Monte Carlo Event Generators

The basic lepton-quark scattering processes have well defined cross section formulae within the electroweak standard model. With the inclusion of parton density distributions and perturbative QCD corrections the problem of practical evaluations become quite complex and analytical calculations are only possible in simplified cases or through approximations. Normal numerical methods are in many cases possible but Monte Carlo simulation is often preferable because of its generality and applicability to complex problems. In the case of the multiparticle hadronic final state the only viable alternative is in fact the Monte Carlo method.

We will deal with two most popular for DIS event generation programs: LEPTO and PYTHIA
Monte Carlo Techniques

First of all one assumes the existence of a random number generator. This is a (Fortran) function which, each time it is called, returns a number $R$ in the range between 0 and 1, such that the inclusive distribution of numbers $R$ is flat in the range, and such that different numbers $R$ are uncorrelated.

Selection From a Distribution

The situation that is probably most common is that we know a function $f(x)$ which is non-negative in the allowed $x$ range $x_{\text{min}} < x < x_{\text{min}}$. We want to select an $x$ ‘at random’ so that the probability in a small interval $dx$ around a given $x$ is proportional to $f(x) \, dx$. Here $f(x)$ might be a distribution function, a differential cross section, or any of a number of distributions.
If it is possible to find a primitive function \( F(x) \) which has a known inverse \( F^{-1}(x) \), an \( x \) can be found as follows (method 1):

\[
\int_{x_{\text{min}}}^{x} f(x) \, dx = R \int_{x_{\text{min}}}^{x_{\text{max}}} f(x) \, dx
\]

\[\implies x = F^{-1}(F(x_{\text{min}}) + R(F(x_{\text{max}}) - F(x_{\text{min}}))) .\]

The statement of the first line is that a fraction \( R \) of the total area under \( f(x) \) should be to the left of \( x \). However, seldom are functions of interest so nice that the method above works.

Special tricks can sometimes be found. Consider e.g. the generation of a Gaussian

\[ f(x) = \exp(-x^2) \]

This function is not integrable, but if we combine it with the same Gaussian distribution of a second variable \( y \), it is possible to transform to polar coordinates

\[ f(x) \, dx \, f(y) \, dy = \exp(-x^2 - y^2) \, dx \, dy = r \exp(-r^2) \, dr \, d\varphi \]

and now the \( r \) and \( \varphi \) distributions may be easily generated and recombined to yield \( x \). At the same time we get a second number \( y \), which can also be used. For the generation of transverse momenta in fragmentation, this is very convenient, since in fact we want to assign two transverse degrees of freedom.
A hit-or-miss method (method 2)

If the maximum of $f(x)$ is known in the $x$ range considered, then do the following:

1. select an $x$ with even probability in the allowed range, i.e. $x = x_{\text{min}} + R(x_{\text{max}} - x_{\text{min}})$;
2. compare a (new) $R$ with the ratio $f(x)/f_{\text{max}}$; if $f(x)/f_{\text{max}} \leq R$, then reject the $x$ value and return to point 1 for a new try;
3. otherwise the most recent $x$ value is retained as final answer.

The probability that $f(x)/f_{\text{max}} > R$ is proportional to $f(x)$; hence the correct distribution of retained $x$ values. The efficiency of this method, i.e. the average probability that an $x$ will be retained, is $(\int f(x) \, dx)/(f_{\text{max}}(x_{\text{max}} - x_{\text{min}}))$. The method is acceptable if this number is not too low, i.e. if $f(x)$ does not fluctuate too wildly.

For a more complex situations as functions with spikes or multidimensional distributions see special literature or PHYTIA manual.
LEPTO -- A Monte Carlo Generator for Deep Inelastic Lepton-Nucleon Scattering

LEPTO is a general and flexible Monte Carlo MC to simulate complete lepton-nucleon scattering events and integrate cross sections. It is based on the leading order electroweak cross sections for the underlying parton level scattering processes. The main emphasis of LEPTO is rather on the hadronic part of the event. QCD corrections are therefore included using exact first order matrix elements and higher orders in the leading $\log Q^2$ parton cascade approach. The fragmentation of produced partons into observable hadrons is performed with the Lund string hadronization model. An arbitrary configuration of a lepton and a nucleon can be defined with constraints on the scattering kinematics and the generated events can be transformed to different frames.
Kinematics

The main kinematic relations given here are for the case of electron-proton scattering, $e + p \rightarrow \ell + H$ where $\ell$ is the scattered lepton and $H$ the final hadron system, but are of course equally valid for any lepton beam or a neutron target. Let $p_e$, $p_\ell$ be the four-vectors of the incoming and scattered lepton, respectively, and $P$ that of the incoming proton. Some basic kinematic relations are then (cf. [12, 13]) (all references are from LEPTO)

\begin{align*}
    s & \equiv (p_e + P)^2 \simeq 4E_eE_p \\
    W^2 & \equiv (q + P)^2 = Q^2 \frac{1 - x}{x} + m_p^2 \\
    Q^2 & \equiv -q^2 = - (p_e - p_\ell)^2 \simeq 4E_eE_\ell \sin^2 \frac{\theta_\ell}{2} \\
    m_p \nu & \equiv P \cdot q \simeq 2E_p(E_e - E_\ell \cos^2 \frac{\theta_\ell}{2}) \\
    x & \equiv \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2m_p \nu} \simeq \frac{E_e E_\ell \sin^2 \frac{\theta_\ell}{2}}{E_p(E_e - E_\ell \cos^2 \frac{\theta_\ell}{2})} \\
    y & \equiv \frac{P \cdot q}{P \cdot p_e} = \frac{\nu}{\nu_{\text{max}}} \simeq \frac{E_e - E_\ell \cos^2 \frac{\theta_\ell}{2}}{E_e}
\end{align*}
Electroweak cross sections

