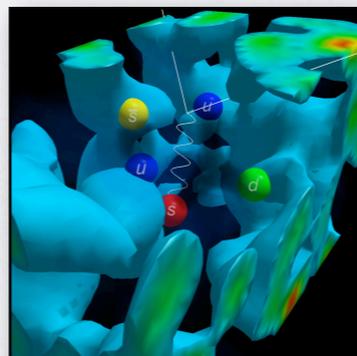


HARD SCATTERINGS AND SOFT GLUONS

Lorenzo Magnea

University of Torino - INFN Torino

QCD@LHC14 - Suzdal - 25/08/14



Outline

- Motivation
- One word about one loop
- A bird's eye over two loops
- Breaking ground at three loops
- Soft gluons to all loops: from theory ...
- ... to phenomenology
- Outlook

MOTIVATION



Where we stand

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- 🎧 In the **coming years** the need will be for **precise predictions** of **increasingly complex** SM processes.
 - * *A great time for the QCD community!*
- 🎧 The **run-up** to the LHC has seen a **vast effort** and **great progress** in **precision phenomenology**: PDF's, jets, hard cross sections, resummations ...
 - * *A continuing effort that will hopefully pay off during Run Two!*

ONE WORD ABOUT ONE LOOP



Done.

Done.

📌 Multi-leg NLO calculations matched with parton showers are now a **commodity**.



Examples of processes calculated with GoSam

GoSam + MadDipole/MadGraph/MadEvent

- $pp \rightarrow W^+W^- + 2 jets$ [Greiner, GH, Mastrolia, Ossola, Reiter, Tramontano '12]
- $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 + jet$ [Cullen, Greiner, GH '12]
- $pp \rightarrow (G \rightarrow \gamma\gamma) + 1 jet$ [Greiner, GH, Reichel, von Soden-Fraunhofen '13]
- $pp \rightarrow \gamma\gamma + 1, 2 jets$ [Gehrmann, Greiner, GH '13]
- $pp \rightarrow HH + 2 jets$ [Dolan, Englert, Greiner, Spannowsky '13]

GoSam + Sherpa

- $pp \rightarrow W^+W^+ + 2 jets$ [Greiner, GH, Luisoni, Mastrolia, Ossola, Reiter, Tramontano '12]
- $pp \rightarrow H + 2 jets$ [van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano '13]
- $pp \rightarrow W^+W^- b\bar{b}$ [GH, Maier, Nisius, Schlenk, Winter '13]
- $pp \rightarrow t\bar{t} + 0, 1 jet$ (includes shower) [Höche, Huang, Luisoni, Schönherr, Winter '13]
- $pp \rightarrow H t\bar{t} + 0, 1 jet$ [van Deurzen, Luisoni, Mastrolia, Mirabella, Ossola, Peraro '13]

GoSam + Powheg (includes shower)

- $pp \rightarrow HW/HZ + 0, 1 jet$ [Luisoni, Nason, Oleari, Tramontano '13]

GoSam + Herwig++/Matchbox (includes shower)

- $pp \rightarrow Z + jet$ [Bellm, Gieseke, Greiner, GH, Plätzer, Reuschle, von Soden-Fraunhofen '13]

GoSam + MadDipole/MadGraph/MadEvent + Sherpa

- $pp \rightarrow H + 3 jets$ [Cullen, van Deurzen, Greiner, Luisoni, Mastrolia, Mirabel

Heinrich

n-Tuple availability

- The n-Tuple files are available
 - On the grid
 - On castor at CERN
- For a range of processes

Process	Pathname	Energy	Jet cut
W+ + 1,2,3,4 jets	Wp<n>j	7TeV	25GeV
W+ + 1,2,3 jets	Wp<n>j	8TeV	20GeV
W- + 1,2,3,4 jets	Wm<n>j	7TeV	25GeV
W- + 1,2,3 jets	Wm<n>j	8TeV	20GeV
Z/gamma* + 1,2 jets	Zee<n>j	7TeV	25GeV
Z/gamma* + 3,4 jets	Zee<n>j	7TeV	20GeV
Z/gamma* + 1,2,3 jets	Zee<n>j	8TeV	20GeV
2,3,4 jets	PureQCD<n>j	7TeV,8TeV	40GeV

From <http://blackhat.hepforge.org/trac/wiki/Availability>

Loopfest 2014, Brooklyn, 18th June

Maître

[J. Alwall et al. 1405.0301]

AUTOMATIC NLO IN THE SM (2014)



Process	Syntax	Cross section (pb)			
		LO 13 TeV		NLO 13 TeV	
Vector-boson pair + jets					
b.1 $pp \rightarrow W^+W^- (4f)$	$p p > w^+ w^-$	$7.335 \pm 0.005 \cdot 10^2$	+5.0% +2.0% -5.1% -2.0%	$1.028 \pm 0.003 \cdot 10^2$	+4.0% +1.0% -4.0% -1.0%
b.2 $pp \rightarrow ZZ$	$p p > z z$	$1.097 \pm 0.002 \cdot 10^2$	+4.0% +1.0% -4.0% -1.0%	$1.415 \pm 0.005 \cdot 10^2$	+3.1% +1.0% -3.1% -1.0%
b.3 $pp \rightarrow ZW^{\pm}$	$p p > z w^{\pm}$	$2.777 \pm 0.003 \cdot 10^2$	+5.0% +2.0% -4.7% -1.0%	$4.487 \pm 0.013 \cdot 10^2$	+4.0% +1.7% -4.0% -1.0%
b.4 $pp \rightarrow \gamma\gamma$	$p p > \gamma \gamma$	$2.510 \pm 0.002 \cdot 10^2$	+10.1% +5.4% -10.0% -3.1%	$6.563 \pm 0.021 \cdot 10^2$	+17.0% +5.0% -16.0% -1.9%
b.5 $pp \rightarrow \gamma Z$	$p p > \gamma z$	$2.523 \pm 0.004 \cdot 10^2$	+9.0% +2.0% -11.2% -1.4%	$3.095 \pm 0.013 \cdot 10^2$	+5.0% +1.0% -7.1% -1.4%
b.6 $pp \rightarrow \gamma W^{\pm}$	$p p > \gamma w^{\pm}$	$2.954 \pm 0.005 \cdot 10^2$	+9.5% +2.0% -11.0% -1.7%	$7.124 \pm 0.026 \cdot 10^2$	+9.7% +1.0% -9.9% -1.0%
b.7 $pp \rightarrow W^+W^- j (4f)$	$p p > w^+ w^- j$	$2.465 \pm 0.003 \cdot 10^2$	+11.0% +1.0% -10.0% -0.8%	$3.730 \pm 0.013 \cdot 10^2$	+4.0% +1.1% -4.0% -0.8%
b.8 $pp \rightarrow ZZ j$	$p p > z z j$	$3.662 \pm 0.003 \cdot 10^2$	+10.3% +1.0% -9.3% -0.8%	$4.830 \pm 0.016 \cdot 10^2$	+5.0% +1.1% -4.0% -0.9%
b.9 $pp \rightarrow ZW^{\pm} j$	$p p > z w^{\pm} j$	$1.805 \pm 0.005 \cdot 10^2$	+11.1% +0.9% -10.0% -0.7%	$2.086 \pm 0.007 \cdot 10^2$	+4.0% +0.9% -4.0% -0.7%
b.10 $pp \rightarrow \gamma\gamma j$	$p p > \gamma \gamma j$	$1.022 \pm 0.001 \cdot 10^2$	+20.3% +1.2% -17.7% -1.0%	$2.292 \pm 0.010 \cdot 10^2$	+11.3% +1.0% -11.3% -1.0%
b.11* $pp \rightarrow \gamma Z j$	$p p > \gamma z j$	$8.310 \pm 0.017 \cdot 10^2$	+14.3% +1.0% -12.0% -1.0%	$1.220 \pm 0.005 \cdot 10^2$	+7.3% +0.9% -7.4% -0.9%
b.12* $pp \rightarrow \gamma W^{\pm} j$	$p p > \gamma w^{\pm} j$	$2.546 \pm 0.010 \cdot 10^2$	+13.7% +0.8% -12.1% -1.0%	$3.713 \pm 0.015 \cdot 10^2$	+7.2% +0.9% -7.1% -1.0%
b.13 $pp \rightarrow W^+W^+ jj$	$p p > w^+ w^+ j j$	$1.484 \pm 0.006 \cdot 10^{-1}$	+26.0% +2.0% -18.0% -1.0%	$2.251 \pm 0.011 \cdot 10^{-1}$	+10.0% +2.0% -10.0% -1.0%
b.14 $pp \rightarrow W^+W^- jj$	$p p > w^+ w^- j j$	$6.752 \pm 0.007 \cdot 10^{-2}$	+26.0% +2.0% -18.0% -1.0%	$1.003 \pm 0.003 \cdot 10^{-1}$	+10.1% +2.0% -10.0% -1.0%
b.15 $pp \rightarrow W^+W^- jj (4f)$	$p p > w^+ w^- j j$	$1.144 \pm 0.002 \cdot 10^1$	+21.2% +0.7% -19.0% -0.5%	$1.295 \pm 0.005 \cdot 10^1$	+5.0% +0.7% -4.8% -0.5%
b.16 $pp \rightarrow ZZ jj$	$p p > z z j j$	$1.344 \pm 0.002 \cdot 10^0$	+26.0% +0.7% -19.0% -0.4%	$1.706 \pm 0.011 \cdot 10^0$	+5.0% +0.8% -7.3% -0.6%
b.17 $pp \rightarrow ZW^{\pm} jj$	$p p > z w^{\pm} j j$	$8.038 \pm 0.009 \cdot 10^0$	+26.0% +0.9% -19.0% -0.5%	$9.130 \pm 0.031 \cdot 10^0$	+3.1% +0.9% -5.1% -0.6%
b.18 $pp \rightarrow \gamma\gamma jj$	$p p > \gamma \gamma j j$	$5.377 \pm 0.029 \cdot 10^0$	+30.3% +0.8% -19.0% -1.0%	$7.501 \pm 0.032 \cdot 10^0$	+6.0% +0.8% -10.1% -1.0%
b.19* $pp \rightarrow \gamma Z jj$	$p p > \gamma z j j$	$3.360 \pm 0.008 \cdot 10^0$	+34.3% +0.8% -18.0% -0.8%	4.342 ± 0.016	+3.1% +0.9% -18.0% -0.8%
b.20* $pp \rightarrow \gamma W^{\pm} jj$	$p p > \gamma w^{\pm} j j$	$1.233 \pm 0.002 \cdot 10^1$	+34.3% +0.8% -18.0% -0.8%	1.448 ± 0.005	+3.1% +0.9% -18.0% -0.8%

Hirschi

First OpenLoops Applications (Higgs and Top phenomenology)

- MEPS@NLO for $\ell\ell\nu\nu+0,1 jets$, Cascioli, Höche, Krauss, Maierhöfer, S. P., Siegert, arXiv:1309.0500
- MC@NLO $pp \rightarrow t\bar{t}b\bar{b}$ with $m_b > 0$, Cascioli, Maierhöfer, Moretti, S. P., Siegert, arXiv:1309.5912
- NLO for $pp \rightarrow W^+W^-b\bar{b}$ with $m_b > 0$, Cascioli, Kallweit, Maierhöfer, S. P., arXiv:1312.0546
- NNLO for $pp \rightarrow \gamma Z$ production, Grazzini, Kallweit, Rathlev, Torre, arXiv:1309.7000
- NLO merging for $pp \rightarrow HH+0,1 jets$, Maierhöfer, Papafystathiou, arXiv:1401.0007
- MEPS@NLO for $t\bar{t}+0,1,2 jets$, Höche, Krauss, Maierhöfer, S. P., Schönherr, Siegert arXiv:1402.6293
- MEPS@NLO for $WWW+0,1 jets$, Höche, Krauss, S. P., Schönherr, Thompson arXiv:1403.7516
- NNLO for $q\bar{q} \rightarrow t\bar{t}$ production, Abelo, Gehrmann-de Ridder, Maierhöfer, S.P., arXiv:1404.6493
- NNLO for $pp \rightarrow ZZ$ production, Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, S.P., Rathlev, Tancredi, Weihs, arXiv:1405.2219

