The Neutrino Magnetic Moment

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Neutrino Electromagnetic Interactions

• Effective Hamiltonian: $\mathcal{H}_{em}^{(\nu)}(x) = j_{\mu}^{(\nu)}(x)A^{\mu}(x) = \sum_{k,j=1} \overline{\nu_k}(x)\Lambda_{\mu}^{kj}\nu_j(x)A^{\mu}(x)$

Effective electromagnetic vertex:

$$\langle \nu_f(p_f)|j^{(\nu)}_{\mu}(0)|\nu_i(p_i)\rangle = \overline{u_f}(p_f)\Lambda^{fi}_{\mu}(q)u_i(p_i)$$

$$q = p_i - p_f$$



Vertex function:

$$\Lambda_{\mu}(q) = (\gamma_{\mu} - q_{\mu} q/q^{2}) \begin{bmatrix} F_{Q}(q^{2}) + F_{A}(q^{2})q^{2}\gamma_{5} \end{bmatrix} - i\sigma_{\mu\nu}q^{\nu} \begin{bmatrix} F_{M}(q^{2}) + iF_{E}(q^{2})\gamma_{5} \end{bmatrix}$$
Lorentz-invariant form factors: charge anapole magnetic electric
$$q^{2} = 0 \implies q \qquad a \qquad \mu \qquad \varepsilon$$
chirality-conserving chirality-flipping

Electromagnetic Vertex Function



▶ Hermitian form factors: $F_Q = F_Q^{\dagger}$, $F_A = F_A^{\dagger}$, $F_M = F_M^{\dagger}$, $F_E = F_E^{\dagger}$

▶ Majorana neutrinos: $F_Q = -F_Q^T$, $F_A = F_A^T$, $F_M = -F_M^T$, $F_E = -F_E^T$ no diagonal charges and electric and magnetic moments in the mass basis!

- Left-handed ultrarelativistic neutrinos: $\gamma_5 \rightarrow -1$:
 - charge and anapole have similar phenomenology
 - magnetic and electric moments have similar phenomenology:

dipole moments $d = \mu - i\varepsilon$

- ► Ultrarelativistic neutrinos: chirality ~ helicity:
 - the charge and anapole terms conserve helicity
 - the magnetic and electric terms invert helicity

Neutrino Electric Charges

- Neutrinos can be millicharged particles in BSM theories.
- There are strong experimental limits:

Limit	Method	Reference
$ q_{ u_e} \lesssim 3 imes 10^{-21} e$	Neutrality of matter	Raffelt (1999)
$ q_{ u_e} \lesssim 3.7 imes 10^{-12} e$	Nuclear reactor	Gninenko et al (2006)
$ q_{ u_e} \lesssim 1.5 imes 10^{-12} e$	Nuclear reactor	Studenikin (2013)
$ q_{ u_{\mu}} \lesssim 3 imes 10^{-8} e$	COHERENT $CE\nu NS$	Cadeddu et al (2020)
$ q_{ u_{\mu au}} \lesssim 2 imes 10^{-8} e$	COHERENT $CE\nu NS$	Cadeddu et al (2020)
$ q_{ u_{\mu}} \lesssim 3 imes 10^{-9} e$	LSND	Das et al (2020)
$ q_{ u_{ au}} \lesssim 4 imes 10^{-6} e$	DONUT	Das et al (2020)
$ q_{ u_{ au}} \lesssim 3 imes 10^{-4} e$	SLAC e^- beam dump	Davidson et al (1991)
$ q_{ u_{ au}} \lesssim 4 imes 10^{-4} e$	BEBC beam dump	Babu et al (1993)
$ q_ u \lesssim 6 imes 10^{-14} e$	Solar cooling (plasmon decay)	Raffelt (1999)
$ q_ u \lesssim 2 imes 10^{-14} e$	Red giant cooling (plasmon decay)	Raffelt (1999)

Neutrino Charge Radius

- In the Standard Model neutrinos are neutral and there are no electromagnetic interactions at the tree-level.
- Radiative corrections generate an effective electromagnetic interaction vertex

In the Standard Model:

