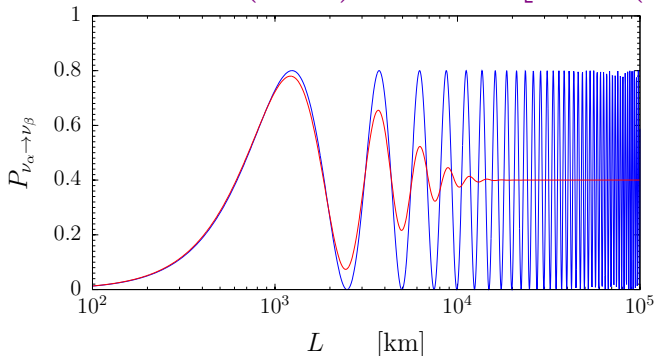
VLBL

$L/E \lesssim 10^5\,\text{eV}^{-2} \Rightarrow \Delta m^2 \gtrsim 10^{-5}\,\text{eV}^2$

Reactor: $L \sim 10^2\,\text{km}$, $E \sim 1\,\text{MeV}$
KamLAND

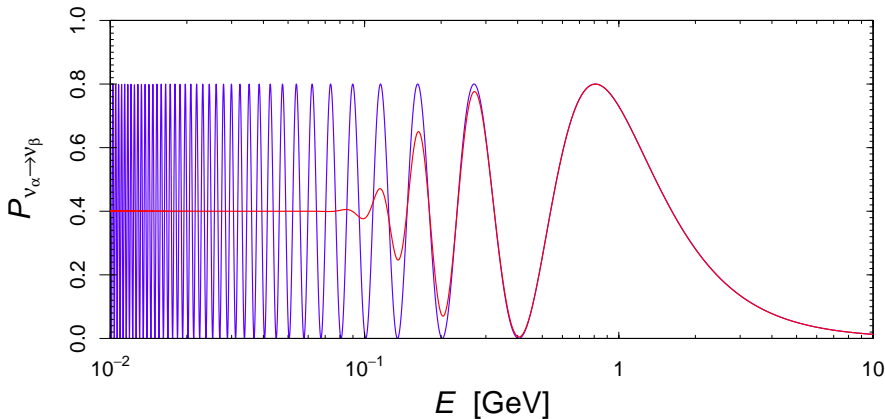# Average over Energy Resolution of the Detector

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) = \frac{1}{2}\,\sin^2 2\vartheta \left[1 - \cos\left(\frac{\Delta m^2 L}{2E}\right)\right]$$

$$\Delta m^2 = 10^{-3}\,\text{eV} \qquad \sin^2 2\vartheta = 0.8 \qquad \langle E \rangle = 1\,\text{GeV} \qquad \sigma_E = 0.1\,\text{GeV}$$
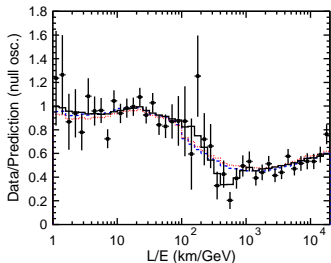
$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2}\,\sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\,\mathrm{d}E\right] \qquad (\alpha \neq \beta)$$
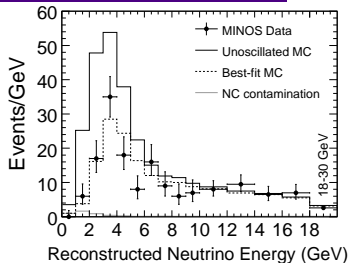
$$\Delta m^2 = 10^{-3}\,\text{eV} \qquad \sin^2 2\vartheta = 0.8 \qquad L = 10^3\,\text{km} \qquad \sigma_E = 0.01\,\text{GeV}$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2}\sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E)\,\mathrm{d}E \right] \qquad (\alpha \neq \beta)$$
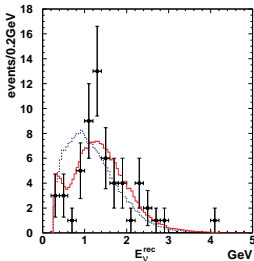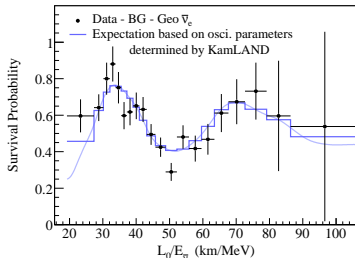
Observations of Neutrino Oscillations

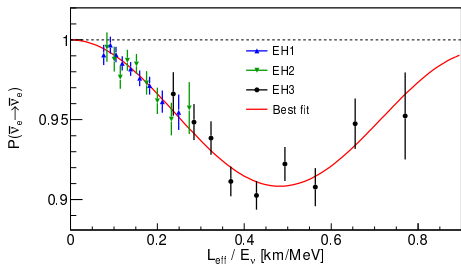[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]

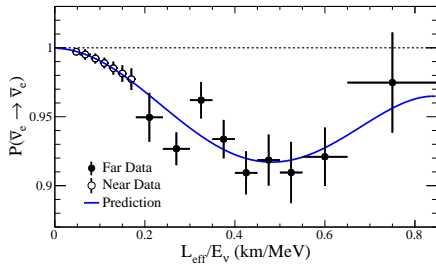[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]

[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]

[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

[Daya Bay, PRL, 112 (2014) 061801, arXiv:1310.6732]   [RENO, arXiv:1511.05849]

Exclusion Curves

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle = \frac{1}{2} \sin^2 2\vartheta \left[1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) \, dE \right] \qquad (\alpha \neq \beta)$$

$$\langle P_{\nu_\alpha \to \nu_\beta}(L, E) \rangle \leq P_{\nu_\alpha \to \nu_\beta}^{\max} \implies \sin^2 2\vartheta \leq \frac{2 P_{\nu_\alpha \to \nu_\beta}^{\max}}{1 - \int \cos\left(\frac{\Delta m^2 L}{2E}\right) \phi(E) \, dE}$$
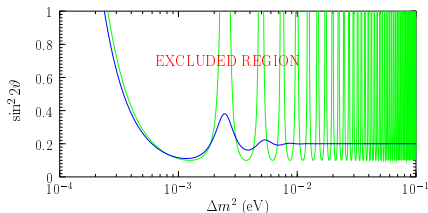
rotate and mirror
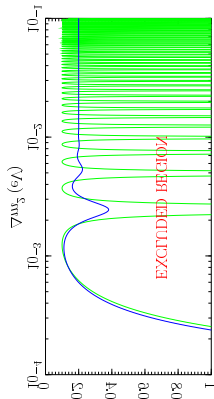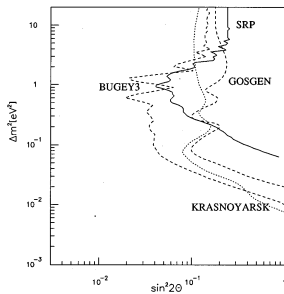
Reactor SBL Experiments: $\bar{\nu}_e \to \bar{\nu}_e$

Accelerator SBL Experiments: $\overset{(-)}{\nu_\mu} \to \overset{(-)}{\nu_e}$
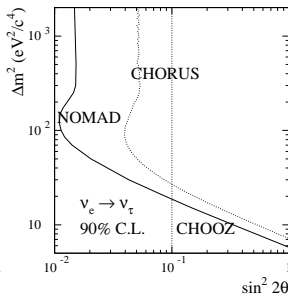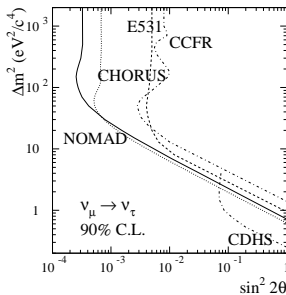
Accelerator SBL Experiments: $\overset{(-)}{\nu_\mu} \to \overset{(-)}{\nu_\tau}$ and $\overset{(-)}{\nu_e} \to \overset{(-)}{\nu_\tau}$