In leading order electroweak theory [2] the differential neutral current (NC) cross section for the scattering of a charged lepton is (neglecting masses) given by

\[
\frac{d^2\sigma_{NC}(e^\pm)}{dx \, dQ^2} = \frac{4\pi\alpha^2}{x Q^4} \left[ y^2 x F_1(x, Q^2) + (1 - y) F_2(x, Q^2) \pm \left( \frac{y^2}{2} \right) x F_3(x, Q^2) \right]
\]

in terms of the nucleon structure functions \( F_1, F_2, F_3 \). Two of these are related through the Callan-Gross relation, \( 2xF_1 = F_2 \), which holds for spin 1/2 quarks when neglecting masses, intrinsic transverse momenta and order \( \alpha_s \) QCD effects. These effects are usually negligibly small, except at small-\( x \) \([19, 20]\), but can optionally be included (cf. section 2.3). More important for the structure of the electroweak theory is to take the lepton beam polarization into account. For a left- \( (L) \) and right-handed \( (R) \) electron one has the differential cross section

\[
\frac{d^2\sigma_{NC}(e^-_{L,R}, p)}{dx \, dQ^2} = \frac{2\pi\alpha^2}{x Q^4} \left[ \left( 1 + (1 - y)^2 \right) F_2^{L,R}(x, Q^2) + \left( 1 - (1 - y)^2 \right) x F_3^{L,R}(x, Q^2) \right]
\]

(8)
The structure functions are in the standard parton model given by

\begin{align}
F_2^{L,R}(x, q^2) &= \sum_q \left[ xq(x, Q^2) + x\bar{q}(x, Q^2) \right] A_q^{L,R}(Q^2) \\
xF_3^{L,R}(x, q^2) &= \sum_q \left[ xq(x, Q^2) - x\bar{q}(x, Q^2) \right] B_q^{L,R}(Q^2)
\end{align}

where the sum is over all quark flavours and \( q(\bar{q}) \) denote the quark (antiquark) density distributions in the nucleon (cf. section 2.4). The coefficients are given by

\begin{align}
A_q^{L,R}(Q^2) &= e_q^2 - 2e_q(v_e \pm a_e)v_qP_Z + (v_e \pm a_e)^2(v_q^2 + a_q^2)P_Z^2 \\
B_q^{L,R}(Q^2) &= \mp 2e_q(v_e \pm a_e)a_qP_Z \pm 2(v_e \pm a_e)^2v_qa_qP_Z^2
\end{align}

With \( f = e, q \) for any fundamental fermion, \( e_f \) is here the electric charge \( (e_e = -1) \), \( v_f = (T_{3f} - 2e_f \sin^2 \theta_W)/\sin 2\theta_W \) and \( a_f = T_{3f}/\sin 2\theta_W \) are the NC vector and axial vector couplings expressed in terms of the third component of the weak isospin \( (T_{3e} = -\frac{1}{2}) \) and the Weinberg angle \( \theta_W \). \( P_Z \) is the ratio of the Z and \( \gamma \) propagators \( P_Z = Q^2/(Q^2 + M_Z^2) \). The corresponding cross sections for positrons, \( e_{L,R}^+ \), are obtained from the above electron formulae, eqs. (8)-(12), by the replacements
\begin{equation}
F_2^{L,R} \rightarrow F_2^{R,L}, \quad xF_3^{L,R} \rightarrow -xF_3^{R,L},
\end{equation}

i.e. a left(right)-handed electron corresponds to a right(left)-handed positron and an additional sign change in $xF_3$. The cross section for an arbitrarily polarized electron/positron beam is simply obtained as a linear combination of these pure left- and right-handed cross sections (cf. [19]).

The pure $\gamma$ exchange term, i.e. the one without a $P_Z$ dependence in eq. (11), dominates completely at low $Q^2$, and the cross section then takes the familiar form measured in fixed target electron and muon beam experiments

\begin{equation}
\frac{d^2\sigma_\gamma(ep)}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}(1 + (1 - y)^2)F_2^{em}(x, Q^2)
\end{equation}

where the electromagnetic structure function is given by

\begin{equation}
F_2^{em}(x, Q^2) = \sum_q e_q^2 \left[ xq(x, Q^2) + x\bar{q}(x, Q^2) \right]
\end{equation}

With increasing $Q^2$ first the $\gamma/Z^0$ interference term (linear in $P_Z$) and then the pure weak term (quadratic in $P_Z$) become important and finally dominate the cross section at large $Q^2$. 

2004, Torino Aram Kotzinian compass
The differential cross sections for charged current (CC) \( ep \) interactions are given by

\[
\frac{d^2 \sigma_{CC}(e^- p)}{d x \, d Q^2} = \frac{(1 - \lambda) \pi \alpha^2}{4 \sin^4 \theta_W \left( Q^2 + M_W^2 \right)^2} \sum_{i,j} \left[ V_{ui} d_j \right] u_i + (1 - y)^2 \left[ V_{uj} d_i \right] \bar{d}_i \tag{16}
\]

\[
\frac{d^2 \sigma_{CC}(e^+ p)}{d x \, d Q^2} = \frac{(1 + \lambda) \pi \alpha^2}{4 \sin^4 \theta_W \left( Q^2 + M_W^2 \right)^2} \sum_{i,j} \left[ V_{ui} d_j \right] \bar{u}_i + (1 - y)^2 \left[ V_{uj} d_i \right] \bar{d}_i \tag{17}
\]

where \( V_{ui} d_j \) are elements of the Kobayashi-Maskawa matrix, \( u_i \) and \( d_j \) denote the parton density functions for the up-type and down-type quark flavours, respectively, and \( i, j \) are family indices. The \( e^\pm \) beam polarization is denoted by \( \lambda \) (\( \pm 1 \) for a right/left-handed state). Considering only four massless quark flavours \( (u, d, s, c) \) and using the unitarity relation \( \sum_j \left| V_{ui} d_j \right|^2 = \sum_j \left| V_{uj} d_i \right|^2 = 1 \) one obtains for any lepton with fixed helicity

\[
\frac{d^2 \sigma_{CC}(\ell p)}{d x \, d Q^2} \sim \frac{G_F^2}{\pi} \left( 1 + \frac{Q^2}{M_W^2} \right)^{-2} \begin{cases} 
(u + c) + (1 - y)^2 (\bar{d} + \bar{s}) & \text{for} \quad \ell = e_L^{-}, \bar{\nu} \\
(\bar{u} + \bar{c}) + (1 - y)^2 (d + s) & \text{for} \quad \ell = e_R^{+}, \nu \\
0 & \text{for} \quad \ell = e_R^{-}, e_L^{+}
\end{cases} \tag{18}
\]

Here, \( G_F = \pi \alpha / (\sqrt{2} \sin^2 \theta_W \, M_W^2) \) is the Fermi coupling constant, \( M_W \) the \( W \)-boson mass and \( u \) denotes the \( u \)-quark density \( u(x, Q^2) \) etc.
In the initialization phase (subroutine LINIT) the simulation variables are defined and their effective limits calculated from applied cuts. The optimization parameters are set and the maximum needed for the weighting is found using an adaptation of MINUIT [22]. Further, the total cross section can be obtained at this stage by numerical integration over the kinematic variables. In the simulation phase, see subroutine LEPTO, phase space points are chosen from the cross section (in subroutine LEPTOX) as discussed above. This Monte Carlo sampling is also used to provide an estimate of the cross section for the process being simulated, see PARL(24) in common LEPTOU. Since the result is updated with each generated event the accuracy depends on the generated statistics as \( \sim 1/\sqrt{N} \).