[Bernabeu et al, PRD 62 (2000) 113012, NPB 680 (2004) 450]

$$\langle r_{\nu_{\ell}}^{2} \rangle_{\rm SM} = -\frac{G_{\rm F}}{2\sqrt{2}\pi^{2}} \left[3 - 2\log\left(\frac{m_{\ell}^{2}}{m_{W}^{2}}\right) \right] \qquad \begin{cases} \langle r_{\nu_{\ell}}^{2} \rangle_{\rm SM} = -8.2 \times 10^{-33} \, {\rm cm}^{2} \\ \langle r_{\nu_{\mu}}^{2} \rangle_{\rm SM} = -4.8 \times 10^{-33} \, {\rm cm}^{2} \\ \langle r_{\nu_{\mu}}^{2} \rangle_{\rm SM} = -3.0 \times 10^{-33} \, {\rm cm}^{2} \end{cases}$$

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Experimental Bounds

Method	Experiment	Limit [cm ²]	CL	Year
Reactor $\bar{\nu}_e e^-$	Krasnoyarsk	$ \langle r^2_{ u_e} angle < 7.3 imes 10^{-32}$	90%	1992
	TEXONO	$-4.2 imes 10^{-32} < \langle r^2_{ u_e} angle < 6.6 imes 10^{-32}$	90%	2009
Accelerator $\nu_e e^-$	LAMPF	$-7.12 imes 10^{-32} < \langle r^2_{ u_e} angle < 10.88 imes 10^{-32}$	90%	1992
	LSND	$-5.94 imes 10^{-32} < \langle r^2_{ u_e} angle < 8.28 imes 10^{-32}$	90%	2001
Accelerator $ u_{\mu} e^{-}$	BNL-E734	$-5.7 imes 10^{-32} < \langle r^2_{ u_{\mu}} angle < 1.1 imes 10^{-32}$	90%	1990
	CHARM-II	$ \langle r^2_{ u_\mu} angle < 1.2 imes 10^{-32}$	90%	1994

[see the review CG, Studenikin, arXiv:1403.6344

and the update in Cadeddu, CG, Kouzakov, Y.F. Li, Studenikin, Y.Y. Zhang, arXiv:1810.05606]

Neutrino Magnetic and Electric Moments

Effective dimension-5 Lagrangian:

$$\mathcal{L}_{\text{mag}} = \frac{1}{2} \sum_{k,j=1}^{\mathcal{N}} \overline{\nu_{Lk}} \, \sigma^{\alpha\beta} \left(\mu_{kj} + \varepsilon_{kj} \, \gamma_5 \right) N_{Rj} \, F_{\alpha\beta} + \text{H.c.}$$

- Note that the magnetic and electric moments (as the charge and anapole) are well-defined in the mass basis.
- ► N = 3, $N_{Rj} = \nu_{Rj}$, and $\Delta L = 0 \implies$ Dirac neutrinos with diagonal and off-diagonal (transition) magnetic and electric moments
- ► N = 3 and $N_{Rj} = \nu_{Lj}^c \implies$ Majorana neutrinos with transition magnetic and electric moments only

► $N > 3 \implies$ active + sterile Dirac ($\Delta L = 0$) or Majorana neutrinos "neutrino dipole portal" or "neutrino magnetic moment portal"

Dirac Neutrinos

[Fujikawa, Shrock, PRL 45 (1980) 963; Pal, Wolfenstein, PRD 25 (1982) 766; Shrock, NPB 206 (1982) 359; Dvornikov, Studenikin, PRD 69 (2004) 073001, JETP 99 (2004) 254]

Simplest extension of the Standard Model with three right-handed neutrinos and $\Delta L = 0$

$$\mathcal{L}_{mag} = \frac{1}{2} \sum_{k,j=1}^{3} \overline{\nu_{Lk}} \, \sigma^{\alpha\beta} \left(\mu_{kj} + \varepsilon_{kj} \, \gamma_5 \right) \nu_{Rj} \, F_{\alpha\beta} + \text{H.c.}$$

$$\begin{aligned} & \mu_{kj}^{\mathsf{D}} \\ & i\varepsilon_{kj}^{\mathsf{D}} \end{aligned} \simeq \frac{3eG_{\mathsf{F}}}{16\sqrt{2}\pi^2} \left(m_k \pm m_j \right) \left(\delta_{kj} - \frac{1}{2} \sum_{\ell=e,\mu,\tau} U_{\ell k}^* U_{\ell j} \frac{m_{\ell}^2}{m_W^2} \right) \end{aligned}$$

- The constraint ΔL = 0 is necessary to forbid a Majorana mass term for the three right-handed neutrinos ν_{1R}, ν_{2R}, ν_{3R}.
- The magnetic and electric moments are proportional to the neutrino masses!
- ▶ This is because Standard Model interactions involve only ν_{1L} , ν_{2L} , ν_{3L} .