Anatomy of Exclusion Plots
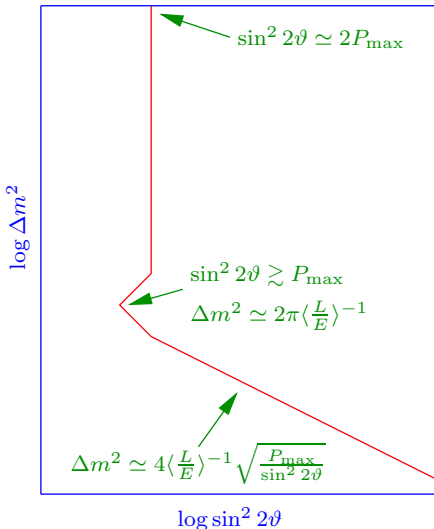
- $\Delta m^2 \gg \langle L/E \rangle^{-1}$

  $P_{\max} \simeq \dfrac{1}{2} \sin^2 2\vartheta \Rightarrow \sin^2 2\vartheta \simeq 2P_{\max}$

- $\text{Min}\left\langle \cos\left(\dfrac{\Delta m^2 L}{2E}\right) \right\rangle \geq -1$

  $\sin^2 2\vartheta = \dfrac{2\,P_{\max}}{1 - \text{Min}\left\langle \cos\left(\frac{\Delta m^2 L}{2E}\right)\right\rangle} \geq P_{\max}$

  $\Delta m^2 \simeq 2\pi \langle L/E \rangle^{-1}$

- $\Delta m^2 \ll 2\pi \langle L/E \rangle^{-1}$

  $\cos\left(\dfrac{\Delta m^2 L}{2E}\right) \simeq 1 - \dfrac{1}{2}\left(\dfrac{\Delta m^2 L}{2E}\right)^2$
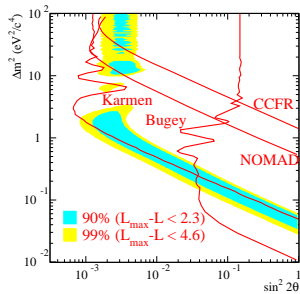
  $\Delta m^2 \simeq 4\left\langle \dfrac{L}{E}\right\rangle^{-1} \sqrt{\dfrac{P_{\max}}{\sin^2 2\vartheta}}$

$\dfrac{1}{2}\sin^2 2\vartheta \left[1 - \left\langle \cos\left(\dfrac{\Delta m^2 L}{2E}\right)\right\rangle\right] = P_{\max}$

Off-Axis Experiments

high-intensity WB beam
detector shifted by a small angle from axis of beam
almost monochromatic neutrino energy
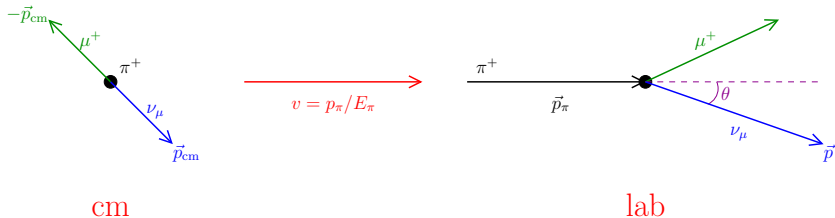
$$E_{\mathsf{cm}} = p_{\mathsf{cm}} = \frac{m_\pi}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 29.79\,\mathsf{MeV}$$

$$\gamma = \left(1 - v^2\right)^{-1/2} = E_\pi/m_\pi \gg 1 \qquad \begin{cases} E = \gamma\left(E_{\mathsf{cm}} + v\,p_{\mathsf{cm}}^z\right) \\ p^z = \gamma\left(v\,E_{\mathsf{cm}} + p_{\mathsf{cm}}^z\right) \end{cases}$$

$$p^z = p\cos\theta = E\cos\theta \qquad \Longrightarrow \qquad E = \frac{E_{\mathsf{cm}}}{\gamma\left(1 - v\cos\theta\right)}$$

$$\cos\theta \simeq 1 - \theta^2/2 \qquad \text{and} \qquad v \simeq 1$$

$$E = \frac{E_{\text{cm}}}{\gamma(1 - v\cos\theta)} \simeq \frac{\gamma(1+v)}{1 + \gamma^2\theta^2 v(1+v)/2} E_{\text{cm}} \simeq \frac{2\gamma}{1 + \gamma^2\theta^2} E_{\text{cm}}$$

$$E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \frac{E_\pi}{1 + \gamma^2\theta^2} = \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \frac{E_\pi\, m_\pi^2}{m_\pi^2 + E_\pi^2\,\theta^2}$$

- $\theta = 0 \implies E \propto E_\pi$ WB beam

- $E_\pi\theta \gg m_\pi \implies E \propto \dfrac{m_\pi^2}{E_\pi\,\theta^2}$ high-energy π^+ give low-energy ν_μ

$$\frac{dE}{dE_\pi} \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \frac{1 - \gamma^2\theta^2}{(1 + \gamma^2\theta^2)^2}$$

$$\frac{dE}{dE_\pi} \simeq 0 \quad \text{for} \quad \theta = \gamma^{-1} = \frac{m_\pi}{E_\pi} \implies E \simeq \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \frac{m_\pi}{2\theta} \simeq \frac{29.79\,\text{MeV}}{\theta}$$

off-axis angle $\quad \theta \simeq m_\pi / \langle E_\pi \rangle \quad \implies \quad E \simeq \dfrac{29.79\,\text{MeV}}{\theta}$
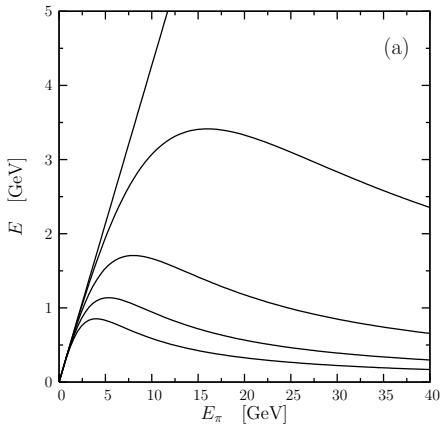
$\theta = 0.0^\circ, 0.5^\circ, 1.0^\circ, 1.5^\circ, 2.0^\circ$

► E can be tuned on oscillation peak $E_{\text{peak}} = \Delta m^2 L / 2\pi$

► small $E \implies$ short $L_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2} \implies$ sensitivity to small values of Δm^2

$$\frac{\phi(\theta)}{\phi(0)} = \frac{1}{4}\left(\frac{2}{1+\gamma^2\,\theta^2}\right)^2$$
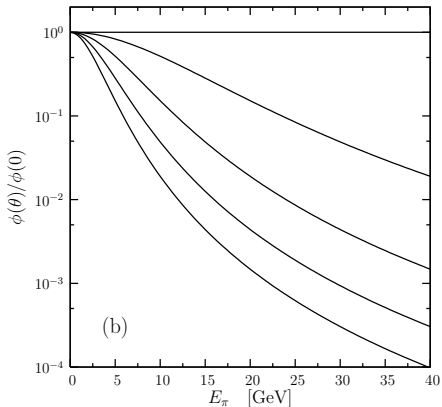
$\theta = 0.0°, 0.5°, 1.0°, 1.5°, 2.0°$

flux suppression requires superbeam

Neutrino Oscillations in Matter

- Neutrino Oscillations

- Neutrino Oscillations in Vacuum

- Two-Neutrino Oscillations

- Neutrino Oscillations in Matter
  - Effective Potentials in Matter
  - Evolution of Neutrino Flavors in Matter
  - Two-Neutrino Mixing
  - Constant Matter Density
  - MSW Effect (Resonant Transitions in Matter)

- Wave-Packet Theory of NuOsc

- Common Question: Do Charged Leptons Oscillate?