To define the parton content of the proton for the cross section formulae above the parton density functions \( q(x, Q^2) \), \( \bar{q}(x, Q^2) \) and \( g(x, Q^2) \) are needed. These give the probability to find a quark or antiquark of specific flavours, or a gluon, respectively, carrying a fraction \( x \) of the proton momentum when probing the proton with a momentum transfer \( Q^2 \). Several parametrizations of these distributions have been obtained using data, in particular from lepton scattering experiments, and with \( Q^2 \)-dependence according to the perturbative QCD evolution equations [25]. The fit to the data provides the \( x \)-dependence and the QCD parameter \( \Lambda \). The choice among many available parton density parametrisations in PYTHIA 5.7 [26] and in PDFLIB [27] is made through the switches LST(15) and LST(16) in common LEPTOU.
Nucleon remnant and hadronization

The remnant system is the target nucleon ‘minus’ the parton entering the hard scattering system (initial parton showers and matrix elements). This interacting parton can be either a valence quark, a sea-quark or a gluon.

When the interacting parton is a valence quark the nucleon remnant is simply a diquark composed of the two left-over valence quarks as spectators. In the Lund model [5] a colour triplet string is stretched between the colour triplet charged struck quark and the diquark which is a colour antitriplet. This system is then hadronized in the usual way [5, 26] by the production of quark-antiquark and diquark-antidiquark pairs from the energy in the field, leading to hadron production. The proton remnant diquark is not a single entity; its two quarks may go into a leading baryon but they can also be separated to produce a leading meson followed by a baryon.

In case the interacting parton is a sea quark ($q_s$) or antiquark the nucleon remnant contains the corresponding antiquark or quark in addition to the three valence quarks ($q_v$). This more complicated four-quark system $q_vq_vq_vq_s$ or $q_vq_vq_vq_s$ must be taken into account to conserve the flavour quantum numbers.
In the conventional way (default in LEPTO version 6.2 and earlier) the following treatment has been used. If $\bar{q}_s = \bar{u}$ or $\bar{d}$ it is cancelled against a corresponding valence quark leaving a simple diquark system to be treated as above. For other flavours of $\bar{q}_s$ it is joined with a valence quark of arbitrary flavour into a meson ($M = q_v \bar{q}_s$). The $\bar{q}_s$ is assumed to have no specific dynamic properties such that this splitting process into a meson and a diquark should be similar to normal hadronization. The meson is then given a fraction $z$ of the remnants energy-momentum ($E + p_z$) along the beam direction from a probability distribution $P(z)$ (cf. LST(14)) and only a small Gaussian $p_{\perp}$ (cf. PARL(14)). The left-over diquark, with longitudinal momentum given by $1 - z$ and equal but opposite $p_{\perp}$, forms a string system with the scattered quark and hadronization proceeds as usual. If an antiquark ($\bar{q}_s$) was scattered the remnant is a four-quark system $q_v q_v q_v q_s$ which is treated similarly to the previous case. Here, the corresponding quark ($q_s$) is combined with a random diquark giving a baryon ($B = q_v q_v q_s$) leaving the remaining valence quark to form a string system with the scattered antiquark. The split of the remnant is as before, taking account of the masses in the distribution for $z$ (cf. LST(14)).
Description of program components

standard FORTRAN77 and should therefore run on any computer with such a compiler. Single precision is normally used, but double precision is being used when required.

Subroutines and functions

SUBROUTINE LIMIT(LFILE,LEPIN,PLZ,PPZ,INTER)

Purpose: to initialize the event generation procedure and, optionally, integrate cross section.

Arguments:
LFILE : logical file number containing weights for first order QCD, see LST(8).
          = 0: the weights are calculated but not save on file, no file is used.
          < 0: the weights are calculated and stored on file number -LFILE.
          > 0: the weights are read from file number LFILE.

Remarks: When using weights from a file, the same conditions (interaction, cuts etc) must hold as when the weights were calculated. The relevant quantities are checked and a mismatch results in an error message, cf. LST(3). There is no strong reason, in this upgraded program, to store weights on a file since their calculation is fast enough to be repeated in each run.

LEPIN : type of lepton, i.e. 11 = e⁻, 12 = νₑ, 13 = μ⁻, 14 = νₘ and negative values for the corresponding antiparticles, i.e. JETSET [26] code.

PLZ, PPZ : momentum (GeV/c) for incoming lepton and nucleon, respectively, along the z-axis (if both non-zero, i.e. colliding beams, they must have opposite signs). Colliding beams which are not along a common axis, or variable beam energies, are possible using LST(17).

INTER : type of interaction to be simulated.
          =1: electromagnetic (EM), i.e. γ exchange.
          =2: weak charged current (CC), i.e. W⁺ exchange.
          =3: weak neutral current, i.e. Z⁰ exchange.
          =4: neutral current (NC), i.e. γ/Z⁰ exchange.
SUBROUTINE **LEPTO**

*Purpose:* to administer the generation of one event of the kind specified by the last LINIT call.

**SUBROUTINE LFRAME(IFRAMER,IPHI)**

*Purpose:* to transform the event between different frames.

*Arguments:*

IFRAMER: specifies the desired frame (as for LST(5)).
- =1: hadronic CM frame, z-axis along exchanged boson.
- =2: lepton-nucleon CM frame, z-axis along lepton.
- =3: lab system as defined by last user call to LINIT.
- =4: as 3, but z-axis along exchanged boson.

IPHI : specifies whether to include a random rotation for the azimuthal angle, φ, of the lepton scattering plane.
- =0: no rotation, scattering plane is x – z plane. A possible earlier rotation is undone.

*Remark:* The present frame is stored in LST(28), LST(29) and is updated by LFRAME. Transforming the event with other routines can therefore cause errors in a following LFRAME call.
Common blocks

Most of the communication between the user and the program is via the switches and parameters in the common blocks. The user need mainly be concerned with common LEPTOU since all others are essentially for internal use. All variables are given sensible default values in block data LEPTOD, as shown by (D=...) below. These values may be changed by the user to modify the behaviour of the program. Note, however, that this should usually be done before calling LINIT.