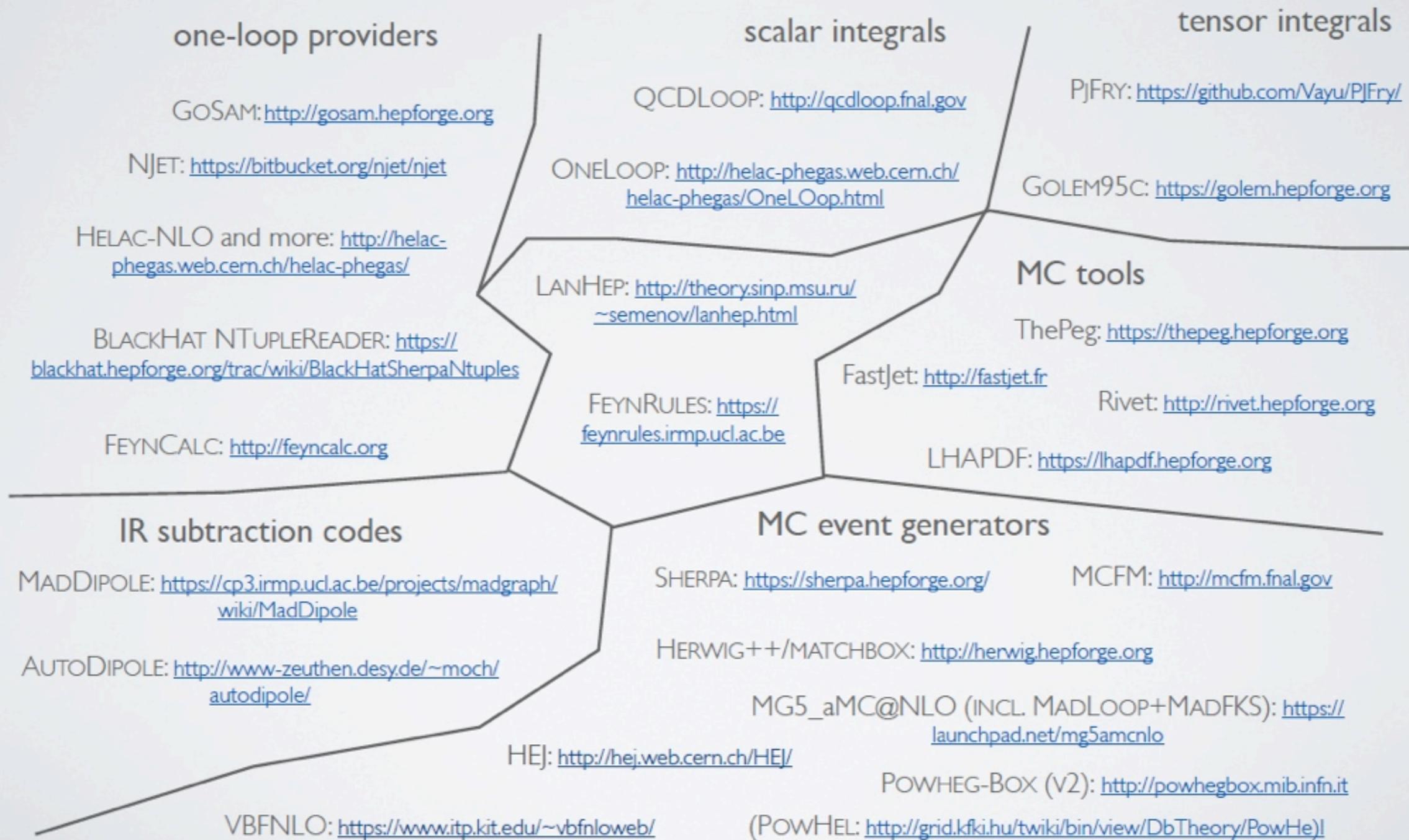
Technical Motivation

- technical stress tests: multi-particle and multi-scale problems, loop-induced processes, multiple resonances, ...
- beyond parton-level NLO: MC@NLO, MEPS@NLO and NNLO

Pozzorini

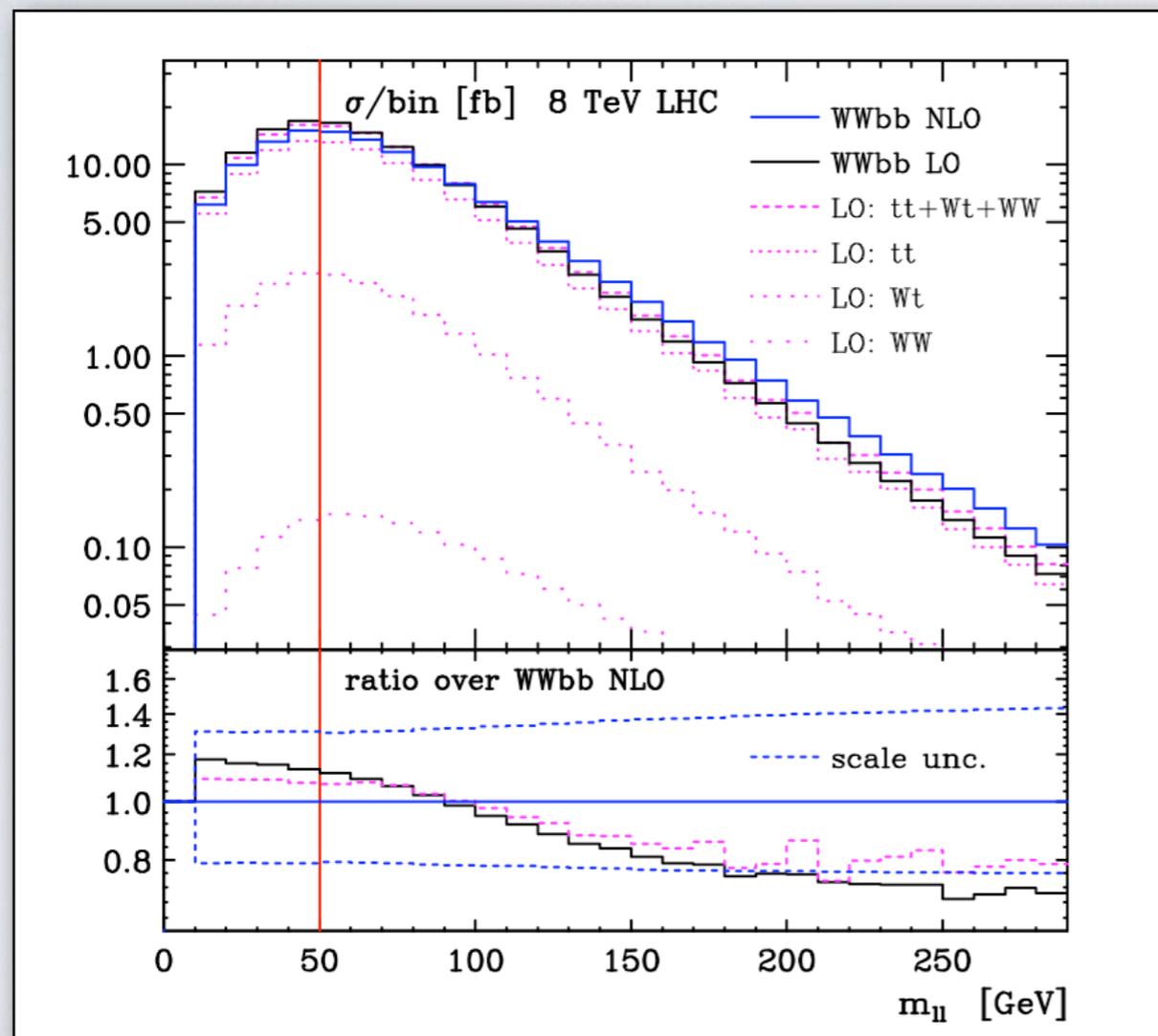
From David Kosower's summary talk at LoopFest 2014

Directory of NLO (and related) tools

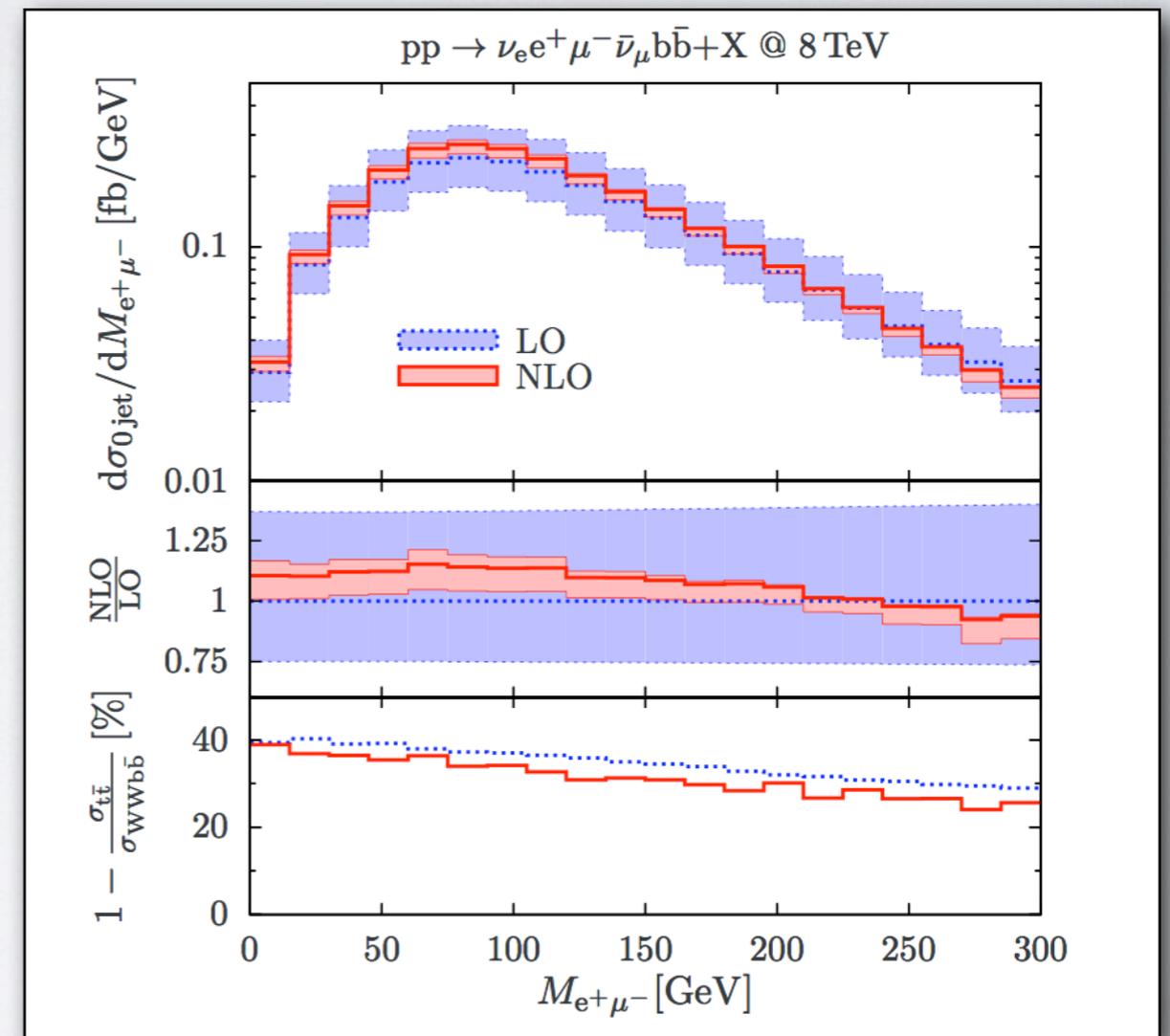


One NLO example

- Higgs decays to WW^* have a large branching ratio but no mass peak and large backgrounds.
- A precise estimate requires computing $pp \rightarrow l\nu l\nu b\bar{b}$ at NLO with massive b quarks.
- This is now done by two groups including off-shell effects and full interference.



Dilepton mass spectrum with MG5_aMC@NLO
from Rikkert Frederix 1311.4893.