Effective Potentials in Matter

coherent interactions with medium: forward elastic CC and NC scattering
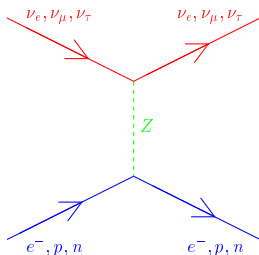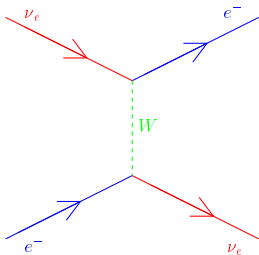
$$V_{\text{CC}} = \sqrt{2}\, G_{\text{F}} N_e \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

only $V_{\text{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\qquad \overline{V}_{\text{CC}} = -V_{\text{CC}} \qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

Evolution of Neutrino Flavors in Matter

▶ Flavor neutrino ν_α with momentum p: $\quad |\nu_\alpha(p)\rangle = \sum_k U^*_{\alpha k} |\nu_k(p)\rangle$

▶ Evolution is determined by Hamiltonian

▶ Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0 |\nu_k(p)\rangle = E_k |\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

▶ Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I |\nu_\alpha(p)\rangle = V_\alpha |\nu_\alpha(p)\rangle$

▶ Schrödinger evolution equation: $i \dfrac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H}|\nu(p,t)\rangle$

▶ Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$

▶ For $t > 0$ the state $|\nu(p,t)\rangle$ is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t)|\nu_\beta(p)\rangle$$

▶ Transition probability: $P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2$

evolution equation of states

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p, t)\rangle = \mathcal{H}|\nu(p, t)\rangle \,, \qquad |\nu(p, 0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes

$$\varphi_\beta(p, t) = \langle \nu_\beta(p)|\nu(p, t)\rangle \,, \qquad \varphi_\beta(p, 0) = \delta_{\alpha\beta}$$

evolution of flavor transition amplitudes

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p, t) = \langle \nu_\beta(p)|\mathcal{H}|\nu(p, t)\rangle$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p, t) = \langle \nu_\beta(p)|\mathcal{H}_0|\nu(p, t)\rangle + \langle \nu_\beta(p)|\mathcal{H}_I|\nu(p, t)\rangle$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p, t) = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p, t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p, t) \rangle$$

$$\langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p, t) \rangle =$$

$$\sum_\rho \sum_{k,j} \underbrace{\langle \nu_\beta(p) | \nu_k(p) \rangle}_{U_{\beta k}} \underbrace{\langle \nu_k(p) | \mathcal{H}_0 | \nu_j(p) \rangle}_{\delta_{kj} E_k} \underbrace{\langle \nu_j(p) | \nu_\rho(p) \rangle}_{U_{\rho j}^*} \underbrace{\langle \nu_\rho(p) | \nu(p, t) \rangle}_{\varphi_\rho(p, t)}$$

$$= \sum_\rho \sum_k U_{\beta k} E_k U_{\rho k}^* \varphi_\rho(p, t)$$

$$\langle \nu_\beta(p) | \mathcal{H}_I | \nu(p, t) \rangle = \sum_\rho \underbrace{\langle \nu_\beta(p) | \mathcal{H}_I | \nu_\rho(p) \rangle}_{\delta_{\beta\rho} V_\beta} \underbrace{\langle \nu_\rho(p) | \nu(p, t) \rangle}_{\varphi_\rho(p, t)}$$

$$= \sum_\rho \delta_{\beta\rho} V_\beta \varphi_\rho(p, t)$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k} E_k U_{\rho k}^* + \delta_{\beta\rho} V_\beta \right) \varphi_\rho}$$

ultrarelativistic neutrinos: $\qquad E_k = p + \dfrac{m_k^2}{2E} \qquad E = p \qquad t = x$

$$V_e = V_{CC} + V_{NC} \qquad\qquad V_\mu = V_\tau = V_{NC}$$

$$i\,\frac{d}{dx}\,\varphi_\beta(p,x) = (p + V_{NC})\,\varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k}\,\frac{m_k^2}{2E}\,U_{\rho k}^* + \delta_{\beta e}\,\delta_{\rho e}\,V_{CC} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x)\, e^{ipx + i\int_0^x V_{NC}(x')\,dx'}$$

$$\Downarrow$$

$$i\,\frac{d}{dx}\,\psi_\beta = e^{ipx + i\int_0^x V_{NC}(x')\,dx'} \left(-p - V_{NC} + i\,\frac{d}{dx} \right) \varphi_\beta$$

$$\boxed{\; i\,\frac{d}{dx}\,\psi_\beta = \sum_\rho \left(\sum_k U_{\beta k}\,\frac{m_k^2}{2E}\,U_{\rho k}^* + \delta_{\beta e}\,\delta_{\rho e}\,V_{CC} \right) \psi_\rho \;}$$

$$\boxed{\; P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2 \;}$$

evolution of flavor transition amplitudes in matrix form

$$i \frac{d}{dx} \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{CC} = 2 E V_{CC} = 2\sqrt{2} E G_F N_e$$

effective mass-squared matrix in vacuum $\quad \mathbb{M}_{VAC}^2 = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \mathbb{V} = \mathbb{M}_{MAT}^2 \quad$ effective mass-squared matrix in matter

potential due to coherent forward elastic scattering

Two-Neutrino Mixing

$\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta\, m_1^2 + \sin^2\vartheta\, m_2^2 & \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) \\ \cos\vartheta\sin\vartheta\left(m_2^2 - m_1^2\right) & \sin^2\vartheta\, m_1^2 + \cos^2\vartheta\, m_2^2 \end{pmatrix}$$

$$= \frac{1}{2}\,\Sigma m^2 + \frac{1}{2}\begin{pmatrix} -\Delta m^2 \cos 2\vartheta & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta \end{pmatrix}$$

$\uparrow$

irrelevant common phase

$$\Sigma m^2 \equiv m_1^2 + m_2^2 \qquad\qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{CC} & \Delta m^2\sin 2\vartheta\\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

$$\text{initial } \nu_e \implies \begin{pmatrix}\psi_e(0)\\\psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2$$
$$P_{\nu_e\to\nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x)$$

Constant Matter Density

$$i\frac{d}{dx}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{CC} & \Delta m^2\sin 2\vartheta\\\Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

$$\frac{dA_{CC}}{dx} = 0$$

diagonalization of effective hamiltonian: $\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_M & \sin\vartheta_M\\-\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}$

$$\begin{pmatrix}\cos\vartheta_M & -\sin\vartheta_M\\\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{CC} & \Delta m^2\sin 2\vartheta\\\Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_M & \sin\vartheta_M\\-\sin\vartheta_M & \cos\vartheta_M\end{pmatrix} =$$

$$= \begin{pmatrix}A_{CC}-\Delta m_M^2 & 0\\0 & A_{CC}+\Delta m_M^2\end{pmatrix}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix} = \left[\frac{A_{CC}}{4E} + \frac{1}{4E}\begin{pmatrix}-\Delta m_M^2 & 0\\0 & \Delta m_M^2\end{pmatrix}\right]\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}$$

irrelevant common phase

Effective Mixing Angle in Matter

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\vartheta - A_{CC})^2 + (\Delta m^2 \sin 2\vartheta)^2}$$

Resonance ($\vartheta_M = \pi/4$)

$$A_{CC}^R = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N_e^R = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_F}$$

$$i\frac{d}{dx}\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m_M^2 & 0\\0 & \Delta m_M^2\end{pmatrix}\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_M & \sin\vartheta_M\\-\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}\quad\Rightarrow\quad\begin{pmatrix}\psi_1^M\\\psi_2^M\end{pmatrix}=\begin{pmatrix}\cos\vartheta_M & -\sin\vartheta_M\\\sin\vartheta_M & \cos\vartheta_M\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

$$\nu_e\to\nu_\mu\quad\Longrightarrow\quad\begin{pmatrix}\psi_e(0)\\\psi_\mu(0)\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}\quad\Longrightarrow\quad\begin{pmatrix}\psi_1^M(0)\\\psi_2^M(0)\end{pmatrix}=\begin{pmatrix}\cos\vartheta_M\\\sin\vartheta_M\end{pmatrix}$$

$$\psi_1^M(x)=\cos\vartheta_M\exp\left(i\frac{\Delta m_M^2 x}{4E}\right)$$

$$\psi_2^M(x)=\sin\vartheta_M\exp\left(-i\frac{\Delta m_M^2 x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x)=|\psi_\mu(x)|^2=\left|-\sin\vartheta_M\psi_1^M(x)+\cos\vartheta_M\psi_2^M(x)\right|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x)=\sin^2 2\vartheta_M\sin^2\left(\frac{\Delta m_M^2 x}{4E}\right)}$$