COMMON /LEPTOU/ CUT(14),LST(40),PARL(30),X,Y,W2,Q2,U

Purpose: contains input switches (LST(1)–LST(20),LST(34),LST(35)) and input parameters (PARL(1)–PARL(20)) to specify physics, kinematic cuts and numerical procedures, as well as output flags (LST(21)–LST(40)) and output variables (PARL(21)–PARL(30)). Overwriting default values should be made before calling LINIT.

Parameters:
CUT(1), CUT(2) : (D= $10^{-4}$, 1.) lower and upper limit of Bjorken-$x$ variable.
CUT(3), CUT(4) : (D= 0., 1.) lower and upper limit of $y$ variable.
CUT(5), CUT(6) : (D= 4., $10^8$) lower and upper limit of $Q^2$ (GeV$^2$).
CUT(7), CUT(8) : (D= 5., $10^8$) lower and upper limit of $W^2$ (GeV$^2$).
CUT(9), CUT(10) : (D= 1., $10^8$) lower and upper limit of variable $\nu$ (GeV).
CUT(11), CUT(12) : (D=1., $10^8$) lower and upper limit of scattered lepton energy (GeV, in frame defined by LINIT call).
LST(7) : (D=1) regulates completeness of Monte Carlo simulation (to speed up program when only partial information is needed).
= −1: only kinematic variables generated.
= 0: kinematic variables and parton level event generated, optionally including QCD effects (cf. LST(8)). Hadronization can be made later by calling LUEXEC.
= 1: full event generated, i.e. as 0 plus hadronization and decays.
LST(8) : (D=12) simulation of QCD effects in hadronic final state.
=0: QCD switched off.
=1: first order QCD matrix elements (ME) for gluon radiation and boson-gluon fusion.
=2: QCD parton cascade evolution from initial and final quark.
=3: QCD parton cascade evolution from initial quark only.
=4: QCD parton cascade evolution from final quark only.
=5: QCD switched off, but target remnant treatment as in cascade case.
=9: set by ARIADNE [49] when simulating parton emission in the colour dipole model [53].
LST(12) : (D=4) maximum flavour used in sea structure function parametrizations.
LST(13) : (D=5) heaviest quark flavour allowed in boson-gluon fusion. A threshold factor is applied to compensate for neglected quark masses in the matrix elements.
LST(14) : (D=4) treatment of target remnant after removing interacting parton, see section 2.7.
=0: remnant approximated by anti-parton of removed parton, i.e. by $q, \bar{q}, g$ for removed $\bar{q}, q, g$. No baryon is produced.
=1: for removed valence quark the remnant is a diquark hadronizing into a baryon with the Lund model. For a removed gluon, sea quark ($q_s$), sea antiquark the remnant is, respectively, a $q_vq_vq_v$, $q_vq_vq_v\bar{q}_s$, $q_vq_vq_vq_s$ which is split into $q_vq_v + q_v$, $q_vq_v + M(q_v\bar{q}_s)$, $q_v + B(q_vq_vq_s)$. The (lighter) part of the remnant containing one random flavour valence quark $q_v$ takes the energy-momentum fraction $z$ given by $P(z) = 2(1-z)$, i.e. $\langle z \rangle = 1/3$.
=2: as 1, but with $P(z) = (a + 1)(1 - z)^a$ with $a$ chosen such that $\langle z \rangle = 1/(a + 2) = m/(m + M)$ where $m$ ($M$) is the mass of the light (heavy) remnant subsystem.
=3: as 2, but using the ‘Peterson’ function $P(z) = N/(z(1 - 1/\bar{z} - c/(1 - z))^2)$ with $c = (m/M)^2$.
=4: using LUZDIS, i.e. the fragmentation function chosen in JETSET.
LST(23): specifies process simulated.
   =1: electromagnetic (EM), i.e. $\gamma$ exchange.
   =2: weak charged current (CC), i.e. $W^\pm$ exchange.
   =3: weak neutral current, i.e. $Z^0$ exchange.
   =4: neutral current (NC), i.e. $\gamma/Z^0$ exchange.

LST(24): specifies first order QCD process in current event.
   =1: $q$-event, i.e. no first order QCD.
   =2: $qg$-event, i.e. gluon radiation in first order QCD.
   =3: $q\bar{q}$-event, i.e. boson-gluon fusion in first order QCD.

LST(25): specifies flavour of struck quark in current event: 1=$d$, 2=$u$, 3=$s$, 4=$c$
         , 5=$b$, $-1=\bar{d}$, $-2=\bar{u}$, $-3=\bar{s}$, $-4=\bar{c}$, $-5=\bar{b}$.

LST(26): entry line in event record of outgoing struck quark. In parton shower case,
         quark at boson vertex before final state shower.

LST(27): signals split of non-trivial nucleon remnant, cf. LST(14).
   =0: no split, simple diquark or LST(14)=0.
   =1: split into parton and particle, $qq+M$ or $q+B$, occurs when sea (anti)quark
      removed through the interaction.
   =2: split into quark and diquark, $q + qq$, occurs when a gluon is removed.
PARL(1) : \( (D=1.) \) number of nucleons in target nucleus, i.e. \( A \).
PARL(2) : \( (D=1.) \) number of protons in target nucleus, i.e. \( Z \).

PARL(3) : \( (D=0.44 \text{ GeV}) \) width of Gaussian distribution for the primordial transverse momentum \( k_\perp \) of partons in the nucleon.
PARL(4) : \( (D=0.75) \) probability that a \( ud \)-diquark in the target remnant has spin and isospin equal zero, i.e. \( I=\overline{S}=0 \).
PARL(5) : \( (D=0.2319) \sin^2 \theta_W \) (Weinberg angle) [21].