A mass insertion is needed to flip chirality:



Diagonal magnetic and electric moments:

$$\begin{split} \mu^{\rm D}_{kk} &\simeq \frac{3eG_{\rm F}m_k}{8\sqrt{2}\pi^2} \\ \varepsilon^{\rm D}_{kk} &= 0 \quad \leftarrow \quad \text{No diagonal electric moments!} \end{split}$$

► Diagonal magnetic moments: $\mu_{kk}^{D} \simeq 3.2 \times 10^{-19} \mu_{B} \left(\frac{m_{k}}{eV}\right)$

Strongly suppressed by small neutrino masses!

$$\mu_{\rm B} \equiv rac{e}{2 \, m_e} \simeq 6 imes 10^{-15} \; rac{{
m MeV}}{{
m Gauss}}$$

► The transition magnetic and electric moments (k ≠ j) are GIM-suppressed:

$$\mu_{kj}^{\mathsf{D}} \\ i\varepsilon_{kj}^{\mathsf{D}} \\ \right\} \simeq -3.9 \times 10^{-23} \mu_{\mathsf{B}} \left(\frac{m_k \pm m_j}{\mathsf{eV}} \right) \sum_{\ell=e,\mu,\tau} U_{\ell k}^* U_{\ell j} \left(\frac{m_\ell}{m_\tau} \right)^2$$

At least four orders of magnitude smaller than the diagonal ones!

Majorana Neutrinos

• Only GIM-suppressed transition magnetic and electric moments $(k \neq j)$:

$$\mu_{kj}^{\mathsf{M}} \simeq -7.8 \times 10^{-23} \mu_{\mathsf{B}} i \left(m_{k} + m_{j} \right) \sum_{\ell=e,\mu,\tau} \operatorname{Im} \left[U_{\ell k}^{*} U_{\ell j} \right] \frac{m_{\ell}^{2}}{m_{W}^{2}}$$
$$\varepsilon_{kj}^{\mathsf{M}} \simeq 7.8 \times 10^{-23} \mu_{\mathsf{B}} i \left(m_{k} - m_{j} \right) \sum_{\ell=e,\mu,\tau} \operatorname{Re} \left[U_{\ell k}^{*} U_{\ell j} \right] \frac{m_{\ell}^{2}}{m_{W}^{2}}$$

[Shrock, NPB 206 (1982) 359]

However, additional model-dependent contributions of the scalar sector can enhance the Majorana transition magnetic and electric moments

[Pal, Wolfenstein, PRD 25 (1982) 766; Barr, Freire, Zee, PRL 65 (1990) 2626; Pal, PRD 44 (1991) 2261]

Left-Right Simmetric Models

Right-handed interactions mediated by W_R avoid the necessity of the mass insertion to flip chirality:



Problem: The same diagrams without the photon line contribute to the neutrino masses: sin ε



- General Problem: difficult to get large magnetic moments and small masses.
- Common Solution: ad-hoc symmetries.

General argument:

Contribution of a BSM diagram to the magnetic moment:

 $\mu_{
u} \sim {eG\over \Lambda} ~~G:~{
m coupling~constants}~{
m and~loop~factors}~~\Lambda:~{
m BSM~energy~scale}$

- The same diagram without photon line gives $\delta m_
u \sim G\Lambda$

• Therefore:
$$\mu_{\nu} \sim 10^{-18} \, \mu_{\rm B} \left(\frac{\delta m_{\nu}}{\rm eV} \right) \left(\frac{\Lambda}{\rm TeV} \right)^{-2}$$

A more quantitative analysis gives:

$$\begin{split} \mu_{\nu}^{\mathrm{D}} \lesssim 3 \times 10^{-15} \, \mu_{\mathrm{B}} \left(\frac{m_{\nu}}{\mathrm{eV}}\right) \left(\frac{\Lambda}{\mathrm{TeV}}\right)^{-2} & \text{[Bell et al, hep-ph/0504134]} \\ \mu_{\ell\ell'}^{\mathrm{M}} \lesssim 4 \times 10^{-9} \, \mu_{\mathrm{B}} \left(\frac{M_{\ell\ell'}^{\mathrm{M}}}{\mathrm{eV}}\right) \left(\frac{\Lambda}{\mathrm{TeV}}\right)^{-2} \left|\frac{m_{\tau}^{2}}{m_{\ell}^{2} - m_{\ell'}^{2}}\right| & \text{[Bell et al, hep-ph/0606248]} \end{split}$$

Majorana magnetic moments are less constrained by the smallness of the neutrino masses because the diagram contribution to the mass is Yukawa suppressed (additional Yukawa couplings are needed to convert the antisymmetric magnetic moment operator into a symmetric mass operator).