MSW Effect (Resonant Transitions in Matter)
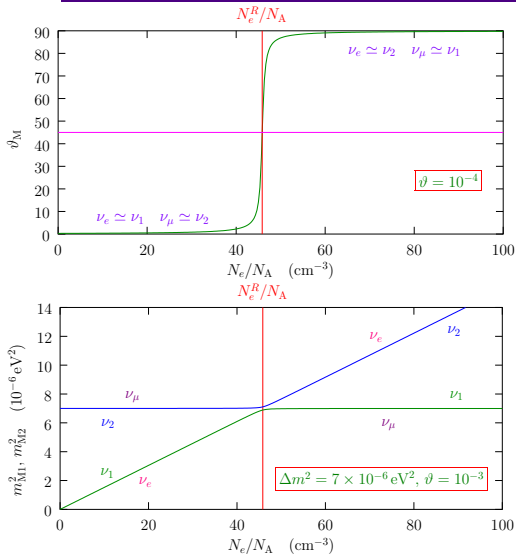
$$\nu_e = \cos\vartheta_{\mathrm{M}}\,\nu_1 + \sin\vartheta_{\mathrm{M}}\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_{\mathrm{M}}\,\nu_1 + \cos\vartheta_{\mathrm{M}}\,\nu_2$$

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m^2_{\mathrm{M}} = \Big[\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \Big]^{1/2}$$

$$i\frac{\mathsf{d}}{\mathsf{d}x}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{\mathsf{CC}}&\Delta m^2\sin 2\vartheta\\\Delta m^2\sin 2\vartheta&\Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

tentative diagonalization: $\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathsf{M}}&\sin\vartheta_{\mathsf{M}}\\-\sin\vartheta_{\mathsf{M}}&\cos\vartheta_{\mathsf{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}$

$$i\frac{\mathsf{d}}{\mathsf{d}x}\begin{pmatrix}\cos\vartheta_{\mathsf{M}}&\sin\vartheta_{\mathsf{M}}\\-\sin\vartheta_{\mathsf{M}}&\cos\vartheta_{\mathsf{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}=$$

$$=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{\mathsf{CC}}&\Delta m^2\sin 2\vartheta\\\Delta m^2\sin 2\vartheta&\Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathsf{M}}&\sin\vartheta_{\mathsf{M}}\\-\sin\vartheta_{\mathsf{M}}&\cos\vartheta_{\mathsf{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}$$

if matter density is not constant $\mathsf{d}\vartheta_{\mathsf{M}}/\mathsf{d}x\neq 0$

$$i\frac{\mathsf{d}}{\mathsf{d}x}\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}=\left[\frac{A_{\mathsf{CC}}}{4E}+\frac{1}{4E}\begin{pmatrix}-\Delta m_{\mathsf{M}}^2&0\\0&\Delta m_{\mathsf{M}}^2\end{pmatrix}+\begin{pmatrix}0&-i\dfrac{\mathsf{d}\vartheta_{\mathsf{M}}}{\mathsf{d}x}\\i\dfrac{\mathsf{d}\vartheta_{\mathsf{M}}}{\mathsf{d}x}&0\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}$$

$\uparrow$

irrelevant common phase

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}=\left[\frac{1}{4E}\begin{pmatrix}-\Delta m_{\mathsf{M}}^2 & 0\\ 0 & \Delta m_{\mathsf{M}}^2\end{pmatrix}+\begin{pmatrix}0 & -i\dfrac{\mathrm{d}\vartheta_{\mathsf{M}}}{\mathrm{d}x}\\ i\dfrac{\mathrm{d}\vartheta_{\mathsf{M}}}{\mathrm{d}x} & 0\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathsf{M}}\\\psi_2^{\mathsf{M}}\end{pmatrix}$$

$\uparrow$

adiabatic

$\uparrow$

non-adiabatic

maximum at resonance

initial conditions:

$$\begin{pmatrix}\psi_1^{\mathsf{M}}(0)\\\psi_2^{\mathsf{M}}(0)\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathsf{M}}^0 & -\sin\vartheta_{\mathsf{M}}^0\\\sin\vartheta_{\mathsf{M}}^0 & \cos\vartheta_{\mathsf{M}}^0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathsf{M}}^0\\\sin\vartheta_{\mathsf{M}}^0\end{pmatrix}$$

solution approximating all non-adiabatic $\nu_1^{\mathsf{M}}\leftrightarrows\nu_2^{\mathsf{M}}$ transitions in resonance

$$\psi_1^{\mathsf{M}}(x)\simeq\left[\cos\vartheta_{\mathsf{M}}^0\exp\left(i\int_0^{x_{\mathsf{R}}}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}_{11}^{\mathsf{R}}+\sin\vartheta_{\mathsf{M}}^0\exp\left(-i\int_0^{x_{\mathsf{R}}}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}_{21}^{\mathsf{R}}\right]$$
$$\times\exp\left(i\int_{x_{\mathsf{R}}}^{x}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)$$
$$\psi_2^{\mathsf{M}}(x)\simeq\left[\cos\vartheta_{\mathsf{M}}^0\exp\left(i\int_0^{x_{\mathsf{R}}}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}_{12}^{\mathsf{R}}+\sin\vartheta_{\mathsf{M}}^0\exp\left(-i\int_0^{x_{\mathsf{R}}}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}_{22}^{\mathsf{R}}\right]$$
$$\times\exp\left(-i\int_{x_{\mathsf{R}}}^{x}\frac{\Delta m_{\mathsf{M}}^2(x')}{4E}\,\mathrm{d}x'\right)$$

Averaged ν_e Survival Probability on Earth

$$\psi_e(x) = \cos\vartheta\,\psi_1^{\mathsf{M}}(x) + \sin\vartheta\,\psi_2^{\mathsf{M}}(x)$$

neglect interference (averaged over energy spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\,\cos^2\vartheta_{\mathsf{M}}^0\,|\mathcal{A}_{11}^{\mathsf{R}}|^2 + \cos^2\vartheta\,\sin^2\vartheta_{\mathsf{M}}^0\,|\mathcal{A}_{21}^{\mathsf{R}}|^2$$
$$+ \sin^2\vartheta\,\cos^2\vartheta_{\mathsf{M}}^0\,|\mathcal{A}_{12}^{\mathsf{R}}|^2 + \sin^2\vartheta\,\sin^2\vartheta_{\mathsf{M}}^0\,|\mathcal{A}_{22}^{\mathsf{R}}|^2$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^{\mathsf{R}}|^2 = |\mathcal{A}_{21}^{\mathsf{R}}|^2 = P_{\mathsf{c}} \qquad\qquad |\mathcal{A}_{11}^{\mathsf{R}}|^2 = |\mathcal{A}_{22}^{\mathsf{R}}|^2 = 1 - P_{\mathsf{c}}$$

$$P_{\mathsf{c}} \equiv \text{crossing probability}$$

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_{\mathsf{c}}\right)\cos 2\vartheta_{\mathsf{M}}^0\,\cos 2\vartheta}$$

[Parke, PRL 57 (1986) 1275]

Crossing Probability

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma\frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $\qquad \gamma = \left.\dfrac{\Delta m_M^2/2E}{2|d\vartheta_M/dx|}\right|_R = \dfrac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta}\left|\dfrac{d\ln A_{CC}}{dx}\right|_R$

$A \propto x$ $\qquad F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $\qquad F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp\left(-x\right)$ $\quad F = 1 - \tan^2\vartheta$ [Pizzochero, PRD 36 (1987) 2293]
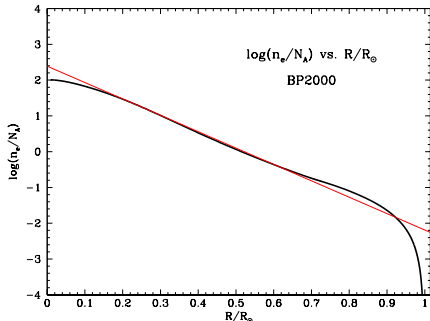
[Toshev, PLB 196 (1987) 170]

[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

Solar Neutrinos

SUN: $\qquad N_e(x) \simeq N_e^c \exp\left(-\dfrac{x}{x_0}\right) \qquad N_e^c = 245\,N_A/\text{cm}^3 \qquad x_0 = \dfrac{R_\odot}{10.54}$