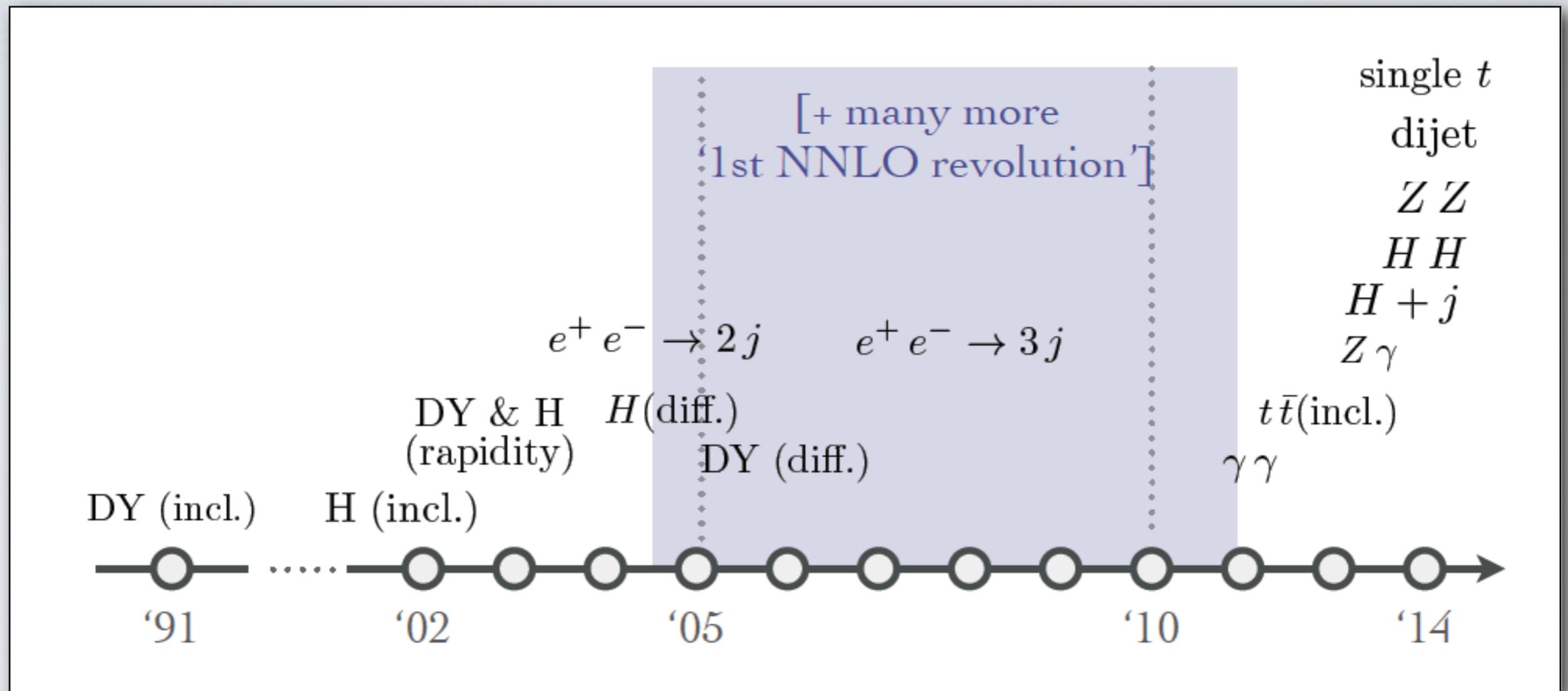
Dilepton mass spectrum with OpenLoops
from Cascioli et al. 1312.0546

A BIRD'S EYE VIEW OVER TWO LOOPS



NNLO revolutions

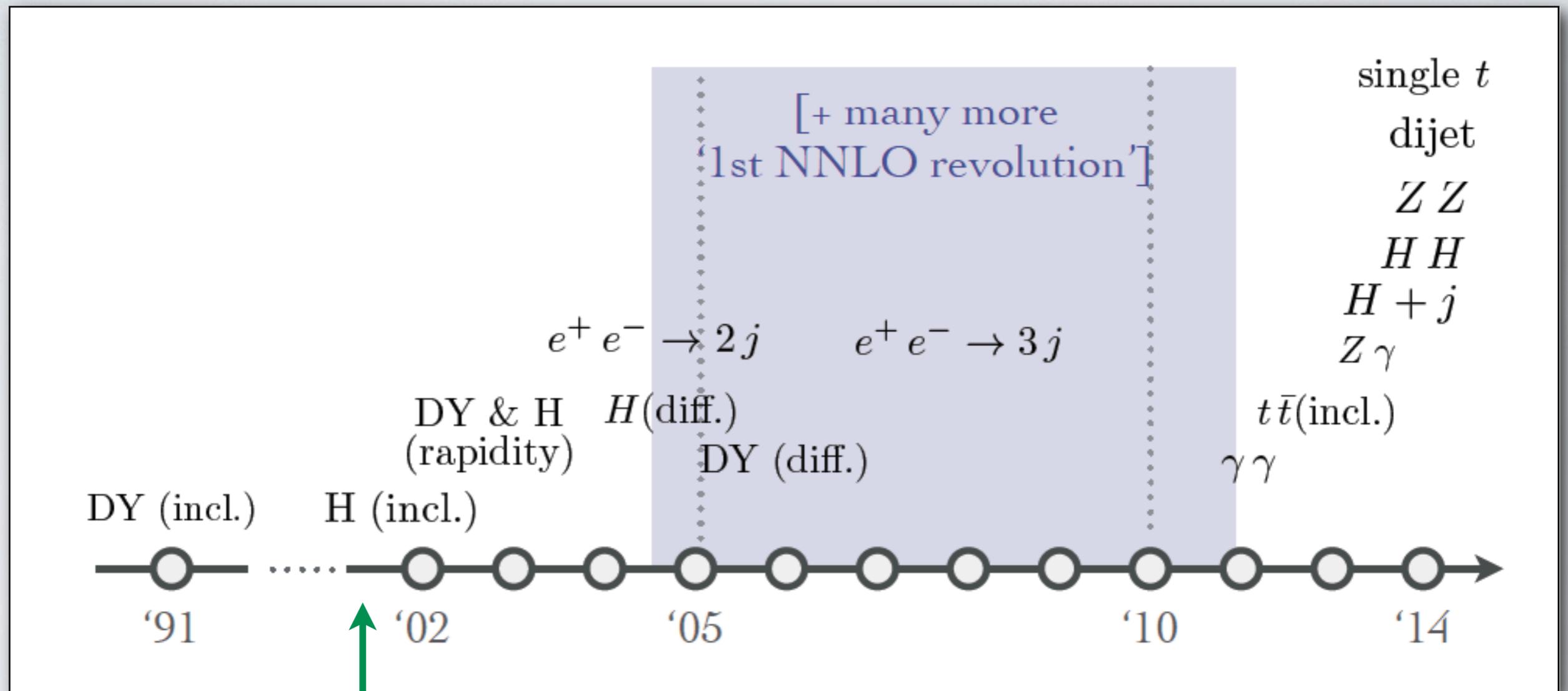
- **Two-loop** calculations are **not yet** a **commodity**: they are largely **custom-made** and **expensive**.
- A major **stumbling block** has been the **subtraction** of **infrared** and **collinear** singularities.
- **Progress** has been slow but is rapidly **speeding up**: **automation** is on the way.



From **Claude Duhr's** talk at **ICHEP 2014**

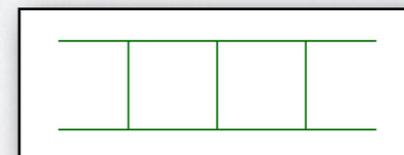
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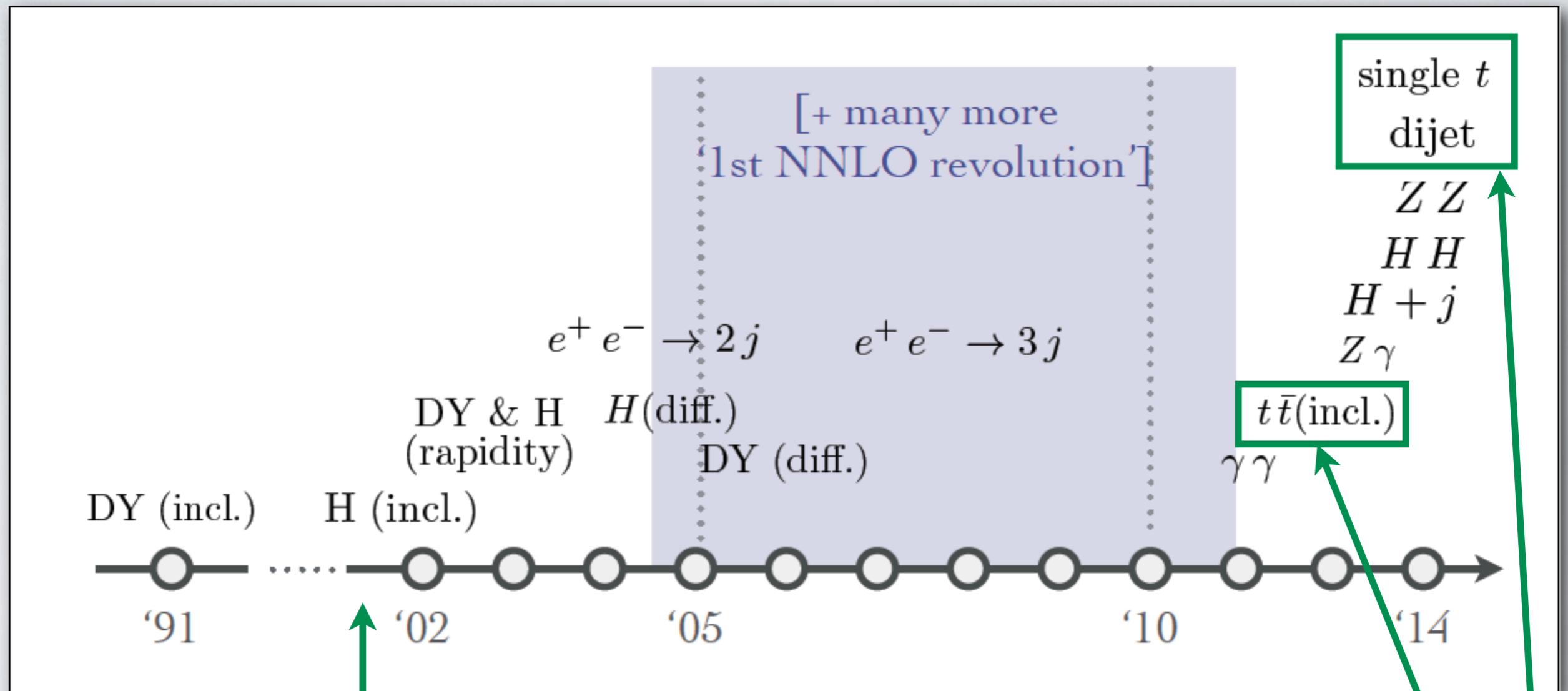
From **Claude Duhr's** talk at **ICHEP 2014**

V. Smirnov [hep-ph/9905323](https://arxiv.org/abs/hep-ph/9905323)



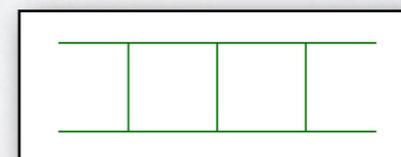
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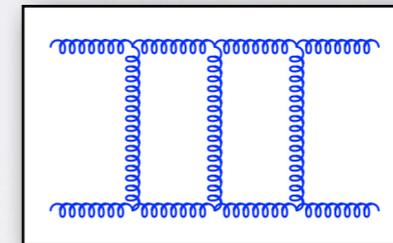
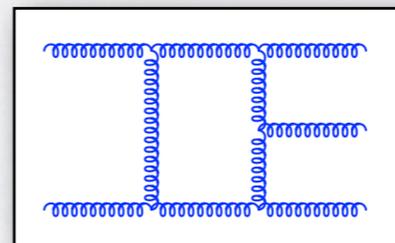
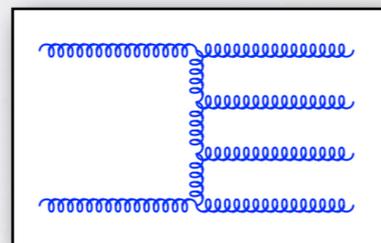
Full color
Masses

The NNLO subtraction problem

- A **well-known** problem: **infrared** and **collinear** divergences **cancel** between final states with different **particle content** and different **phase spaces**.
- The **cancellation** must be **performed locally** in phase space to allow for **generic observables**.
- “**Simple**” subtraction **counterterms** must be constructed in **each** phase space.
- A **surprisingly hard problem**, on the table for more than a **decade**.

$$d\hat{\sigma}_{NNLO} \sim \int_{d\Phi_{m+2}} d\hat{\sigma}_{NNLO}^{RR} + \int_{d\Phi_{m+1}} d\hat{\sigma}_{NNLO}^{RV} + \int_{d\Phi_m} d\hat{\sigma}_{NNLO}^{VV}$$

Different final-state multiplicities conspire to cancel infrared and collinear poles



	analytic	FS colour	IS colour	local
antenna subtraction	✓	✓	✓	✗
STRIPPER	✗	✓	✓	✓
q_T subtraction	✓	✗	✓	✓
reverse unitarity	✓	✗	✓	-
Trócsányi et al	✗	✓	✗	✓