[Balantekin, Vassh, arXiv:1312.6858]

Method	Experiment	Limit $[\mu_B]$	CL	Year
Reactor $\bar{\nu}_e e^-$	Krasnoyarsk	$\mu_{ u_e} < 2.4 imes 10^{-10}$	90%	1992
	Rovno	$\mu_{ u_e} < 1.9 imes 10^{-10}$	95%	1993
	MUNU	$\mu_{ u_e} < 9 imes 10^{-11}$	90%	2005
	TEXONO	$\mu_{ u_e} < 7.4 imes 10^{-11}$	90%	2006
	GEMMA	$\mu_{ u_e} < 2.9 imes 10^{-11}$	90%	2012
Accelerator $\nu_e e^-$	LAMPF	$\mu_{ u_e} < 1.1 imes 10^{-9}$	90%	1992
Accelerator $(u_{\mu}, ar{ u}_{\mu}) e^{-}$	BNL-E734	$\mu_{ u_{\mu}} < 8.5 imes 10^{-10}$	90%	1990
	LAMPF	$\mu_{ u_\mu} < 7.4 imes 10^{-10}$	90%	1992
	LSND	$\mu_{ u_\mu} < 6.8 imes 10^{-10}$	90%	2001
Accelerator $(u_{ au}, ar{ u}_{ au}) e^-$	DONUT	$\mu_{ u_ au} < 3.9 imes 10^{-7}$	90%	2001
Solar $\nu_e e^-$	Super-Kamiokande	$\mu_{S}(\textit{E}_{ u}\gtrsim5MeV) < 1.1 imes10^{-10}$	90%	2004
	Borexino	$\mu_{\sf S}({\it E}_ u\lesssim 1{\sf MeV}) < 2.8 imes 10^{-11}$	90%	2017

[see the review CG, Studenikin, arXiv:1403.6344]

Gap of about 8 orders of magnitude between the experimental limits and the $\leq 10^{-19} \mu_{\rm B}$ prediction of the minimal Standard Model extensions.

▶ $\mu_{\nu} \gg 10^{-19} \mu_{\rm B}$ discovery \Rightarrow non-minimal new physics beyond the SM.

Neutrino spin-flavor precession in a magnetic field

[Lim, Marciano, PRD 37 (1988) 1368; Akhmedov, PLB 213 (1988) 64]

Borexino

[arXiv:1707.09355]



$$\left(\frac{d\sigma_{\nu e^{-}}}{dT_{e}}\right) = \frac{\pi\alpha^{2}}{m_{e}^{2}} \left(\frac{1}{T_{e}} - \frac{1}{E_{\nu}}\right) \left(\frac{\mu_{\text{eff}}}{\mu_{\text{B}}}\right)^{2}$$

Taking into account neutrino oscillations:

$$\mu_{\text{eff}}^2 = \sum_{k,j=1}^3 P_{\nu_e \to \nu_k} |\mu_{kj}|^2$$

 All the positive magnetic moment contributions can be constrained.

$$\cdot$$
 At 90% CL, in units of $10^{-11}\,\mu_{
m B}$:

XENON1T

[arXiv:2006.09721]



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 μ_{ν} is not directly comparable to GEMMA μ_{ν_e}

$$u_{\nu_e}^2 = \sum_{j} \left| \sum_{k} U_{ek}^* \left(\mu_{jk} - i\varepsilon_{jk} \right) \right|^2$$

[see CG. Studenikin, arXiv:1403.6344]

Neglecting the electric moments, we have

$$\mu_{\nu_e}^2 = \sum_{i,j} U_{ei} \, \mu_{ij}^2 \, U_{ej}^*$$



Jinping neutrino experiment 2400 m underground water-based liquid scintillator 4 kton fiducial target mass 5 kton total mass 10-year exposure



Active-to-Sterile ν Transition Dipole Moment

[Shoemaker, Tsai, Wyenberg, arXiv:2007.05513]



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Active-to-Sterile ν Transition Dipole Moment

[Shoemaker, Tsai, Wyenberg, arXiv:2007.05513]