$\log(n_e/N_A)$ vs. $R/R_\odot$

BP2000

$$\overline{P}^{\text{sun}}_{\nu_e \to \nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta_M^0 \cos 2\vartheta$$

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

$$\gamma = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{\text{CC}}}{dx}\right|_R}$$

$$F = 1 - \tan^2\vartheta$$

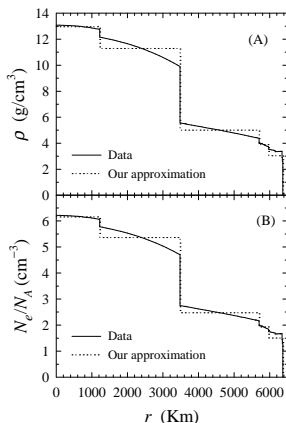$$A_{\text{CC}} = 2\sqrt{2}EG_F N_e$$

practical prescription:

[Lisi et al., PRD 63 (2001) 093002]

$$\left\{\begin{array}{ll} \text{numerical } |d\ln A_{\text{CC}}/dx|_R & \text{for } x \leq 0.904 R_\odot \\ |d\ln A_{\text{CC}}/dx|_R \to \dfrac{18.9}{R_\odot} & \text{for } x > 0.904 R_\odot \end{array}\right.$$

Electron Neutrino Regeneration in the Earth

$$P^{\text{sun+earth}}_{\nu_e \to \nu_e} = \overline{P}^{\text{sun}}_{\nu_e \to \nu_e} + \frac{\left(1 - 2\overline{P}^{\text{sun}}_{\nu_e \to \nu_e}\right)\left(P^{\text{earth}}_{\nu_2 \to \nu_e} - \sin^2\vartheta\right)}{\cos 2\vartheta}$$

[Mikheev, Smirnov, Sov. Phys. Usp. 30 (1987) 759], [Baltz, Weneser, PRD 35 (1987) 528]

$P^{\text{earth}}_{\nu_2 \to \nu_e}$ is usually calculated numerically approximating the Earth density profile with a step function.

Effective massive neutrinos propagate as plane waves in regions of constant density.

Wave functions of flavor neutrinos are joined at the boundaries of steps.

Solar Neutrino Oscillations

LMA (Large Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-5}$ eV^2,	$\tan^2 \vartheta \sim 0.8$
LOW (LOW Δm^2):	$\Delta m^2 \sim 7 \times 10^{-8}$ eV^2,	$\tan^2 \vartheta \sim 0.6$
SMA (Small Mixing Angle):	$\Delta m^2 \sim 5 \times 10^{-6}$ eV^2,	$\tan^2 \vartheta \sim 10^{-3}$
QVO (Quasi-Vacuum Oscillations):	$\Delta m^2 \sim 10^{-9}$ eV^2,	$\tan^2 \vartheta \sim 1$
VAC (VACuum oscillations):	$\Delta m^2 \lesssim 5 \times 10^{-10}$ eV^2,	$\tan^2 \vartheta \sim 1$
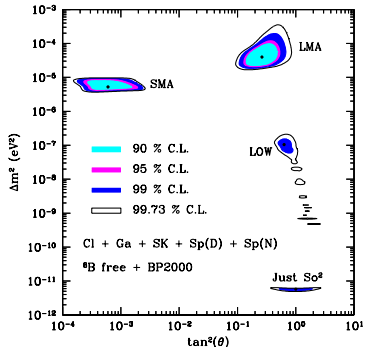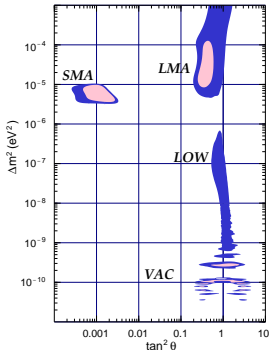
[de Gouvea, Friedland, Murayama, PLB 490 (2000) 125]

[Bahcall, Krastev, Smirnov, JHEP 05 (2001) 015]
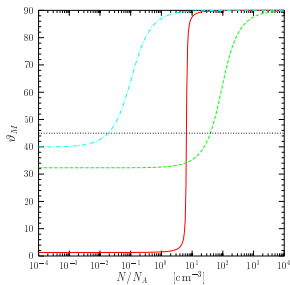
solid line: $\Delta m^2 = 5 \times 10^{-6}\,\text{eV}^2$
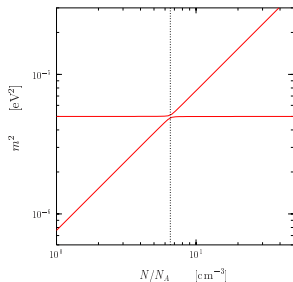(typical SMA) $\tan^2 \vartheta = 5 \times 10^{-4}$

dashed line: $\Delta m^2 = 7 \times 10^{-5}\,\text{eV}^2$
(typical LMA) $\tan^2 \vartheta = 0.4$

dash-dotted line: $\Delta m^2 = 8 \times 10^{-8}\,\text{eV}^2$
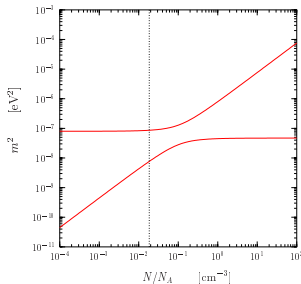(typical LOW) $\tan^2 \vartheta = 0.7$
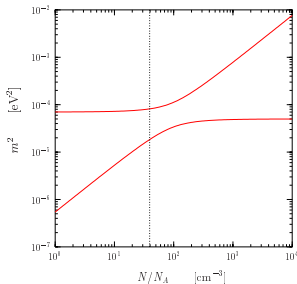
typical SMA   typical LMA   typical LOW
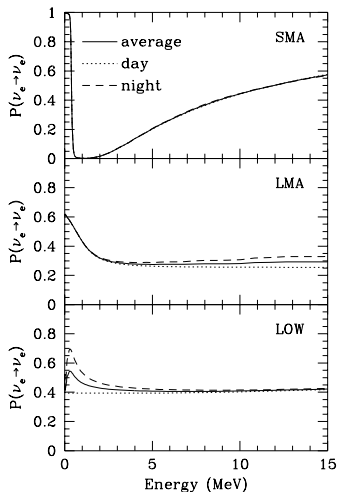
SMA: $\Delta m^2 = 5.0 \times 10^{-6}$ eV^2 $\sin^2 2\vartheta = 3.5 \times 10^{-3}$
LMA: $\Delta m^2 = 1.6 \times 10^{-5}$ eV^2 $\sin^2 2\vartheta = 0.57$
LOW: $\Delta m^2 = 7.9 \times 10^{-8}$ eV^2 $\sin^2 2\vartheta = 0.95$
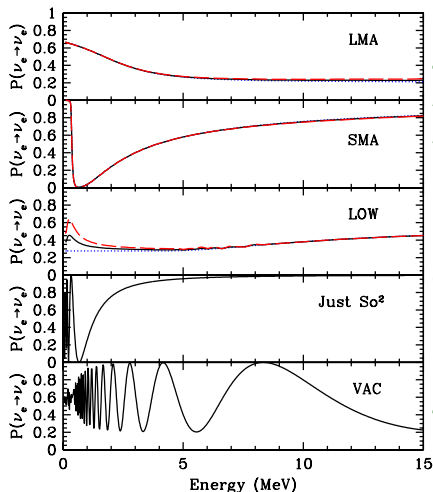
LMA: $\Delta m^2 = 4.2 \times 10^{-5}$ eV^2 $\tan^2 \vartheta = 0.26$
SMA: $\Delta m^2 = 5.2 \times 10^{-6}$ eV^2 $\tan^2 \vartheta = 5.5 \times 10^{-4}$
LOW: $\Delta m^2 = 7.6 \times 10^{-8}$ eV^2 $\tan^2 \vartheta = 0.72$
Just So^2: $\Delta m^2 = 5.5 \times 10^{-12}$ eV^2 $\tan^2 \vartheta = 1.0$
VAC: $\Delta m^2 = 1.4 \times 10^{-10}$ eV^2 $\tan^2 \vartheta = 0.38$