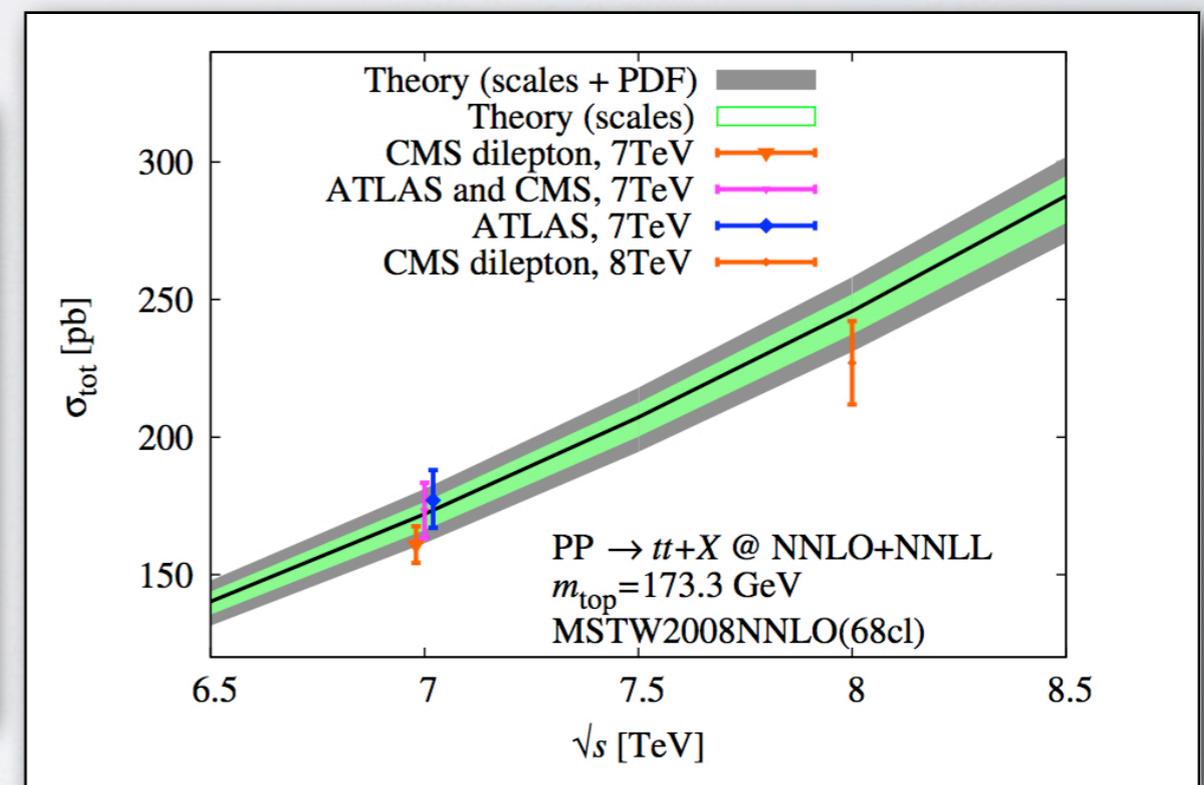
- Several** solutions are now **available**.
- Analytical** vs. **numerical** approaches.
- Dedicated** vs. **general** algorithms.
- Several groups** at work.
- No silver bullet** yet.

Top pair production

- The **first complete** QCD calculation of a **NNLO** cross section involving **four colored partons**. Full complexity of **color exchange** comes into play. A **major achievement**.
- Highly **relevant** for phenomenology: the **heaviest** particle lies **closest** to the new physics.
- The **subtraction** of IR and collinear poles is performed **purely numerically** (**STRIPPER**).
- The structure of **singularities** is **slightly simplified** with respect to the massless case.
- Partial **analytic results** are available (**R. Bonciani et al. 1309.4450**): a **challenging** calculation with **interesting analytical features**.

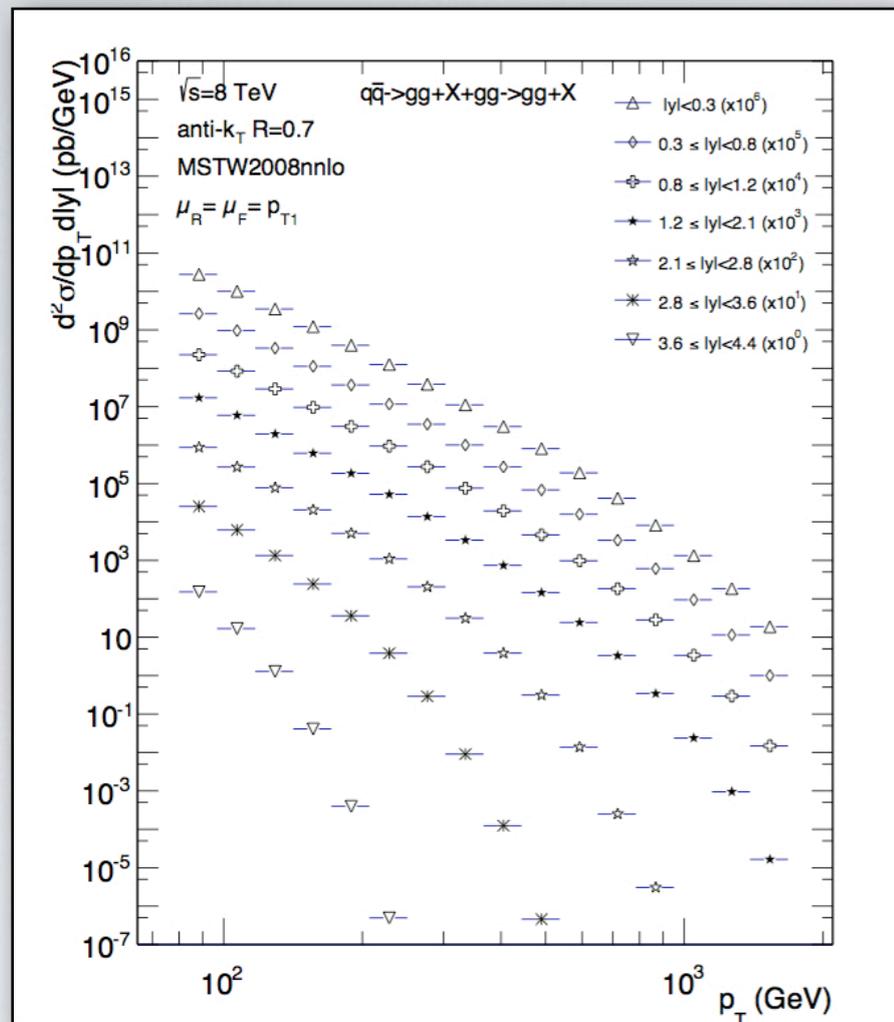
TABLE II. Pure NNLO theoretical predictions for various colliders and c.m. energies.

Collider	σ_{tot} [pb]	scales [pb]	PDF [pb]
Tevatron	7.009	+0.259(3.7%) -0.374(5.3%)	+0.169(2.4%) -0.121(1.7%)
LHC 7 TeV	167.0	+6.7(4.0%) -10.7(6.4%)	+4.6(2.8%) -4.7(2.8%)
LHC 8 TeV	239.1	+9.2(3.9%) -14.8(6.2%)	+6.1(2.5%) -6.2(2.6%)
LHC 14 TeV	933.0	+31.8(3.4%) -51.0(5.5%)	+16.1(1.7%) -17.6(1.9%)



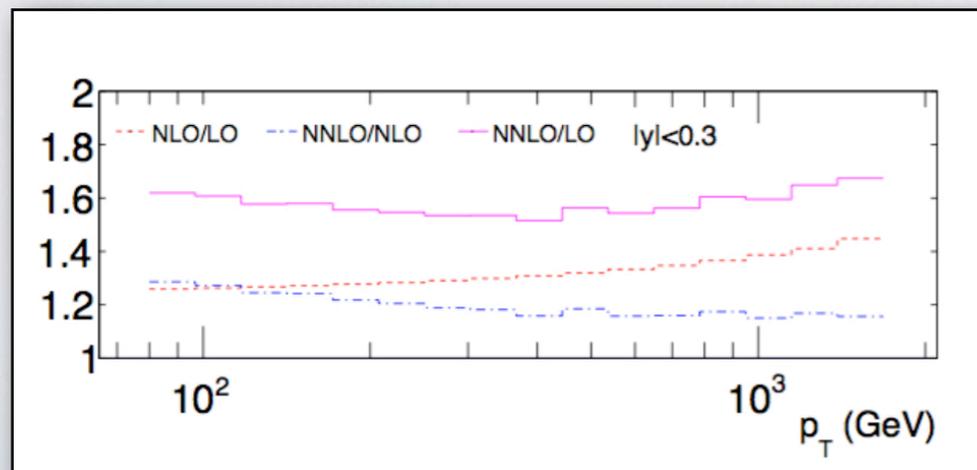
Numerical results and comparison with LHC data for the NNLO top pair production cross section, from M. Czakon, P. Fiendler and A. Mitov, 1303.6254

Dijet cross sections

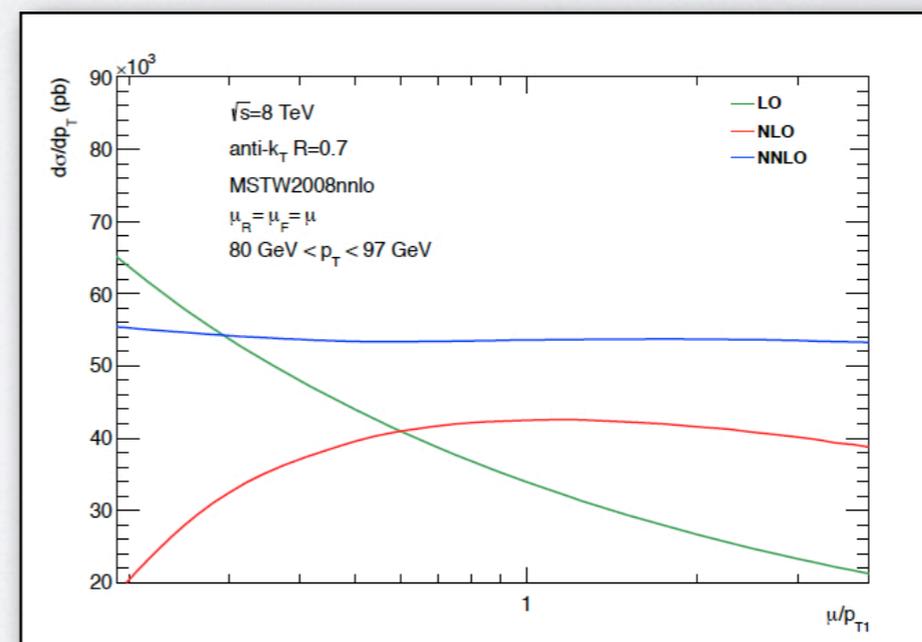


- A **multi-million dollar** calculation, spanning a **decade**.
- Some partonic channel **completed**: **gluon-gluon** full color, **q-qbar** leading color.
- Some partonic channels **on the way**: **quark-gluon** is phenomenologically **important**.
- Analytic** subtraction of singularities using **'antennas'** lead to **highly complex** calculations.
- Important** for a range of phenomenological issues: **parton distributions**, α_s , high-energy probes. Jets enter in essentially **all LHC cross sections**.

Single inclusive NNLO jet cross section at LHC 8 for fixed p_T and y , anti- k_T algorithm with $R = 0.7$. Gluon jets only. From J. Currie et al. 1407.5558.



Variations between LO, NLO and NNLO at central rapidities



Scale variations of the LO, NLO and NNLO jet cross sections, gluons only. From J. Currie's talk at LoopFest 2014.