PARL(28), PARL(29), PARL(30) : values of \( x_p, z_q \) and \( \phi \) in first order massless QCD matrix elements, section 2.5, for current event if it is a \( qg \)- or \( q\overline{q} \)-event, see LST(24). For a \( q^- \)-event they are set to 1.0, 1.0 and 0.0.

\[ X : \text{Bjorken-}x, \text{ i.e. } x = Q^2/2P \cdot q. \]
\[ Y : \text{standard } y \text{ variable, i.e. } y = P \cdot q/P \cdot p_e. \]
\[ W^2 : \text{mass-square of hadronic system, i.e. } W^2 = (P + q)^2. \]
\[ Q^2 : \text{momentum transfer squared, i.e. } Q^2 = -q^2 = -(p_e - p_\ell)^2. \]
\[ U : \text{energy transfer variable } \nu = P \cdot q/m_p. \]
COMMON /LUJETS/  N,K(4000,5),P(4000,5),V(4000,5)
COMMON /LUDAT1/ MSTU(200),PARU(200),MSTJ(200),PARJ(200)

Purpose: LUJETS contains the record of the generated event and is essential for using the results. LUDAT1 contains switches and parameters that are, e.g., essential to control final state parton showers, $\alpha_s$ evaluation and hadronization. These common blocks are described in the JETSET manual [26].

N : number of lines in the K, P and V matrices occupied by the current event. N is continuously updated as the definition of the original configuration and the treatment of fragmentation and decay proceed. In the following, the individual parton/particle number, running between 1 and N, is called I.

K(I,2) : particle KF code, as described in section 5.1.
K(I,3) : line number of parent particle, where known, otherwise 0. Note that the assignment of a particle to a given parton in a parton system is unphysical, and what is given there is only related to the way the fragmentation was generated.
K(I,4) : normally the line number of the first daughter; it is 0 for an undecayed particle or unfragmented parton.
\( P(I,1) \): momentum in the \( x \) direction, in GeV/c.
\( P(I,2) \): momentum in the \( y \) direction, in GeV/c.
\( P(I,3) \): momentum in the \( z \) direction, in GeV/c.
\( P(I,4) \): energy, in GeV.
\( P(I,5) \): mass, in GeV/c\(^2\). In parton showers, with space-like virtualities, i.e. where \( Q^2 = -m^2 > 0 \), one puts \( P(I,5) = -Q \).

\( V(I,1) \): \( x \) position of production vertex, in mm.
\( V(I,2) \): \( y \) position of production vertex, in mm.
\( V(I,3) \): \( z \) position of production vertex, in mm.
\( V(I,4) \): time of production, in mm/c (\( \approx 3.33 \times 10^{-12} \) s).
\( V(I,5) \): proper lifetime of particle, in mm/c (\( \approx 3.33 \times 10^{-12} \) s). If the particle is not expected to decay, \( V(I,5) = 0 \). A line with \( K(I,1) = 4 \), i.e. a particle that could have decayed, but did not within the allowed region, has the proper non-zero \( V(I,5) \).

In the absence of electric or magnetic fields, or other disturbances, the decay vertex \( VP \) of an unstable particle may be calculated as
\[ VP(j) = V(I,j) + V(I,5) * P(I,j) / P(I,5), \ j = 1-4. \]
PYTHIA and JETSET

Hadronization in LEPTO is performed by JETSET

The PYTHIA program is frequently used for event generation in high-energy physics. The emphasis is on multiparticle production in collisions between elementary particles. This in particular means hard interactions in $e^+e^-$, pp and ep colliders, although also other applications are envisaged. The program is intended to generate complete events, in as much detail as experimentally observable ones, within the bounds of our current understanding of the underlying physics. Many of the components of the program represents original research, in the sense that models have been developed and implemented for a number of aspects not covered by standard theory.

Event generators often have a reputation for being ‘black boxes’

The Monte Carlo program is built as a slave system, i.e. you, the user, have to supply the main program. From this the various subroutines are called on to execute specific tasks, after which control is returned to the main program.

Our aim is to try to understand as much as possible the underlying physics included in LEPTO and PYTHIA
The Event Record

The event record is the central repository for information about the particles produced in the current event: flavours, momenta, event history, and production vertices. It plays a very central rôle: without a proper understanding of what the record is and how information is stored, it is meaningless to try to use PYTHIA. The record is stored in the common block PYJETS. Almost all the routines that the user calls can be viewed as performing some action on the record: fill a new event, let partons fragment or particles decay, boost it, list it, find clusters, etc.

The Particle Data Group particle code is used consistently throughout the program.

<table>
<thead>
<tr>
<th>KF</th>
<th>Name</th>
<th>Printed</th>
<th>KF</th>
<th>Name</th>
<th>Printed</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>d</td>
<td>d</td>
<td>11</td>
<td>e⁻</td>
<td>e⁻</td>
</tr>
<tr>
<td>2</td>
<td>u</td>
<td>u</td>
<td>12</td>
<td>νₑ</td>
<td>nu_e</td>
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<tr>
<td>3</td>
<td>s</td>
<td>s</td>
<td>13</td>
<td>μ⁻</td>
<td>mu⁻</td>
</tr>
<tr>
<td>4</td>
<td>c</td>
<td>c</td>
<td>14</td>
<td>νᵢμ</td>
<td>nu_mu</td>
</tr>
<tr>
<td>5</td>
<td>b</td>
<td>b</td>
<td>15</td>
<td>τ⁻</td>
<td>tau⁻</td>
</tr>
<tr>
<td>6</td>
<td>t</td>
<td>t</td>
<td>16</td>
<td>νᵢτ</td>
<td>nu_tau</td>
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<td>b'</td>
<td>17</td>
<td>τ'</td>
<td>tau'</td>
</tr>
<tr>
<td>8</td>
<td>t'</td>
<td>t'</td>
<td>18</td>
<td>νᵢ'</td>
<td>nu'_tau</td>
</tr>
<tr>
<td>9</td>
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<td></td>
<td>19</td>
<td></td>
<td></td>
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<td>10</td>
<td></td>
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Table 6: Gauge boson and other fundamental boson codes.

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<th>KF</th>
<th>Name</th>
<th>Printed</th>
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<td>(g)</td>
<td>(g)</td>
<td>31</td>
<td>(Z^0)</td>
<td>(Z'0)</td>
</tr>
<tr>
<td>22</td>
<td>(\gamma)</td>
<td>gamma</td>
<td>32</td>
<td>(Z'^0)</td>
<td>(Z''0)</td>
</tr>
<tr>
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<td>(Z^0)</td>
<td>33</td>
<td>(Z''^0)</td>
<td>(Z''^0)</td>
</tr>
<tr>
<td>24</td>
<td>(W^+)</td>
<td>(w^+)</td>
<td>34</td>
<td>(W'^+)</td>
<td>(w'^+)</td>
</tr>
</tbody>
</table>

Diquark codes, Table 8.