 $\leftarrow \begin{cases} m_4 = 640 \text{ keV} \\ d = 2.2 \times 10^{-9} \mu_{\text{P}} \end{cases}$ Best Fit "neutrino dipole portal" Dipole moment: $d = \mu - i \varepsilon$ $\blacktriangleright \mathcal{L} \supset d \bar{\nu}_I \sigma^{\mu\nu} N_R F_{\mu\nu} + \text{H.c.}$ • Upscattering $\nu_{solar} + e \rightarrow e + N$ $\frac{d\sigma}{dE_R} = d^2 \alpha \left| \frac{1}{E_R} - \frac{m_4^2}{2E_\nu E_R m_e} \left(1 - \frac{E_R}{2E_\nu} + \frac{m_e}{2E_\nu} \right) \right|$ $-\frac{1}{E_{\nu}}+\frac{m_4^4(E_R-m_e)}{8E_{\nu}^2E_R^2m_e^2}$

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The Neutrino Magnetic Moment Portal



[Brdar, Greljo, Kopp, Opferkuch, arXiv:2007.15563]

Electron Anti-Neutrinos from the Sun



The interpretation of results in terms of a magnetic moment depends on the unknown magnetic field in the Sun: [Akhmedov, Pulido, hep-ph/0209192]

$$P_{
u_{eL} o ar{
u}_{eR}} \simeq 1.8 imes 10^{-10} \sin^2 2 artheta_{12} \left(rac{\mu_{12}}{10^{-12} \, \mu_{
m B}} \, rac{B_{\perp}(0.05 R_{\odot})}{10 \, {
m kG}}
ight)^2$$

Neutrino Magnetic Moments in CE ν NS

• Neutrino magnetic (and electric) moment contributions to CE ν NS $\nu_{\ell} + \mathcal{N} \rightarrow \sum_{\nu_{\ell'}} \nu_{\ell'} + \mathcal{N}$:

$$\frac{d\sigma_{\nu_{\ell}-\mathcal{N}}}{dT}(E_{\nu},T) = \frac{G_{\mathsf{F}}^{2}M}{\pi} \left(1 - \frac{MT}{2E_{\nu}^{2}}\right) \left[g_{V}^{n}NF_{N}(|\vec{q}|^{2}) + g_{V}^{p}ZF_{Z}(|\vec{q}|^{2})\right]^{2} + \frac{\pi\alpha^{2}}{m_{e}^{2}} \left(\frac{1}{T} - \frac{1}{E_{\nu}}\right) Z^{2}F_{Z}^{2}(|\vec{q}|^{2}) \sum_{\ell'\neq\ell} \frac{|\mu_{\ell\ell'}|^{2}}{\mu_{\mathsf{B}}^{2}}$$
$$g_{V}^{n} = -\frac{1}{2} \qquad g_{V}^{p} = \frac{1}{2} - 2\sin^{2}\vartheta_{W} = 0.0227 \pm 0.0002$$

- The magnetic moment interaction adds incoherently to the weak interaction because it flips helicity.
- The m_e is due to the definition of the Bohr magneton: $\mu_B = e/2m_e$.

COHERENT Constraints on ν Magnetic Moments

[Cadeddu et al, arXiv:2005.01645]



The sensitivity to |µ_{ν_e}| is not competitive with that of reactor experiments:

 $|\mu_{
u_e}| < 2.9 imes 10^{-11} \, \mu_{
m B} \quad \mbox{(90\% CL)} \ {}_{[{
m Gemma, Ahep 2012 (2012) 350150}]}$

The constraint on |μ_{νμ}| is not too far from the best current laboratory limit:

Conclusions

- Neutrino Electromagnetic Interactions are expected in the Standard Model (charge radii) and in BSM theories: dipole magnetic and electric moments, non-standard charge radii, and millicharges
- The existence of neutrino magnetic moments is related to the existence of neutrino masses through chirality flipping BSM operators.
- \blacktriangleright Current laboratory limits are at the level of $10^{-11}-10^{-10}\,\mu_{\rm B}$
- Conjectural theoretical expectations:
 - $\mu \lesssim 10^{-14} \,\mu_{\rm B}$ for Dirac neutrinos.
 - Maybe larger for Majorana neutrinos.
- Interesting XENON1T hint for $\mu \approx 2 \times 10^{-11} \mu_{\rm B}$.
- If there are BSM sterile neutrinos the active-sterile transition magnetic moments may be a dipole portal to the Dark Sector.