A good NNLO harvest



- 📌 **Preliminary** results for differential distributions in **Higgs + one jet** production (Boughezal, Caola, Melnikov, Petriello, Schultze; Chen, Gehrmann, Glover, Jaquier).
 - ◆ *Note: the two groups use different subtraction techniques.*
- 📌 Differential distributions for **t-channel single top** production in the 'structure function' approximation (Brucherseifer, Caola, Melnikov).
 - ◆ *Non-factorizable contribution is color-subleading.*
- 📌 Differential distributions for **ZZ** production (using **q_T** subtraction) (Cascioli et al.).
 - ◆ *The method is generalizable to all EW di-boson final states.*
- 📌 **Preliminary** results for **γ* γ*** production presented by Lazopoulos at LoopFest
 - ◆ *A stepping stone to a general code for electroweak final states.*
- 📌 **Very recent!** Differential distributions for **associated ZH** production (Ferrera, Grazzini, Tramontano, I407.4747).
- 📌 **Even more recent!** **Virtual** corrections to **NNLO HH** production (effectively a 2.5 loop calculation) (Grigo, Melnikov, Steinhauser, I408.2422).
- 📌 **All master integrals** required for **pp->VV** two-loop amplitudes now **known** (Caola, Henn, Melnikov, Smirnov, I404.5590).
- 📌 **Progress** towards the construction of a **general basis** for two-loop **master integrals** (Mastrolia et al.; Badger, Frellesvig, Zhang, I407.3133)

BREAKING GROUND AT THREE LOOPS



“Drell-Yan” at N³LO

- After the **landmark** calculation of **three-loop DIS structure functions** by Moch, Vermaseren and Vogt a decade ago, the next great **PQCD challenge** is the computation of a **cross section without an OPE** at three loops. The “Drell-Yan” process is the best candidate.
- At **LHC**, “Drell-Yan” means **vector boson production** and **Higgs production** via **gluon fusion**. The phenomenological **impact** is **evident**, especially given the **large corrections** to **Higgs** production at one and two loops.
- Approximate** three-loop results using **threshold** and **Regge** limits **exist** (Moch, Vogt, 2005; LM, Laenen, 2005; Ball, Bonvini, Forte, Marzani, Ridolfi, 2013).
- The **full calculation** is now being tackled **step by step** (Anastasiou, Duhr, Dulat, Herzog, Mistiberger 1311.1425; Kilgore 1312.1296; Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistiberger 1403.4616).
- The **leading term** in the **threshold** expansion is now **known**

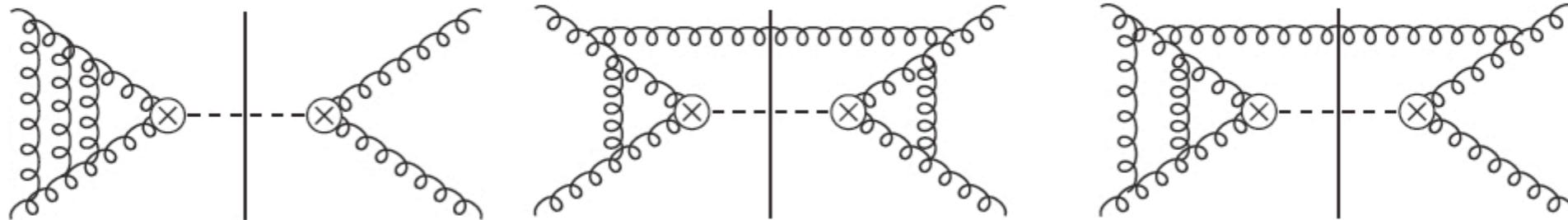
$$\hat{\sigma} = \hat{\sigma}(z), \quad z = \frac{Q^2}{\hat{s}}, \quad \hat{\sigma}(z) = \hat{\sigma}_{SV} + \hat{\sigma}_0 + (1-z)\hat{\sigma}_1 + \mathcal{O}[(1-z)^2]$$

- The **three-loop soft-virtual** contribution is **fully predicted** by threshold **resummation** **except** for $\delta(1-z)$ contributions which are the **new result**.
- The “Drell-Yan” **timeline**: **1979 - 1991 - 2002 - 2015?**

“Drell-Yan” at N³LO

- A **massive** calculation with **many ingredients**, and $O(10^3)$ master **integrals** to evaluate.

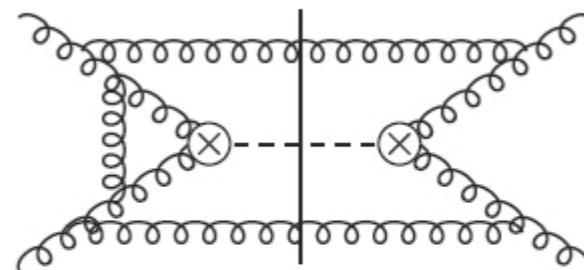
● At N³LO, there are 5 contributions:



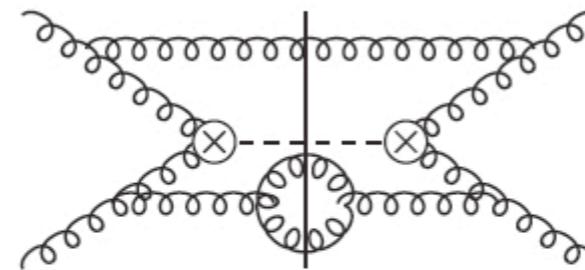
Triple virtual

Real-virtual
squared

Double virtual
real



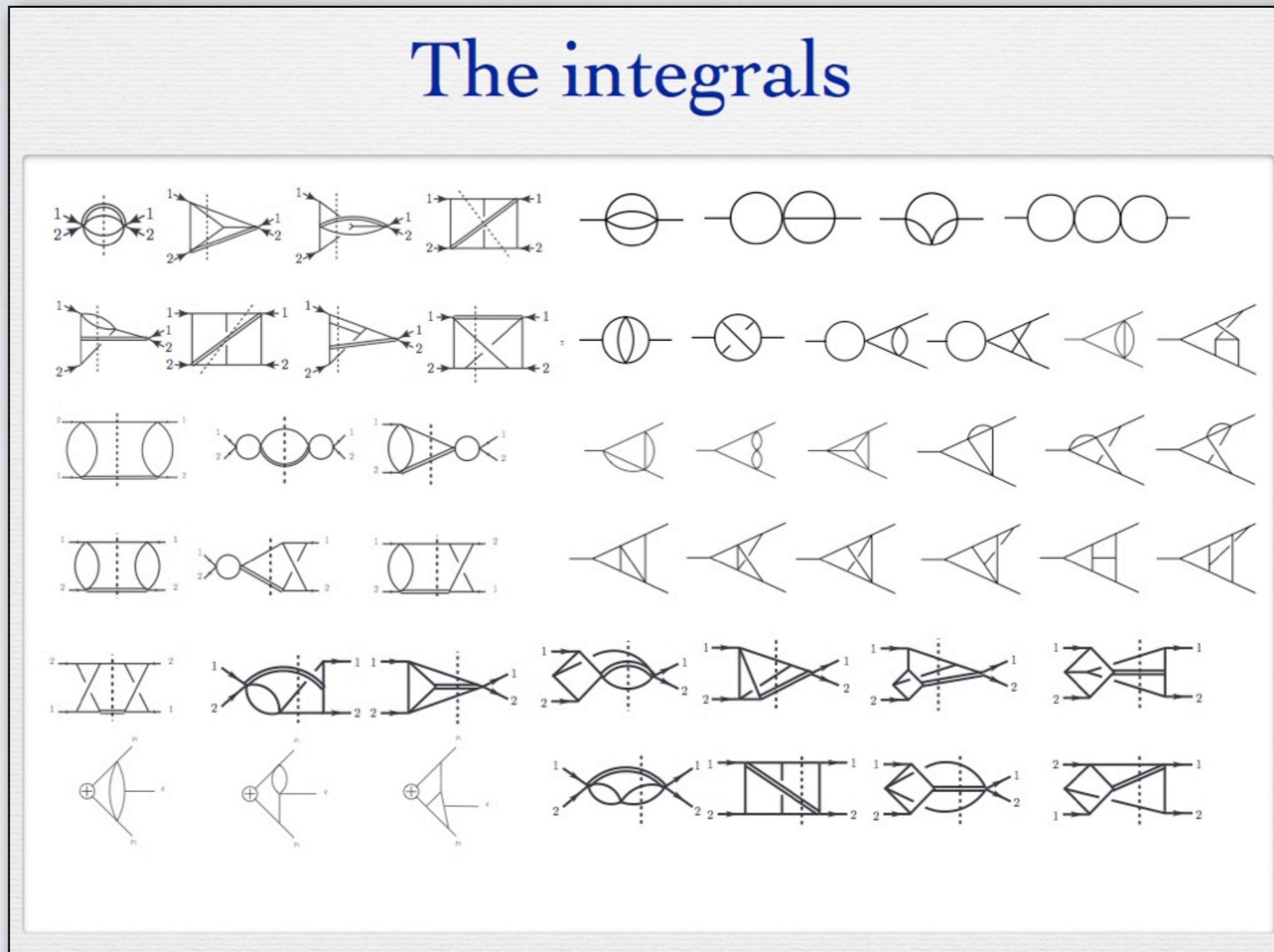
Double real
virtual



Triple real

“Drell-Yan” at N³LO

- A **massive** calculation even in the **soft-virtual** approximation, with **50** master **integrals**.



From **Claude Duhr's** talk at **Loops&Legs 2014**

“Drell-Yan” at N³LO

- The three-loop **soft-virtual** approximation to Higgs production in gluon fusion.

$$\begin{aligned}
 \hat{\eta}^{(3)}(z) = & \delta(1-z) \left\{ C_A^3 \left(-\frac{2003}{48} \zeta_6 + \frac{413}{6} \zeta_3^2 - \frac{7579}{144} \zeta_5 + \frac{979}{24} \zeta_2 \zeta_3 - \frac{15257}{864} \zeta_4 - \frac{819}{16} \zeta_3 + \frac{16151}{1296} \zeta_2 + \frac{215131}{5184} \right) \right. \\
 & + N_F \left[C_A^2 \left(\frac{869}{72} \zeta_5 - \frac{125}{12} \zeta_3 \zeta_2 + \frac{2629}{432} \zeta_4 + \frac{1231}{216} \zeta_3 - \frac{70}{81} \zeta_2 - \frac{98059}{5184} \right) \right. \\
 & \quad \left. + C_A C_F \left(\frac{5}{2} \zeta_5 + 3 \zeta_3 \zeta_2 + \frac{11}{72} \zeta_4 + \frac{13}{2} \zeta_3 - \frac{71}{36} \zeta_2 - \frac{63991}{5184} \right) + C_F^2 \left(-5 \zeta_5 + \frac{37}{12} \zeta_3 + \frac{19}{18} \right) \right] \\
 & + N_F^2 \left[C_A \left(-\frac{19}{36} \zeta_4 + \frac{43}{108} \zeta_3 - \frac{133}{324} \zeta_2 + \frac{2515}{1728} \right) + C_F \left(-\frac{1}{36} \zeta_4 - \frac{7}{6} \zeta_3 - \frac{23}{72} \zeta_2 + \frac{4481}{2592} \right) \right] \left. \right\} \\
 & + \left[\frac{1}{1-z} \right]_+ \left\{ C_A^3 \left(186 \zeta_5 - \frac{725}{6} \zeta_3 \zeta_2 + \frac{253}{24} \zeta_4 + \frac{8941}{108} \zeta_3 + \frac{8563}{324} \zeta_2 - \frac{297029}{23328} \right) + N_F^2 C_A \left(\frac{5}{27} \zeta_3 + \frac{10}{27} \zeta_2 - \frac{58}{729} \right) \right. \\
 & \quad \left. + N_F \left[C_A^2 \left(-\frac{17}{12} \zeta_4 - \frac{475}{36} \zeta_3 - \frac{2173}{324} \zeta_2 + \frac{31313}{11664} \right) + C_A C_F \left(-\frac{1}{2} \zeta_4 - \frac{19}{18} \zeta_3 - \frac{1}{2} \zeta_2 + \frac{1711}{864} \right) \right] \right\} \\
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 & \quad \left. + N_F \left[C_A^2 \left(\frac{46}{3} \zeta_3 + \frac{94}{9} \zeta_2 - \frac{4211}{324} \right) + C_A C_F \left(6 \zeta_3 - \frac{63}{8} \right) \right] \right\} \\
 & + \left[\frac{\log^2(1-z)}{1-z} \right]_+ \left\{ C_A^3 \left(181 \zeta_3 + \frac{187}{3} \zeta_2 - \frac{1051}{27} \right) + N_F \left[C_A^2 \left(-\frac{34}{3} \zeta_2 + \frac{457}{54} \right) + \frac{1}{2} C_A C_F \right] - \frac{10}{27} N_F^2 C_A \right\} \\
 & + \left[\frac{\log^3(1-z)}{1-z} \right]_+ \left\{ C_A^3 \left(-56 \zeta_2 + \frac{925}{27} \right) - \frac{164}{27} N_F C_A^2 + \frac{4}{27} N_F^2 C_A \right\} \\
 & + \left[\frac{\log^4(1-z)}{1-z} \right]_+ \left(\frac{20}{9} N_F C_A^2 - \frac{110}{9} C_A^3 \right) + \left[\frac{\log^5(1-z)}{1-z} \right]_+ 8 C_A^3.
 \end{aligned}$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

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- The three-loop **soft-virtual** approximation to Higgs production in gluon fusion.