In Neutrino Oscillations Dirac = Majorana

[Bilenky, Hosek, Petcov, PLB 94 (1980) 495; Doi, Kotani, Nishiura, Okuda, Takasugi, PLB 102 (1981) 323]

[Langacker, Petcov, Steigman, Toshev, NPB 282 (1987) 589]

Evolution of Amplitudes:
$$i \frac{d\psi_\alpha}{dx} = \frac{1}{2E} \sum_\beta \left(U M^2 U^\dagger + 2EV \right)_{\alpha\beta} \psi_\beta$$

difference:
$$\begin{cases} \text{Dirac:} & U^{(D)} \\ \text{Majorana:} & U^{(M)} = U^{(D)} D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \implies DM^2 = M^2 D \implies DM^2 D^\dagger = M^2$$

$$U^{(M)} M^2 (U^{(M)})^\dagger = U^{(D)} D M^2 D^\dagger (U^{(D)})^\dagger = U^{(D)} M^2 (U^{(D)})^\dagger$$

Wave-Packet Theory of NuOsc

$$t \simeq x = L \iff \text{Wave Packets}$$

Space-Time uncertainty	$\iff$	Localization of production and detection processes

Energy-Momentum uncertainty	$\iff$	Coherent creation and detection of different massive neutrinos

[Kayser, PRD 24 (1981) 110]   [CG, FPL 17 (2004) 103]
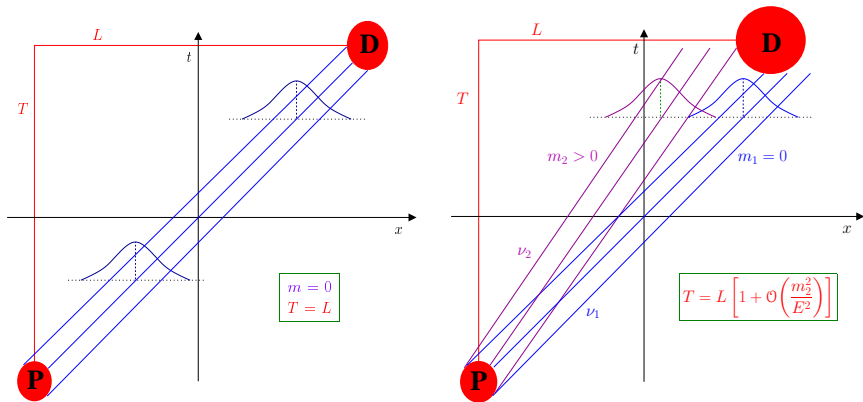
The size of the massive neutrino wave packets is determined by the coherence time δt_{P} of the Production Process

($\delta t_{\mathrm{P}} \gtrsim \delta x_{\mathrm{P}}$, because the coherence region must be causally connected)

velocity of neutrino wave packets: $\qquad v_k = \dfrac{p_k}{E_k} \simeq 1 - \dfrac{m_k^2}{2E^2}$

Coherence Length

[Nussinov, PLB 63 (1976) 201] [Kiers, Nussinov, Weiss, PRD 53 (1996) 537]

Wave Packets have different velocities and separate

different massive neutrinos can interfere if and only if wave packets arrive with $\delta t_{kj} \lesssim \sqrt{(\delta t_P)^2 + (\delta t_D)^2}$ $\Rightarrow$ $L \lesssim L_{kj}^{\mathrm{coh}}$
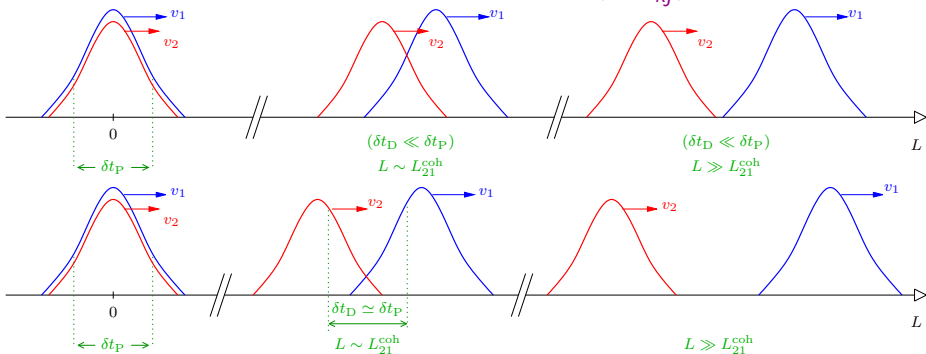
$$|\delta t_{kj}| \simeq |v_k - v_j|\, T \simeq \frac{|\Delta m_{kj}^2|}{2E^2} L \implies L_{kj}^{\mathrm{coh}} \sim \frac{2E^2}{|\Delta m_{kj}^2|} \sqrt{(\delta t_P)^2 + (\delta t_D)^2}$$

Quantum Mechanical Wave Packet Model

neglecting mass effects in amplitudes of production and detection processes

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* \int \mathrm{d}p \, \psi_k^P(p) \, |\nu_k(p)\rangle \qquad |\nu_\beta\rangle = \sum_k U_{\beta k}^* \int \mathrm{d}p \, \psi_k^D(p) \, |\nu_k(p)\rangle$$

$$\mathcal{A}_{\alpha\beta}(x,t) = \langle\nu_\beta| \, e^{-i\widehat{E}t + i\widehat{P}x} \, |\nu_\alpha\rangle$$

$$= \sum_k U_{\alpha k}^* \, U_{\beta k} \int \mathrm{d}p \, \psi_k^P(p) \, \psi_k^{D*}(p) \, e^{-iE_k(p)t + ipx}$$

Gaussian Approximation of Wave Packets

$$\psi_k^P(p) = \left(2\pi\sigma_{pP}^2\right)^{-1/4} \exp\left[-\frac{(p - p_k)^2}{4\sigma_{pP}^2}\right]$$

$$\psi_k^D(p) = \left(2\pi\sigma_{pD}^2\right)^{-1/4} \exp\left[-\frac{(p - p_k)^2}{4\sigma_{pD}^2}\right]$$

the value of p_k is determined by the production process (causality)

$$\mathcal{A}_{\alpha\beta}(x,t) \propto \sum_k U^*_{\alpha k} U_{\beta k} \int dp \exp\left[-iE_k(p)t + ipx - \frac{(p-p_k)^2}{4\sigma_p^2}\right]$$

global energy-momentum uncertainty: $\boxed{\dfrac{1}{\sigma_p^2} = \dfrac{1}{\sigma_{pP}^2} + \dfrac{1}{\sigma_{pD}^2}}$

sharply peaked wave packets

$$\sigma_p \ll E_k^2(p_k)/m_k \implies E_k(p) = \sqrt{p^2 + m_k^2} \simeq E_k + v_k(p - p_k)$$

$$E_k = E_k(p_k) = \sqrt{p_k^2 + m_k^2} \qquad v_k = \left.\frac{\partial E_k(p)}{\partial p}\right|_{p=p_k} = \frac{p_k}{E_k} \quad \text{group velocity}$$

$$\mathcal{A}_{\alpha\beta}(x,t) \propto \sum_k U^*_{\alpha k} U_{\beta k} \exp\left[-iE_k t + ip_k x - \underbrace{\frac{(x - v_k t)^2}{4\sigma_x^2}}\right]$$

suppression factor
for $|x - v_k t| \gtrsim \sigma_x$

$\boxed{\sigma_x \sigma_p = \dfrac{1}{2}}$ global space-time uncertainty: $\boxed{\sigma_x^2 = \sigma_{xP}^2 + \sigma_{xD}^2}$
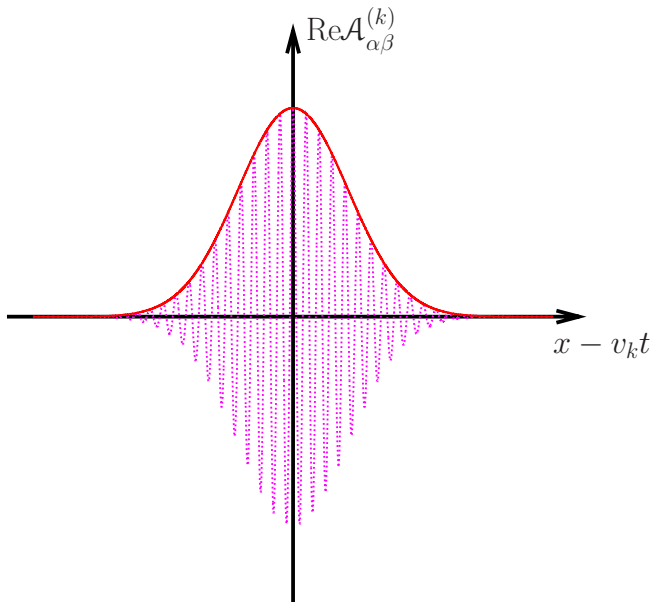
ν_k–ν_j interferene only if $\mathcal{A}_{\alpha\beta}^{(k)}$ and $\mathcal{A}_{\alpha\beta}^{(j)}$ overlap at detection