A diquark made up of a quark with code \(i\) and another with code \(j\), where \(i \geq j\), and with total spin \(s\), is given the code

\[
KF = 1000i + 100j + 2s + 1 ,
\]

(16)

Table 8: Diquark codes. For brevity, diquarks containing \(c\) or \(b\) quarks are not listed, but are defined analogously.

<table>
<thead>
<tr>
<th>KF</th>
<th>Name</th>
<th>Printed</th>
<th>KF</th>
<th>Name</th>
<th>Printed</th>
</tr>
</thead>
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<td>ud_0</td>
<td>1103</td>
<td>dd_1</td>
<td>dd_1</td>
</tr>
<tr>
<td>3101</td>
<td>sd_0</td>
<td>sd_0</td>
<td>2103</td>
<td>ud_1</td>
<td>ud_1</td>
</tr>
<tr>
<td>3201</td>
<td>su_0</td>
<td>su_0</td>
<td>2203</td>
<td>uu_1</td>
<td>uu_1</td>
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<td></td>
<td></td>
<td></td>
<td>3103</td>
<td>sd_1</td>
<td>sd_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3203</td>
<td>su_1</td>
<td>su_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>ss_1</td>
<td>ss_1</td>
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Table 9: Meson codes, part 1.

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<th>Name</th>
<th>Printed</th>
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<td>pi+</td>
<td>213</td>
<td>$\rho^+$</td>
<td>rho+</td>
</tr>
<tr>
<td>311</td>
<td>$K^0$</td>
<td>K0</td>
<td>313</td>
<td>$K^{*0}$</td>
<td>K*0</td>
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<td>321</td>
<td>$K^+$</td>
<td>K+</td>
<td>323</td>
<td>$K^{*+}$</td>
<td>K++</td>
</tr>
<tr>
<td>411</td>
<td>$D^+$</td>
<td>D+</td>
<td>413</td>
<td>$D^{*+}$</td>
<td>D++</td>
</tr>
<tr>
<td>421</td>
<td>$D^0$</td>
<td>D0</td>
<td>423</td>
<td>$D^{*0}$</td>
<td>D*0</td>
</tr>
<tr>
<td>431</td>
<td>$D_{s}^+$</td>
<td>D_{s}+</td>
<td>433</td>
<td>$D_{s}^{*+}$</td>
<td>D_{s}*+</td>
</tr>
<tr>
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<td>B0</td>
<td>513</td>
<td>$B^{*0}$</td>
<td>B*0</td>
</tr>
<tr>
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<td>$B^+$</td>
<td>B+</td>
<td>523</td>
<td>$B^{*+}$</td>
<td>B++</td>
</tr>
<tr>
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<td>B_{s}0</td>
<td>533</td>
<td>$B_{s}^{*0}$</td>
<td>B_{s}*0</td>
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<td>541</td>
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<td>B_{c}+</td>
<td>543</td>
<td>$B_{c}^{*+}$</td>
<td>B_{c}*+</td>
</tr>
<tr>
<td>111</td>
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<td>pi0</td>
<td>113</td>
<td>$\rho^{0}$</td>
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</tr>
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<tr>
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<td>J/psi</td>
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<td>$\eta_b$</td>
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<td>553</td>
<td>$\Upsilon$</td>
<td>Upsilon</td>
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<td>130</td>
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<td>K_{L}0</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>K_{S}0</td>
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</table>
Table 11: Baryon codes. For brevity, some states with b quarks or multiple c ones are omitted from this listing, but are defined in the program.

<table>
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<th>KF</th>
<th>Name</th>
<th>Printed</th>
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<td>Delta⁻</td>
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<td>p</td>
<td>p⁺</td>
<td>2214</td>
<td>Δ⁺</td>
<td>Delta⁺</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>2224</td>
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<td>Delta⁺⁺</td>
</tr>
<tr>
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<td>Sigma⁻</td>
<td>3114</td>
<td>Σ⁺⁻</td>
<td>Sigma⁺⁻</td>
</tr>
<tr>
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<td>Lambda⁰</td>
<td></td>
<td></td>
<td></td>
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<td>Sigma⁰</td>
<td>3214</td>
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<td>Σ⁺</td>
<td>Sigma⁺</td>
<td>3224</td>
<td>Σ⁺⁺</td>
<td>Sigma⁺⁺</td>
</tr>
<tr>
<td>3312</td>
<td>Ξ⁻</td>
<td>Xi⁻</td>
<td>3314</td>
<td>Ξ⁺⁻</td>
<td>Xi⁺⁻</td>
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<tr>
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<td>Xi⁺⁰</td>
</tr>
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<td></td>
<td></td>
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<td>Omega⁻</td>
</tr>
<tr>
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<td>Sigma⁺₀</td>
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<td>Σ⁺₀</td>
<td>Sigma⁺₀</td>
</tr>
<tr>
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<td>Lambda⁺₀</td>
<td></td>
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<tr>
<td>4212</td>
<td>Σ⁺⁺</td>
<td>Sigma⁺⁺</td>
<td>4214</td>
<td>Σ⁺⁺</td>
<td>Sigma⁺⁺</td>
</tr>
</tbody>
</table>
Where to study hadronization

Electron-positron annihilation

Best to study a simple system:
No color in initial state
How to Model Real Collisions

- What if we want to know more details
- Cannot calculate hadronization processes directly in QCD
  - There is no Feynman diagram for $q \rightarrow \pi \pi \pi$
- Instead, we have models based on general properties of QCD
  - Generally called “fragmentation” models
    - Independent (ISAJET) – “Pure” Fragmentation
    - Strings (JETSET, PYTHIA) – String Fragmentation
    - Cluster (HERWIG) – Phase Space Fragmentation
- As time goes by, more and more QCD is incorporated into these models
  - Gluon radiation down to a soft momentum scale $Q_0$
  - Hadronization becomes less important as scale decreases
Basic Idea of String Models

Nature of strong force confines field lines to a narrow “flux tube”

Potential is Coulomb + Linear \( V(\, r \, ) = -\frac{A}{r} + Br \)

\[ \pi^- \quad \pi^+ \]

proton

Anti-proton
Fragmentation

The main fragmentation option in Pythia is the Lund string scheme, but independent fragmentation options are also available. These latter options should not be taken too seriously, since we know that independent fragmentation does not provide a consistent alternative, but occasionally one may like to compare string fragmentation with something else.