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 \end{aligned}$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

Not predicted
by resummation

From Claude Duhr's talk at **Loops&Legs 2014**

Iterated integrals

- A **large class** of integrals arising from Feynman diagrams (**but not all!**) can be expressed as “**iterated integrals**”, yielding functions in the **class of polylogarithms**. At **one** loop

$$\log z = - \int_0^{1-z} \frac{dt}{1-t}, \quad \text{Li}_2(z) = \int_0^z \frac{dt}{t} \int_0^t \frac{du}{1-u}$$

- At **higher orders** one encounters **more general** examples, such as **Harmonic Polylogarithms** or **Goncharov Polylogarithms**

$$G_{a_1, \dots, a_n}(z) \equiv \int_0^z \frac{dt}{t - a_1} G_{a_2, \dots, a_n}(t),$$

- **Notice** that **all** these integrals are of a “**d log**” form: at each step one integrates **over the logarithm** of a simple (here linear) function of the integration variables.
- The **parameters** a_n are the **locations** of **singular** points and have **physical meaning**.
- **Iterated integrals** are organized by a **powerful underlying algebraic structure**, described by the “**Symbol**” map or by a Hopf algebra with a notion of “**Co-product**” (**Duhr**).
- In particular **each** such function can be assigned a “**weight**” w , equal to the **number of iterations**. For example $\text{Li}_2(z)$ has weight $w = 2$, and $\zeta(n)$ has weight $w = n$.
- These structures were **uncovered** in the context of studies of **N=4 Super Yang-Mills theory** amplitudes, where they have played a **pivotal role**.
- We **now** see **powerful new applications** to ordinary **QCD** (**Henn, Smirnov, Von Manteuffel**)

A basis of pure functions

Iterated integrals are the centerpiece of a recent breakthrough (J. Henn, 1304.1806) in the calculation of master integrals for general gauge theory amplitudes.

Consider the standard method of evaluation for multi-loop scattering amplitudes.

- Reduce the integrals arising from Feynman diagrams to a set of master integrals.
- Use IBP and Lorentz invariance identities to derive a system of differential equations coupling all master integrals.
- Solve for the master integrals using simple configurations for boundary conditions.

For master integrals $f_i(x_n)$, and with $\epsilon = 2 - d/2$, the system takes the form

$$\frac{\partial}{\partial x_m} f_i(\epsilon, x_n) = A_{ij}^{(m)}(\epsilon, x_n) f_j(\epsilon, x_n)$$

There is a large, not previously exploited freedom to choose the basis of MI's at will.

Using iterated integrals, J. Henn suggested that an appropriate choice of basis involving uniform weight functions, can lead to a striking double simplification.

$$\frac{\partial}{\partial x_m} f_i(\epsilon, x_n) = \epsilon A_{ij}^{(m)}(x_n) f_j(\epsilon, x_n) = \epsilon \sum_k \left[A_k^{(m)} \right]_{ij} \frac{\partial}{\partial x_m} [\log \alpha_k(x_n)] f_j(\epsilon, x_n)$$

The system is now easily solved order by order in ϵ in terms of iterated integrals.

A key role is played by the alphabet of functions α_k , which encode the kinematic singularities of the amplitude.

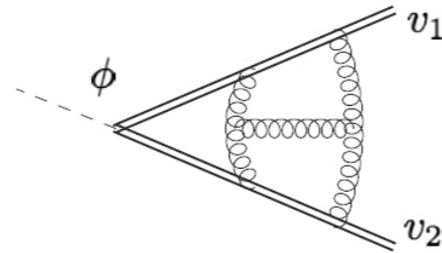
The existence of such a basis of master integrals is not proved in general.

Whenever this basis of uniform weight functions exists the evaluation of MI's is remarkably simplified.

A wealth of applications

- 8 integral families, e.g.

$$\cos \phi = \frac{v_1 \cdot v_2}{\sqrt{v_1^2} \sqrt{v_2^2}}, \quad x = e^{i\phi}$$

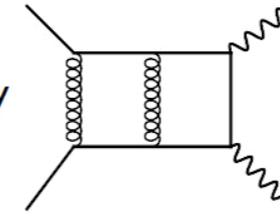


- alphabet $\alpha = \{x, 1+x, 1-x\}$
- 71 master integrals
- application: **QCD cusp anomalous dimension**

Towards the three-loop QCD cusp anomalous dimension:
Grozin, Henn, Korchemsky, Marquard, 1406.7828.

Vector boson production integrals $pp \rightarrow VV'$

- sample integral family



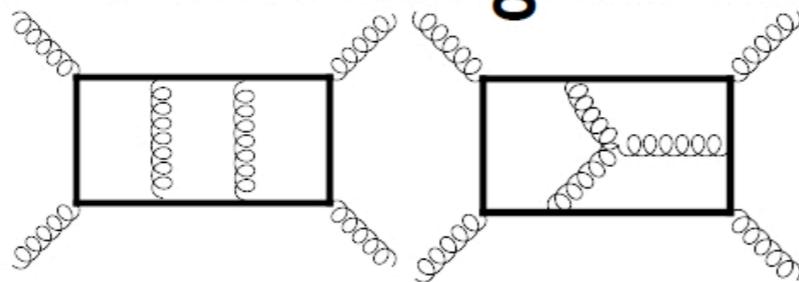
[JMH, Melnikov, V. Smirnov, JHEP 1430 (2014)]
[JMH, Caola, Melnikov, V. Smirnov, 1404.5590]

- variables S, T, M_3^2, M_4^2
- parametrization $\frac{S}{M_3^2} = (1+x)(1+xy), \quad \frac{T}{M_3^2} = -xz, \quad \frac{M_4^2}{M_3^2} = x^2y$
- physical region $0 < x, \quad 0 < y < z < 1$
- alphabet $\alpha = \{x, y, z, 1+x, 1-y, 1-z, 1+xy, z-y, 1+y(1+x)-z, xy+z, 1+x(1+y-z), 1+xz, 1+y-z, z+x(z-y)+xyz, z-y+yz+xyz\}$.

All integrals for virtual two-loop double vector boson production:
Caola, Henn, Melnikov, Smirnov, 1404.5590.

Massive integrals for light-by-light scattering

[Caron-Huot, J.M.H., 2014]



- variables m^2, s, t

3 loops and 3 scales!

- full set of 2-loop master integrals (at 3 loops: all finite master integrals in D=4)

All planar two-loop and finite three-loop integrals for light-by-light scattering
Henn, Smirnov, Smirnov 1312.2588 ; Caron-Huot, Henn, 1404.2922.

SOFT GLUONS TO ALL LOOPS



The virtues of large logs

- **Multi-scale** problems in **renormalizable** quantum field theories have perturbative corrections of the form $\alpha_s^n \log^k (Q_i^2/Q_j^2)$, which may **spoil** the reliability of the perturbative expansion. However, they **carry important** physical **information!**
 - **Renormalization** and **factorization** logs: $\alpha_s^n \log^n (Q^2/\mu^2)$
 - **High-energy logs**: $\alpha_s^n \log^{n-1} (s/t)$
 - **Sudakov** logs: $\alpha_s^n \log^{2n-1} (1-z)$, $1-z = W^2/Q^2, 1-M^2/\hat{s}, Q_\perp^2/Q^2, \dots$

- **Sudakov** logs are **universal**: they originate from **infrared and collinear singularities**: they **exponentiate** and can be **resummed**

$$\underbrace{\frac{1}{\epsilon}}_{\text{virtual}} + \underbrace{(Q^2)^\epsilon \int_0^{m^2} \frac{dk^2}{(k^2)^{1+\epsilon}}}_{\text{real}} \implies \ln(m^2/Q^2)$$

- For **inclusive** observables: **analytic** resummation to high logarithmic accuracy.
 - For **exclusive** final states: **parton shower** event generators, (N)LL accuracy.
- **Resummation** probes the **all-order structure** of perturbation theory.
 - **Power-suppressed** corrections to QCD cross sections can be studied.
 - Links to the **strong coupling** regime can be established for SUSY gauge theories.

The perturbative exponent

A classic way to **organize** Sudakov logarithms is in terms of the **Mellin (Laplace) transform** of the momentum space cross section (**Catani et al. 93**),

$$\begin{aligned} d\sigma(\alpha_s, N) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{2n} c_{nk} \log^k N + \mathcal{O}(1/N) \\ &= H(\alpha_s) \exp \left[\log N g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s g_3(\alpha_s \log N) + \dots \right] + \mathcal{O}(1/N) \end{aligned}$$

This displays the main **features of Sudakov resummation**

- Predictive:** a **k**-loop calculation determines **g_k** and thus a whole **tower** of logarithms to all orders in perturbation theory.
- Effective:**
 - the **range of applicability** of perturbation theory is **extended** (finite order: **α_s log²N** small. NLL resummed: **α_s** small);
 - the renormalization **scale dependence** is naturally **reduced**.
- Theoretically interesting:** resummation **ambiguities** related to the **Landau pole** give access to non-perturbative **power-suppressed corrections**.
- Well understood:**
 - **NLL** Sudakov resummations **exist** for most **inclusive** observables at hadron colliders, **NNLL** and approximate **N³LL** in simple cases.
 - Different **'schools'** (**USA, Italian, SCET** ...) compete, complacency is not an option, active and lively debate.

Color singlet hard scattering

A well-established formalism exists for **distributions** in processes that are **electroweak at tree level** (Gardi, Grunberg 07). For an observable r **vanishing in the two-jet limit**

$$\frac{d\sigma}{dr} = \delta(r) [1 + \mathcal{O}(\alpha_s)] + C_R \frac{\alpha_s}{\pi} \left\{ \left[-\frac{\log r}{r} + \frac{b_1 - d_1}{r} \right]_+ + \mathcal{O}(r^0) \right\} + \mathcal{O}(\alpha_s^2)$$

The Mellin (Laplace) transform, $\sigma(N) = \int_0^1 dr (1-r)^{N-1} \frac{d\sigma}{dr}$

exhibits **log N** singularities that can be organized in **exponential form**

$$\sigma(\alpha_s, N, Q^2) = H(\alpha_s) \mathcal{S}(\alpha_s, N, Q^2) + \mathcal{O}(1/N)$$

where the exponent of the **'Sudakov factor'** is in turn a Mellin transform

$$\mathcal{S}(\alpha_s, N, Q^2) = \exp \left\{ \int_0^1 \frac{dr}{r} \left[(1-r)^{N-1} - 1 \right] \mathcal{E}(\alpha_s, r, Q^2) \right\}$$

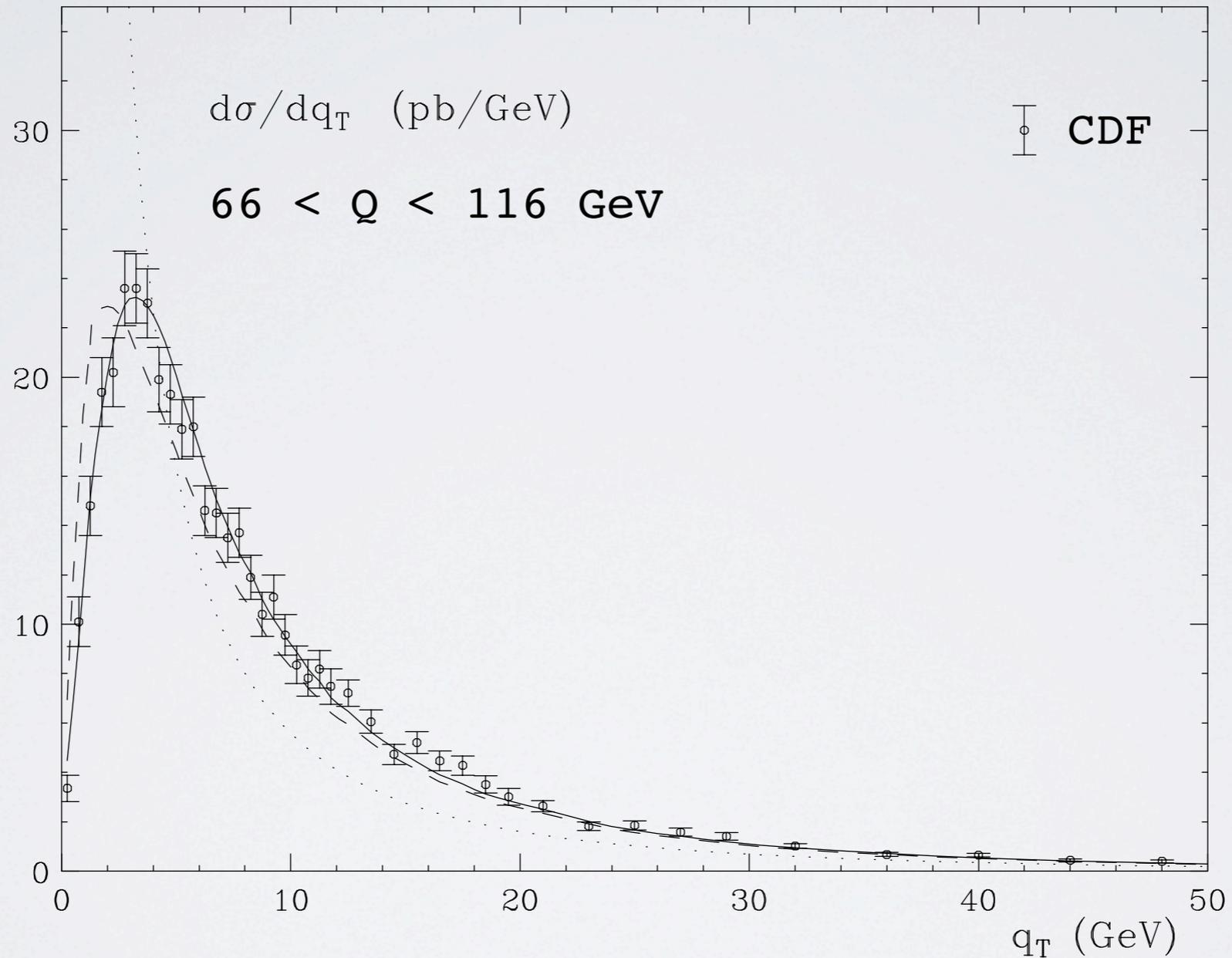
and the general form of the **kernel** is