$$-E_k t + p_k x = -(E_k - p_k) x + E_k (x - t) = -\frac{E_k^2 - p_k^2}{E_k + p_k} x + E_k (x - t)$$

$$= -\frac{m_k^2}{E_k + p_k} x + E_k (x - t) \simeq -\frac{m_k^2}{2E} x + E_k (x - t)$$

$$\mathcal{A}_{\alpha\beta}(x, t) \propto \sum_k U_{\alpha k}^* U_{\beta k} \exp \left[\underbrace{-i \frac{m_k^2}{2E} x}_{\substack{\text{standard} \\ \text{phase} \\ \text{for } t = x}} \underbrace{+ iE_k (x - t)}_{\substack{\text{additional} \\ \text{phase} \\ \text{for } t \neq x}} - \frac{(x - v_k t)^2}{4\sigma_x^2} \right]$$

Space-Time Flavor Transition Probability

$$P_{\alpha\beta}(x,t) \propto \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left[\underbrace{-i\,\frac{\Delta m^2_{kj}\,x}{2E}}_{\substack{\text{standard}\\\text{phase}\\\text{for } t=x}} + \underbrace{i\,(E_k - E_j)\,(x-t)}_{\substack{\text{additional}\\\text{phase}\\\text{for } t \neq x}} \right]$$

$$\times \exp\left[\underbrace{-\frac{(x - \overline{v}_{kj}t)^2}{4\sigma^2_x}}_{\substack{\text{suppression}\\\text{factor for}\\|x - \overline{v}_{kj}t| \gtrsim \sigma_x}} - \underbrace{\frac{(v_k - v_j)^2\,t^2}{8\sigma^2_x}}_{\substack{\text{suppression}\\\text{factor}\\\text{due to}\\\text{separation of}\\\text{wave packets}}} \right]$$

$$v_k = \frac{p_k}{E_k} \simeq 1 - \frac{m^2_k}{2E^2} \qquad \overline{v}_{kj} = \frac{v_k + v_j}{2} \simeq 1 - \frac{m^2_k + m^2_j}{4E^2}$$

Oscillations in Space:
$$P_{\alpha\beta}(L) \propto \int \mathrm{d}t\, P_{\alpha\beta}(L,t)$$

Gaussian integration over $\mathrm{d}t$

$$P_{\alpha\beta}(L) \propto \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\frac{\Delta m_{kj}^2 L}{2E}\right]$$

$$\times \underbrace{\sqrt{\frac{2}{v_k^2 + v_j^2}}}_{\simeq 1} \exp\left[-\underbrace{\frac{(v_k - v_j)^2}{v_k^2 + v_j^2}\frac{L^2}{4\sigma_x^2}}_{\simeq \left(\Delta m_{kj}^2\right)^2/8E^4} - \underbrace{\frac{(E_k - E_j)^2}{v_k^2 + v_j^2}\sigma_x^2}_{\simeq \xi^2\left(\Delta m_{kj}^2\right)^2/8E^2}\right]$$

$$\times \exp\left[i\underbrace{(E_k - E_j)\left(1 - \frac{2\overline{v}_{kj}^2}{v_k^2 + v_j^2}\right)L}_{\ll \Delta m_{kj}^2/2E}\right]$$

Ultrarelativistic Neutrinos:
$$p_k \simeq E - (1-\xi)\frac{m_k^2}{2E} \qquad E_k \simeq E + \xi\frac{m_k^2}{2E}$$

$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-i\frac{\Delta m_{kj}^2 L}{2E}\right]$$

$$\times \exp\left[-\left(\frac{\Delta m_{kj}^2 L}{4\sqrt{2}E^2\sigma_x}\right)^2 - 2\xi^2\left(\frac{\Delta m_{kj}^2 \sigma_x}{4E}\right)^2\right]$$

Oscillation
Lengths

$$L_{kj}^{\text{osc}} = \frac{4\pi E}{\Delta m_{kj}^2}$$

Coherence
Lengths

$$L_{kj}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{kj}^2|}\sigma_x$$

$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left[-2\pi i\frac{L}{L_{kj}^{\text{osc}}}\right]$$

$$\times \exp\left[-\left(\frac{L}{L_{kj}^{\text{coh}}}\right)^2 - 2\pi^2\xi^2\left(\frac{\sigma_x}{L_{kj}^{\text{osc}}}\right)^2\right]$$

new localization term:

$$\exp\left[-2\pi^2\xi^2\left(\frac{\sigma_x}{L_{kj}^{\mathrm{osc}}}\right)^2\right]$$

interference is suppressed for $\sigma_x \gtrsim L_{kj}^{\mathrm{osc}}$

equivalent to neutrino mass measurement

uncertainty of neutrino mass measurement:

$$m_k^2 = E_k^2 - p_k^2 \implies \delta m_k^2 \simeq \sqrt{(2\,E_k\,\delta E_k)^2 + (2\,p_k\,\delta p_k)^2} \sim 4\,E\,\sigma_p$$

$$\sigma_p = \frac{1}{2\,\sigma_x} \qquad E = \frac{|\Delta m_{kj}^2|L_{kj}^{\mathrm{osc}}}{4\pi} \implies \delta m_k^2 \sim \frac{|\Delta m_{kj}^2|L_{kj}^{\mathrm{osc}}}{\sigma_x}$$

$$\boxed{\sigma_x \gtrsim L_{kj}^{\mathrm{osc}} \implies \delta m_k^2 \lesssim |\Delta m_{kj}^2|} \implies \text{only one massive neutrino!}$$

Decoherence in Two-Neutrino Mixing

$\Delta m^2 = 10^{-3} \, \text{eV}^2$ $\qquad \sin^2 2\vartheta = 1$ $\qquad E = 1 \, \text{GeV}$ $\qquad \sigma_p = 50 \, \text{MeV}$

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2480 \, \text{km} \qquad\qquad L^{\text{coh}} = \frac{4\sqrt{2} E^2}{|\Delta m^2|} \, \sigma_x = 11163 \, \text{km}$$
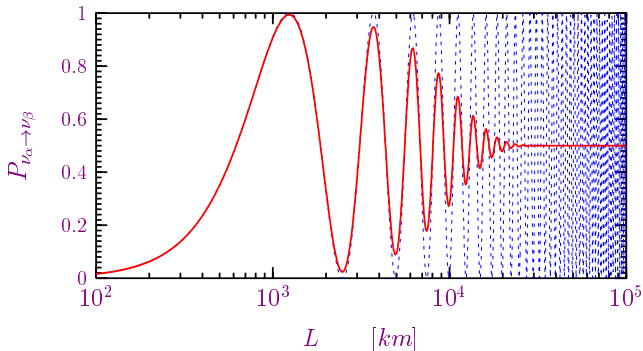
Decoherence for $L \gtrsim L^{\text{coh}} \sim 10^4 \, \text{km}$

Achievements of the QM Wave Packet Model

► Confirmed Standard Oscillation Length: $L_{kj}^{\text{osc}} = 4\pi E/\Delta m_{kj}^2$

► Derived Coherence Length: $L_{kj}^{\text{coh}} = 4\sqrt{2}E^2\sigma_x/|\Delta m_{kj}^2|$

► The localization term quantifies the conditions for coherence

problem

flavor states in production and detection processes have to be assumed

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* \int \mathrm{d}p\, \psi_k^P(p)\, |\nu_k(p)\rangle \qquad |\nu_\beta\rangle = \sum_k U_{\beta k}^* \int \mathrm{d}p\, \psi_k^D(p)\, |\nu_k(p)\rangle$$

calculation of neutrino production and detection?