In either string or independent fragmentation, an iterative approach is used to describe the fragmentation process. Given an initial quark $q = q_0$, it is assumed that a new $q_1\bar{q}_1$ pair may be created, such that a meson $q_0\bar{q}_1$ is formed, and a $q_1$ is left behind. This $q_1$ may at a later stage pair off with a $\bar{q}_2$, and so on. What need be given is thus the relative probabilities to produce the various possible $q_i\bar{q}_i$ pairs, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, etc., and the relative probabilities that a given $q_{i-1}\bar{q}_i$ quark pair combination forms a specific meson, e.g. for $u\bar{d}$ either $\pi^+$, $\rho^+$ or some higher state.

In order to generate the quark–antiquark pairs $q_i\bar{q}_i$ which lead to string breakups, the Lund model invokes the idea of quantum mechanical tunnelling, as follows. If the $q_i$ and $\bar{q}_i$ have no (common) mass or transverse momentum, the pair can classically be created at one point and then be pulled apart by the field. If the quarks have mass and/or transverse momentum, however, the $q_i$ and $\bar{q}_i$ must classically be produced at a certain distance so that the field energy between them can be transformed into the sum of the two transverse masses $m_\perp$. Quantum mechanically, the quarks may be created in one point (so as to keep the concept of local flavour conservation) and then tunnel out to the classically allowed region. In terms of a common transverse mass $m_\perp$ of the $q_i$ and the $\bar{q}_i$, the tunnelling probability is given by
\[
\exp \left( -\frac{\pi m_\perp^2}{\kappa} \right) = \exp \left( -\frac{\pi m_i^2}{\kappa} \right) \exp \left( -\frac{\pi p_\perp^2}{\kappa} \right)
\]

The factorization of the transverse momentum and the mass terms leads to a flavour-independent Gaussian spectrum for the \(p_x\) and \(p_y\) components of \(q_i \bar{q}_i\) pairs. Since the string is assumed to have no transverse excitations, this \(\nu\) is locally compensated between the quark and the antiquark of the pair. The \(p_\perp\) of a meson \(q_{i-1} \bar{q}_i\) is given by the vector sum of the \(p_\perp\):s of the \(q_{i-1}\) and \(\bar{q}_i\) constituents, which implies Gaussians in \(p_x\) and \(p_y\) with a width \(\sqrt{2}\) that of the quarks themselves. The assumption of a Gaussian shape may be a good first approximation, but there remains the possibility of non-Gaussian tails, that can be important in some situations.

The formula also implies a suppression of heavy quark production \(u : d : s : c \approx 1 : 1 : 0.3 : 10^{-11}\). Charm and heavier quarks are hence not expected to be produced in the soft fragmentation. Since the predicted flavour suppressions are in terms of quark masses, which are notoriously difficult to assign (should it be current algebra, or constituent, or maybe something in between?), the suppression of \(s \bar{s}\) production is left as a free parameter in the program: \(u \bar{u} : d \bar{d} : s \bar{s} = 1 : 1 : \gamma_s\), where by default \(\gamma_s = 0.3\). At least qualitatively, the experimental value agrees with theoretical prejudice. There is no production at all of heavier flavours in the fragmentation process, but only in the hard process or as part of the shower evolution.
Once the flavours $q_{i-1}$ and $\bar{q}_i$ have been selected, a choice is made between the possible multiplets. The relative composition of different multiplets is not given from first principles, but must depend on the details of the fragmentation process. To some approximation one would expect a negligible fraction of states with radial excitations or non-vanishing orbital angular momentum. Spin counting arguments would then suggest a 3:1 mixture between the lowest lying vector and pseudoscalar multiplets.

In the program, six meson multiplets are included. If the nonrelativistic classification scheme is used, i.e. mesons are assigned a valence quark spin $S$ and an internal orbital angular momentum $\mathbf{L}$, with the physical spin $s$ denoted $J, \mathbf{J} = \mathbf{L} + \mathbf{S}$, then the multiplets are:

- $L = 0$, $S = 0$, $J = 0$: the ordinary pseudoscalar meson multiplet;
- $L = 0$, $S = 1$, $J = 1$: the ordinary vector meson multiplet;
- $L = 1$, $S = 0$, $J = 1$: an axial vector meson multiplet;
- $L = 1$, $S = 1$, $J = 0$: the scalar meson multiplet;
- $L = 1$, $S = 1$, $J = 1$: another axial vector meson multiplet; and
- $L = 1$, $S = 1$, $J = 2$: the tensor meson multiplet.

For the flavour-diagonal meson states $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, it is also necessary to include

\[
\eta = \frac{1}{2}(u\bar{u} + d\bar{d}) - \frac{1}{\sqrt{2}}s\bar{s}; \quad \omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})
\]
\[
\eta' = \frac{1}{2}(u\bar{u} + d\bar{d}) + \frac{1}{\sqrt{2}}s\bar{s}; \quad \phi = s\bar{s}.
\]
Baryon production

Diquark picture

Baryon production may, in its simplest form, be obtained by assuming that any flavour \( q_i \) given above could represent either a quark or an antidiquark in a colour triplet state. Then the same basic machinery can be run through as above, supplemented with the probability to produce various diquark pairs. In principle, there is one parameter for each diquark, but if tunnelling is still assumed to give an effective description, mass relations can be used to reduce the effective number of parameters. There are three main ones appearing in the program:

- the relative probability to pick a \( \overline{q}q \) diquark rather than a \( q \);
- the extra suppression associated with a diquark containing a strange quark (over and above the ordinary s/u suppression factor \( \gamma_s \)); and
- the suppression of spin 1 diquarks relative to spin 0 ones (apart from the factor of 3 enhancement of the former based on counting the number of spin states).

The extra strange diquark suppression factor comes about since what appears in the exponent of the tunnelling formula is \( m^2 \) and not \( m \), so that the diquark and the strange quark suppressions do not factorize.