$$\mathcal{E}(\alpha_s, r, Q^2) = \int_{r^2 Q^2}^{rQ^2} \frac{d\xi^2}{\xi^2} A(\alpha_s(\xi^2)) + B(\alpha_s(rQ^2)) + D(\alpha_s(r^2 Q^2))$$

where **A** is the **cusp** anomalous dimension, and **B** and **D** have **distinct physical characters**.

Impact of resummation

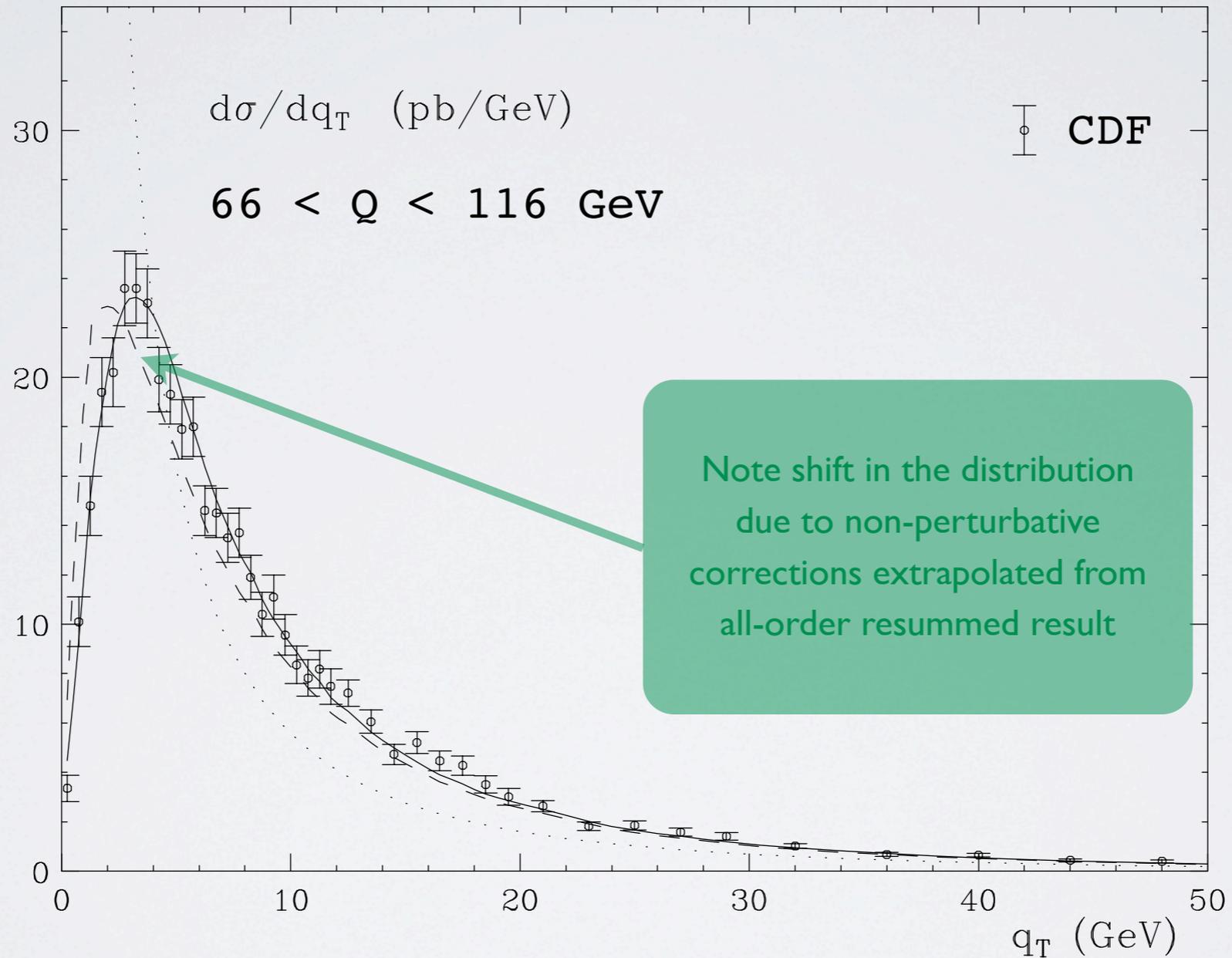
Z-boson q_T spectrum at Tevatron (Kulesza et al. 03)



CDF data on Z production compared with QCD predictions at fixed order (dotted), with joint resummation (dashed), and with the inclusion of power corrections (solid).

Impact of resummation

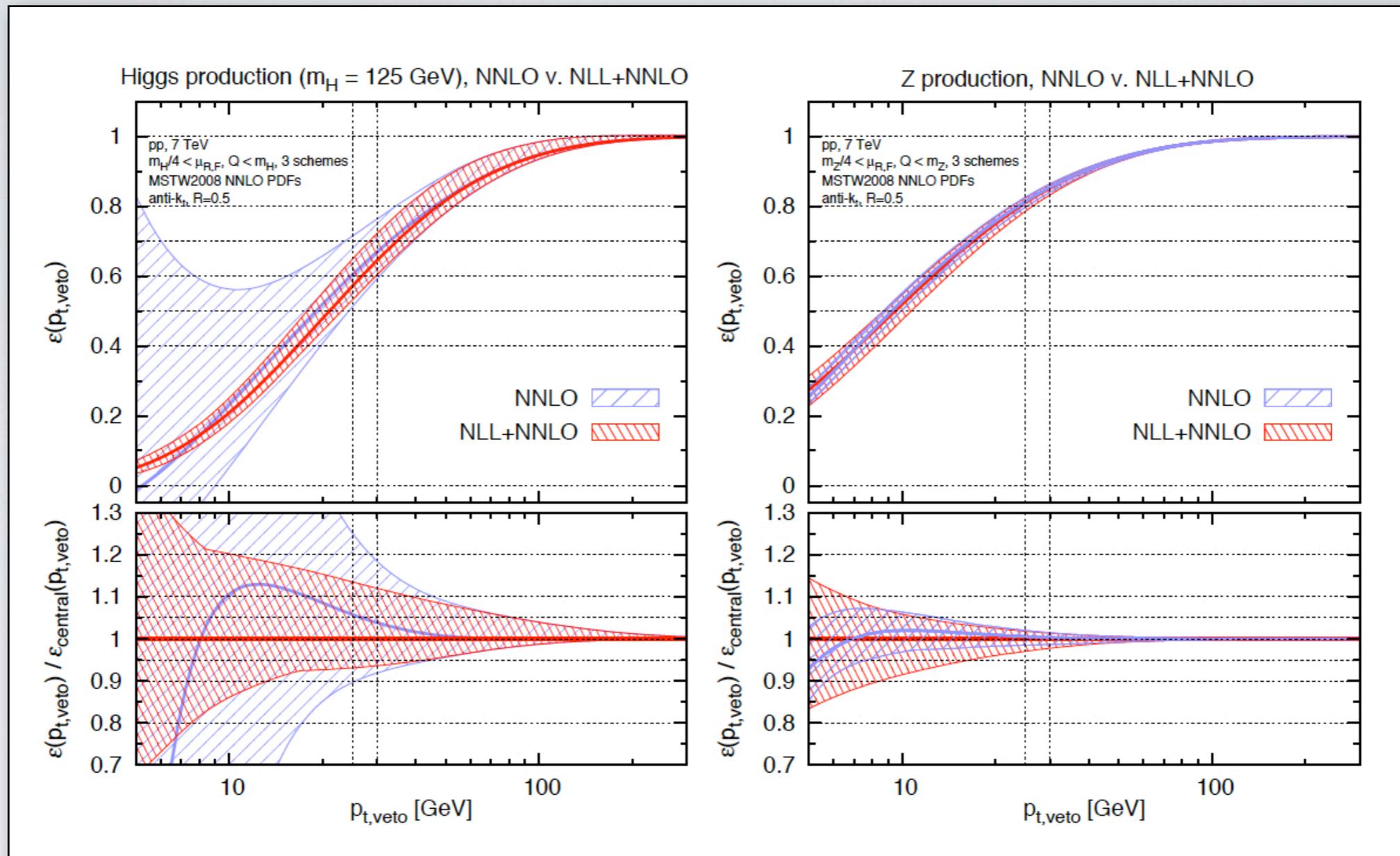
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Complex observables

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)

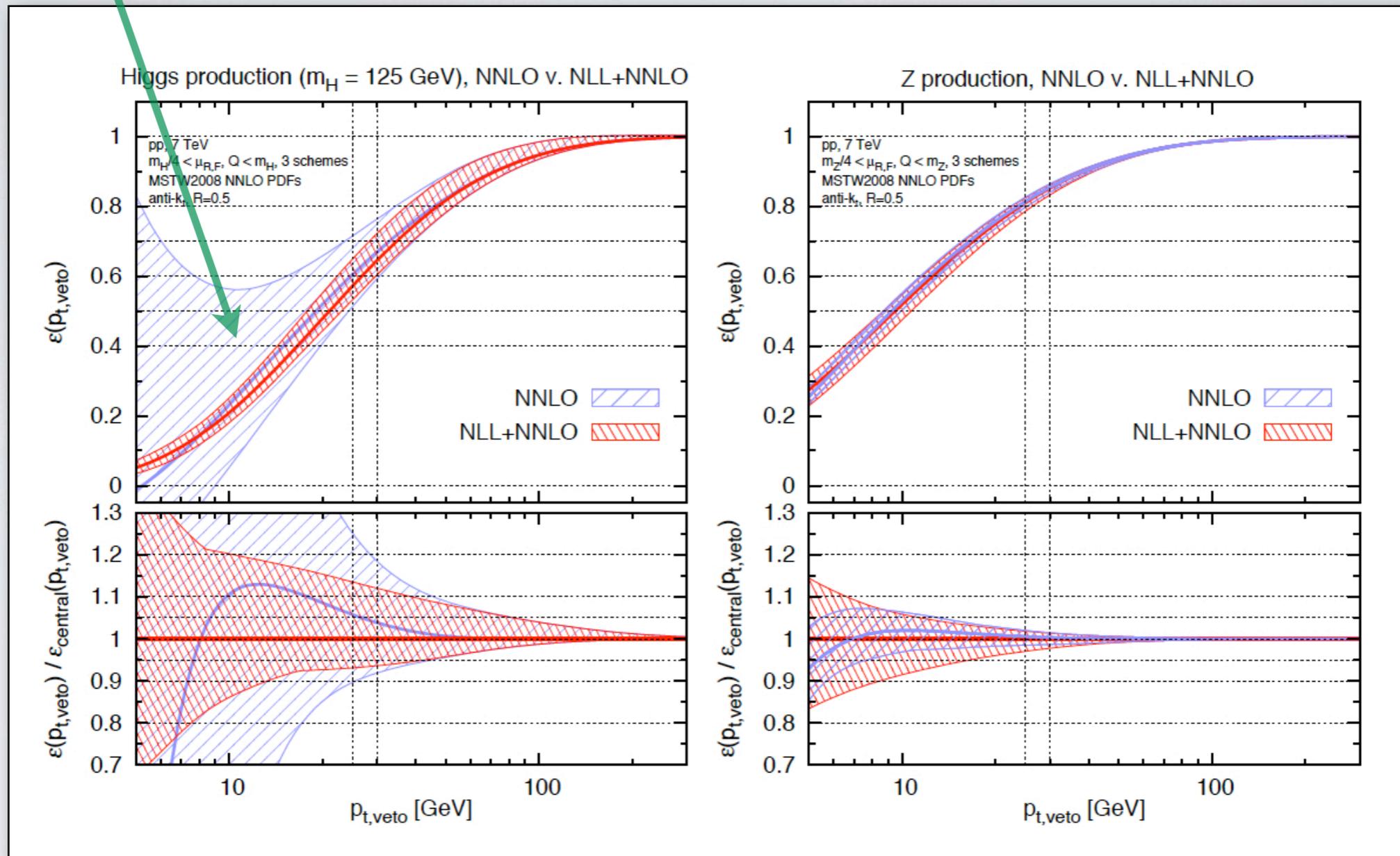


Comparison of NNLO fixed order results and matched resummed NLL-NNLO results for Higgs production with a jet veto (left) and Z production with a jet veto (right). Subsequent improvements include NNLL accuracy (also in SCET, by Becher, Neubert, Rothen, 1307.0025), and exact treatment of quark masses (1308.4634).