⇑

Quantum Field Theoretical Wave Packet Model

[CG, Kim, Lee, Lee, PRD 48 (1993) 4310] [CG, Kim, Lee, PLB 421 (1998) 237] [Kiers, Weiss, PRD 57 (1998) 3091]

[Zralek, Acta Phys. Polon. B29 (1998) 3925] [Cardall, PRD 61 (2000) 07300]

[Beuthe, PRD 66 (2002) 013003] [Beuthe, Phys. Rep. 375 (2003) 105] [CG, JHEP 11 (2002) 017]

Estimates of Coherence Length

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2.5 \, \frac{(E/\text{MeV})}{(\Delta m^2/\text{eV}^2)} \, \text{m}$$

$$L^{\text{coh}} \sim \frac{4\sqrt{2}E^2}{|\Delta m^2|}\sigma_x = 10^{12} \, \frac{(E^2/\text{MeV}^2)}{(|\Delta m^2|/\text{eV}^2)} \left(\frac{\sigma_x}{\text{m}}\right) \text{m}$$

Process	$|\Delta m^2|$	L^{osc}	σ_x	L^{coh}
$\pi \to \mu + \nu$ at rest in vacuum: $E \simeq 30\,\text{MeV}$ natural linewidth	$2.5 \times 10^{-3}\,\text{eV}^2$	$30\,\text{km}$	$\tau_\pi \sim 10\,\text{m}$	$\sim 10^{16}\,\text{km}$
$\pi \to \mu + \nu$ at rest in matter: $E \simeq 30\,\text{MeV}$ collision broadening	$2.5 \times 10^{-3}\,\text{eV}^2$	$30\,\text{km}$	$\tau_{\text{col}} \sim 10^{-5}\,\text{m}$	$\sim 10^{10}\,\text{km}$
$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ at rest in matter: $E \leq 50\,\text{MeV}$ collision broadening	$1\,\text{eV}^2$	$\leq 125\,\text{m}$	$\tau_{\text{col}} \sim 10^{-10}\,\text{m}$	$\lesssim 10^2\,\text{km}$
$^7\text{Be} + e^- \to\ ^7\text{Li} + \nu_e$ in solar core: $E \simeq 0.86\,\text{MeV}$ collision broadening	$7 \times 10^{-5}\,\text{eV}^2$	$31\,\text{km}$	$\tau_{\text{col}} \sim 10^{-9}\,\text{m}$	$\sim 10^4\,\text{km}$

Common Question: Do Charged Leptons Oscillate?

▶ Mass is the only property which distinguishes e, μ, τ.

▶ The flavor of a charged lepton is defined by its mass!

▶ By definition, the flavor of a charged lepton cannot change.

THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

a misleading argument

[Sassaroli, Srivastava, Widom, hep-ph/9509261, EPJC 2 (1998) 769] [Srivastava, Widom, hep-ph/9707268]

in $\pi^+ \to \mu^+ + \nu_\mu$ the final state of the antimuon and neutrino is entangled

$\Downarrow$

if the probability to detect the neutrino oscillates as a function of distance, also the probability to detect the muon must oscillate

WRONG!

the probability to detect the neutrino (as ν_μ or ν_τ or ν_e) does not oscillate as a function of distance, because

$$\sum_{\beta=e,\mu,\tau} P_{\nu_\mu \to \nu_\beta} = 1 \qquad \text{conservation of probability (unitarity)}$$

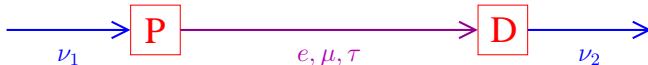[Dolgov, Morozov, Okun, Shchepkin, NPB 502 (1997) 3] [CG, Kim, FPL 14 (2001) 213]

Λ oscillations from $\pi^- + p \to \Lambda + K^0$

[Widom, Srivastava, hep-ph/9605399] [Srivastava, Widom, Sassaroli, PLB 344 (1995) 436]

refuted in [Lowe et al., PLB 384 (1996) 288] [Burkhardt, Lowe, Stephenson, Goldman, PRD 59 (1999) 054018]

Correct definition of Charged Lepton Oscillations

[Pakvasa, Nuovo Cim. Lett. 31 (1981) 497]

Analogy

▶ Neutrino Oscillations: massive neutrinos propagate unchanged between production and detection, with a difference of mass (flavor) of the charged leptons involved in the production and detection processes.

▶ Charged-Lepton Oscillations: massive charged leptons propagate unchanged between production and detection, with a difference of mass of the neutrinos involved in the production and detection processes.

NO FLAVOR CONVERSION!

The propagating charged leptons must be ultrarelativistic, in order to be produced and detected coherently (if τ is not ultrarelativistic, only e and μ contribute to the phase).

Practical Problems

▶ The initial and final neutrinos must be massive neutrinos of known type: precise neutrino mass measurements.

▶ The energy of the propagating charged leptons must be extremely high, in order to have a measurable oscillation length

$$\frac{4\pi E}{(m_\mu^2 - m_e^2)} \simeq \frac{4\pi E}{m_\mu^2} \simeq 2 \times 10^{-11} \left(\frac{E}{\text{GeV}} \right) \text{cm}$$

detailed discussion: [Akhmedov, JHEP 09 (2007) 116, arXiv:0706.1216]

Mistake: Oscillation Phase Larger by a Factor of 2

[Field, hep-ph/0110064, hep-ph/0110066, EPJC 30 (2003) 305, EPJC 37 (2004) 359, Annals Phys. 321 (2006) 627]

$K^0 - \bar{K}^0$: [Srivastava, Widom, Sassaroli, ZPC 66 (1995) 601, PLB 344 (1995) 436] [Widom, Srivastava, hep-ph/9605399]

massive neutrinos: $\quad v_k = \dfrac{p_k}{E_k} \quad \Longrightarrow \quad t_k = \dfrac{L}{v_k} = \dfrac{E_k}{p_k} L$
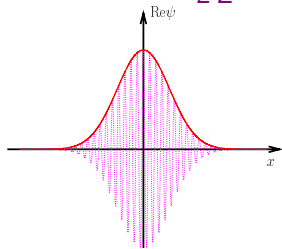
$$\widetilde{\Phi}_k = p_k L - E_k t_k = p_k L - \frac{E_k^2}{p_k} L = \frac{p_k^2 - E_k^2}{p_k} L = \frac{m_k^2}{p_k} L \simeq \frac{m_k^2}{E} L$$

$$\Delta\widetilde{\Phi}_{kj} = -\frac{\Delta m_{kj}^2 \, L}{E} \quad \text{twice the standard phase} \quad \Delta\Phi_{kj} = -\frac{\Delta m_{kj}^2 \, L}{2\,E}$$

WRONG!

group velocities are irrelevant for the phase!

the group velocity is the velocity of the factor
which modulates the amplitude of the wave
packet

in the plane wave approximation the interference
of different massive neutrino contribution must be calculated
at a definite space distance L and after a definite time interval T

[Nieto, hep-ph/9509370] [Kayser, Stodolsky, PLB 359 (1995) 343] [Lowe et al., PLB 384 (1996) 288] [Kayser, hep-ph/9702327]

[CG, Kim, FPL 14 (2001) 213] [CG, Physica Scripta 67 (2003) 29] [Burkhardt et al., PLB 566 (2003) 137]

$$\Delta\widetilde{\Phi}_{kj} = (p_k - p_j)\, L - (E_k - E_j)\, t_k \qquad \text{WRONG!}$$

$$\Delta\Phi_{kj} = (p_k - p_j)\, L - (E_k - E_j)\, T \qquad \text{CORRECT!}$$

no factor of 2 ambiguity claimed in

[Lipkin, PLB 348 (1995) 604, hep-ph/9901399] [Grossman, Lipkin, PRD 55 (1997) 2760]

[De Leo, Ducati, Rotelli, MPLA 15 (2000) 2057]

[De Leo, Nishi, Rotelli, hep-ph/0208086, hep-ph/0303224, IJMPA 19 (2004) 677]