Only two baryon multiplets are included, i.e. there are no \( L = 1 \) excited states. The two multiplets are:

- \( S = J = 1/2 \): the ‘octet’ multiplet of \( \text{SU}(3) \);
- \( S = J = 3/2 \): the ‘decuplet’ multiplet of \( \text{SU}(3) \).
Simple popcorn

A more general framework for baryon production is the ‘popcorn’ one [And85], in which diquarks as such are never produced, but rather baryons appear from the successive production of several $q_i\overline{q}_i$ pairs. The picture is the following. Assume that the original $q$ is red $r$ and the $\overline{q}$ is $\overline{r}$. Normally a new $q_1\overline{q}_1$ pair produced in the field would also be $r\overline{r}$, so that the $\overline{q}_1$ is pulled towards the q end and vice versa, and two separate colour-singlet systems $q\overline{q}_1$ and $q_1\overline{q}$ are formed. Occasionally, the $q_1\overline{q}_1$ pair may be e.g. $g\overline{g}$ ($g =$ green), in which case there is no net colour charge acting on either $q_1$ or $\overline{q}_1$. Therefore, the pair cannot gain energy from the field, and normally would exist only as a fluctuation. If $q_1$ moves towards $q$ and $\overline{q}_1$ towards $\overline{q}$, the net field remaining between $q_1$ and $\overline{q}_1$ is $\overline{b}b$ ($b =$ blue; $g + r = \overline{b}$ if only colour triplets are assumed). In this central field, an additional $q_2\overline{q}_2$ pair can be created, where $q_2$ now is pulled towards $qq_1$ and $\overline{q}_2$ towards $\overline{q}\overline{q}_1$, with no net colour field between $q_2$ and $\overline{q}_2$. If this is all that happens, the baryon $B$ will be made up out of $q_1$, $q_2$ and some $q_4$ produced between $q$ and $q_1$, and $\overline{B}$ of $\overline{q}_1$, $\overline{q}_2$ and some $\overline{q}_5$, i.e. the $B$ and $\overline{B}$ will be nearest neighbours in rank and share two quark pairs. Specifically, $q_1$ will gain energy from $q_2$ in order to end up on mass shell, and the tunnelling formula for an effective $q_1q_2$ diquark is recovered.

In total, the flavour iteration procedure therefore contains the following possible sub-processes (plus, of course, their charge conjugates):

- $q_1 \rightarrow q_2 + (q_1\overline{q}_2)$ meson;
- $q_1 \rightarrow \overline{q}_2q_3 + (q_1q_2q_3)$ baryon;
- $q_1q_2 \rightarrow \overline{q}_3 + (q_1q_2q_3)$ baryon;
- $q_1q_2 \rightarrow q_1q_3 + (q_2\overline{q}_3)$ meson;
What remains to be determined is the energy and longitudinal momentum of the hadron. In fact, only one variable can be selected independently, since the momentum of the hadron is constrained by the already determined hadron transverse mass $m_\perp$,

$$(E + p_z)(E - p_z) = E^2 - p_z^2 = m_\perp^2 = m^2 + p_x^2 + p_y^2.$$ 

In an iteration from the quark end, one is led (by the desire for longitudinal boost invariance and other considerations) to select the $z$ variable as the fraction of $E + p_z$ taken by the hadron, out of the available $E + p_z$. As hadrons are split off, the $E + p_z$ (and $E - p_z$) left for subsequent steps is reduced accordingly:

$$(E + p_z)_{\text{new}} = (1 - z)(E + p_z)_{\text{old}},$$

$$(E - p_z)_{\text{new}} = (E - p_z)_{\text{old}} - \frac{m_\perp^2}{z(E + p_z)_{\text{old}}}.$$ 

The fragmentation function $f(z)$, which expresses the probability that a given $z$ is picked, could in principle be arbitrary — indeed, several such choices can be used inside the program, see below.

If one, in addition, requires that the fragmentation process as a whole should look the same, irrespectively of whether the iterative procedure is performed from the $q$ end or the $\bar{q}$ one, ‘left–right symmetry’, the choice is essentially unique [And83a]: the ‘Lund symmetric fragmentation function’,

2004, Torino

Aram Kotzinian

compass

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\[ f(z) \propto \frac{1}{z} z^{a_{\alpha}} \left( \frac{1 - z}{z} \right)^{a_{\beta}} \exp \left( -\frac{b m_{\perp}^2}{z} \right) \]

There is one separate parameter \( a \) for each flavour, with the index \( \alpha \) corresponding to the ‘old’ flavour in the iteration process, and \( \beta \) to the ‘new’ flavour. It is customary to put all \( a_{\alpha,\beta} \) the same, and thus arrive at the simplified expression

\[ f(z) \propto z^{-1} (1 - z)^a \exp(-b m_{\perp}^2 / z) . \]

In the program, only two separate \( a \) values can be given, that for quark pair production and that for diquark one. In addition, there is the \( b \) parameter, which is universal.

The explicit mass dependence in \( f(z) \) implies a harder fragmentation function for heavier hadrons. The asymptotic behaviour of the mean \( z \) value for heavy hadrons is

\[ \langle z \rangle \approx 1 - \frac{1 + a}{b m_{\perp}^2} . \]

*Cluster decay.* If a string is produced with a small invariant mass, maybe only two-body final states are kinematically accessible. The traditional iterative Lund scheme is then not applicable. We call such a low-mass string a cluster, and consider it separately from above. The modelling is still intended to give a smooth match on to the standard string scheme in the high-cluster-mass limit.
QCD Coherence

Hard to radiate soft gluons at large angles

At large angle, if photon wavelength too large, sees zero net charge: can’t see structure of dipole

Limits photon emission to inside opening angle of e+e-

Would not be the case if e+ and e- emitted incoherently

Same effect in QCD, hard to emit large-angle soft gluons!

“Angular Ordering”

AO naturally creates angular structure of events – jet cones!
Basic idea:

- Outgoing partons radiate gluons & quarks
- Radiation constrained by QCD coherence
  - “Angular ordering” is a major part of this
- At end of chain, gather partons into color singlet “clusters”
- Clusters decay by phase space
  - Small clusters $\rightarrow$ 1 hadron
  - Big clusters $\rightarrow$ 2 clusters
  - “Just Right” $\rightarrow$ 2 hadrons
What’s Better?

When making measurements, you need to simulate real events
- Detector response
- Physics acceptance
- Fluctuations

String & Cluster fragmentation are both used heavily by experiments – both are adequate

Simple reason:
- Models are *tuned* → adjust parameters to fit data
- Much of the QCD-related physics is similar
  - DGLAP evolution is a common feature
  - More reliable at higher energies
- Not surprising, as we use more pQCD, results are less dependent on details of npQCD part!

Still, there are results which favor one model over the other (e.g. charge-rapidity correlations)
Tuning of LUND Fragmentation Model

Default JETSET settings don’t work for HERMES

Use hadron production ratios and measured hadron multiplicities $N^h/N^{DIS}$ in (iterative) tuning procedure