Complex observables

Note the sharp reduction of the theoretical uncertainty upon resummation

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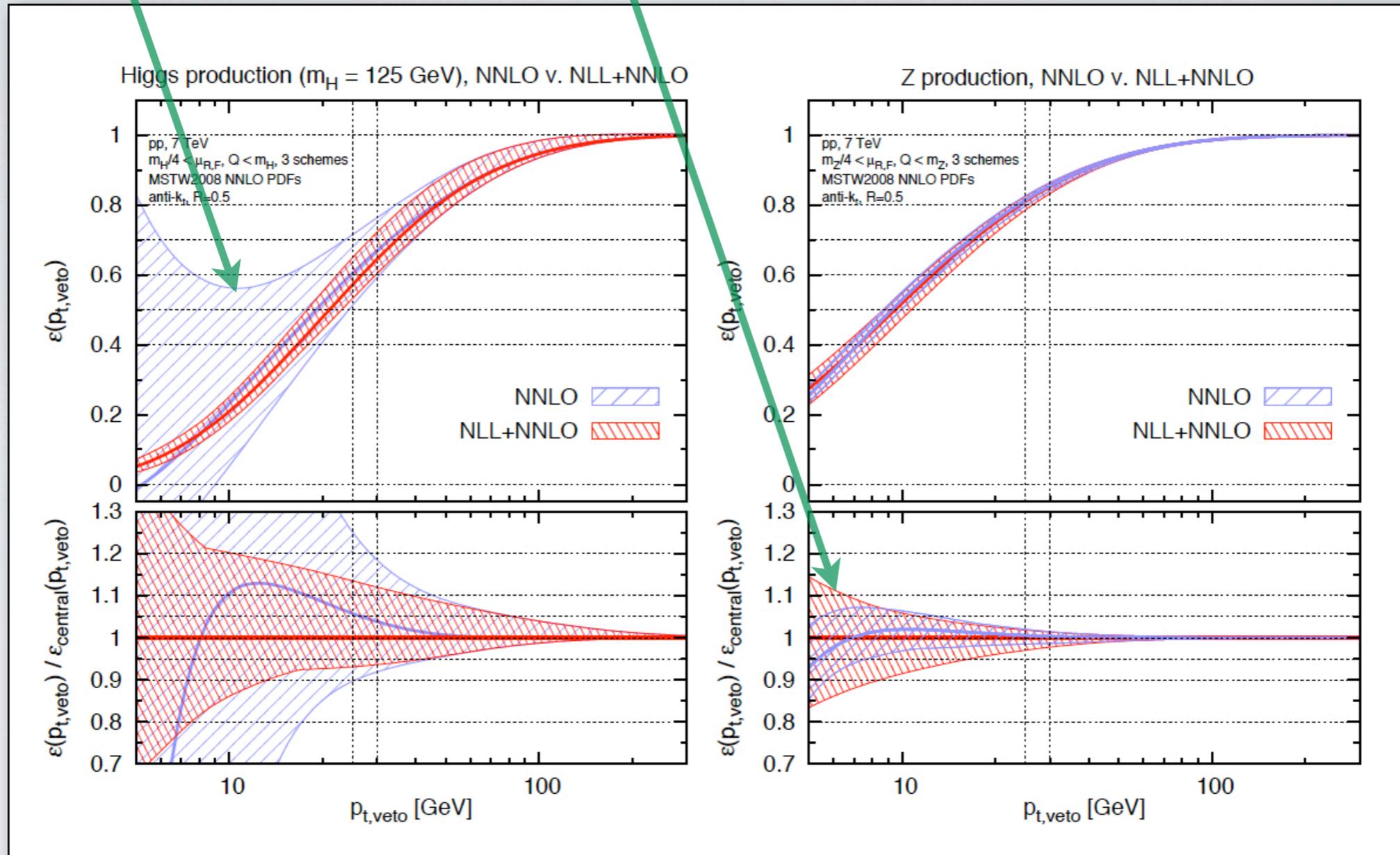
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Complex observables

Note the sharp reduction of the theoretical uncertainty upon resummation ...

... which does not always take place!

Jet veto efficiency in Higgs and Z production (Banfi et al., 03/12)



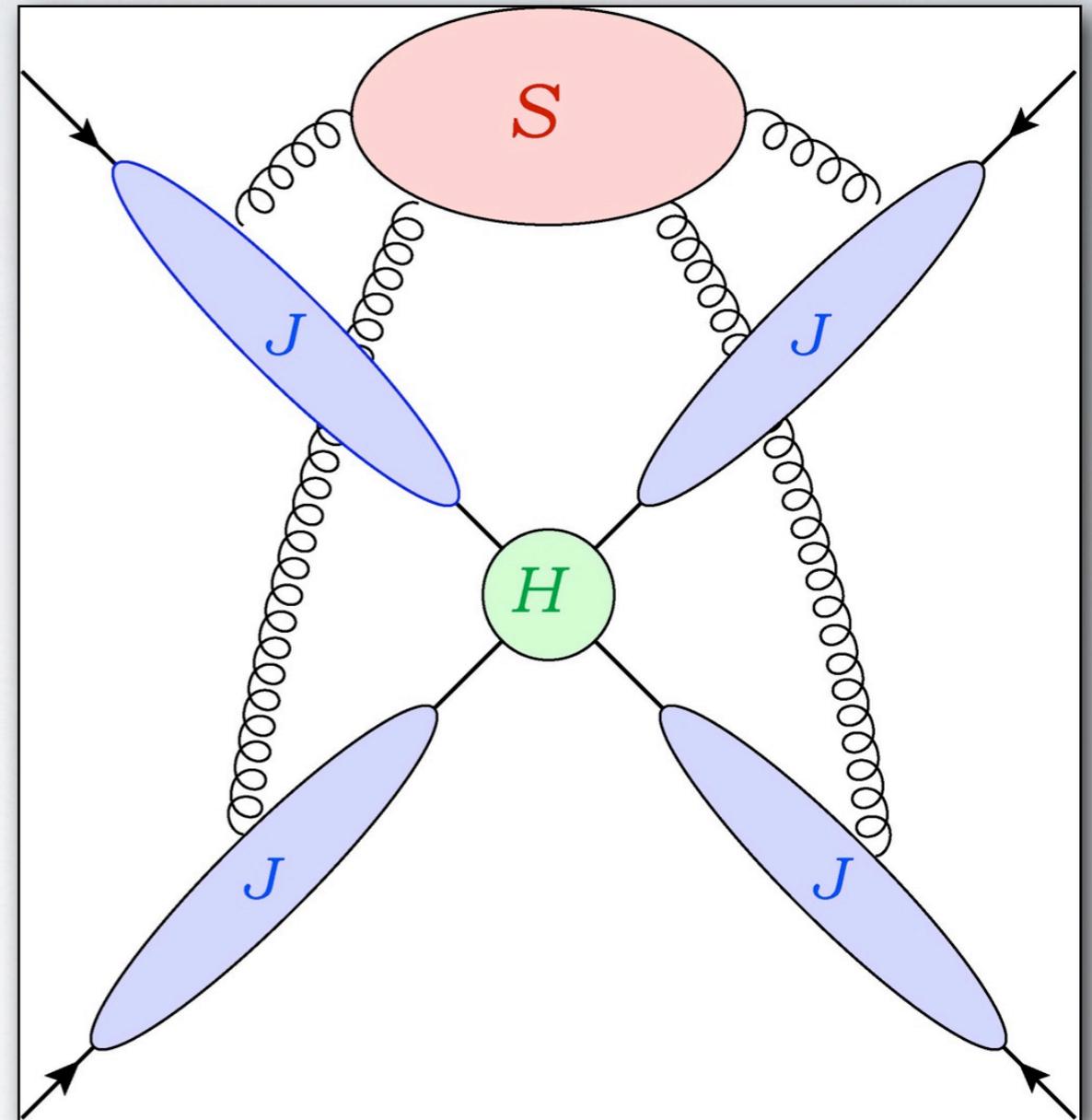
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TOOLS FOR LARGE LOGS



Soft-collinear factorization

- **Sudakov logarithms** are **remainders** of infrared and collinear **divergences**.
- **Divergences** arise in **scattering** amplitudes from **leading regions** in loop momentum space.
- **Power-counting** arguments show that **soft** gluons decouple from the **hard** subgraph.
- **Ward identities** decouple **soft** gluons from **jets** and **restrict** color transfer to the **hard** part.
- **Jet functions** J represent **color singlet** evolution of **external** hard partons.
- The **soft function** S is a **matrix** mixing the available **color representations**.
- In the **planar limit** soft exchanges are confined to **wedges**: S is proportional to the **identity**.
- **Beyond** the planar limit S is determined by an **anomalous dimension matrix** Γ_S .
- The **matrix** Γ_S correlates **color** exchange with **kinematic** dependence.

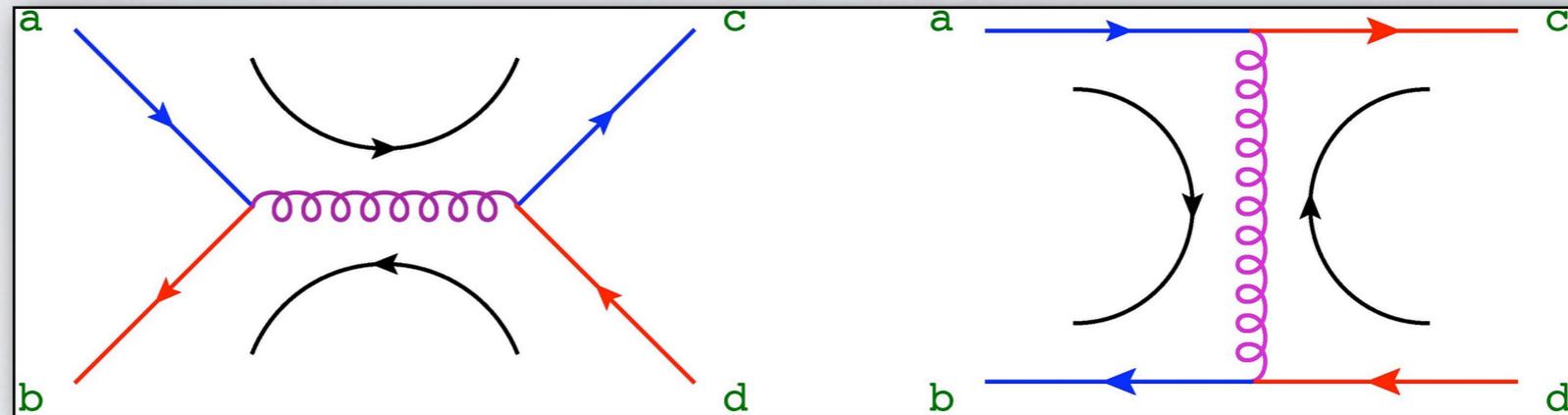


Leading integration regions in loop momentum space for soft-collinear factorization

Color flow

In order to understand the **matrix structure** of the **soft function** it is sufficient to consider the simple case of **quark-antiquark** scattering.

At tree level



Tree-level diagrams and color flows for quark-antiquark scattering

For this process only **two color structures** are possible. A **basis** in the space of available color tensors is

$$c_{abcd}^{(1)} = \delta_{ab}\delta_{cd}, \quad c_{abcd}^{(2)} = \delta_{ac}\delta_{bd}$$

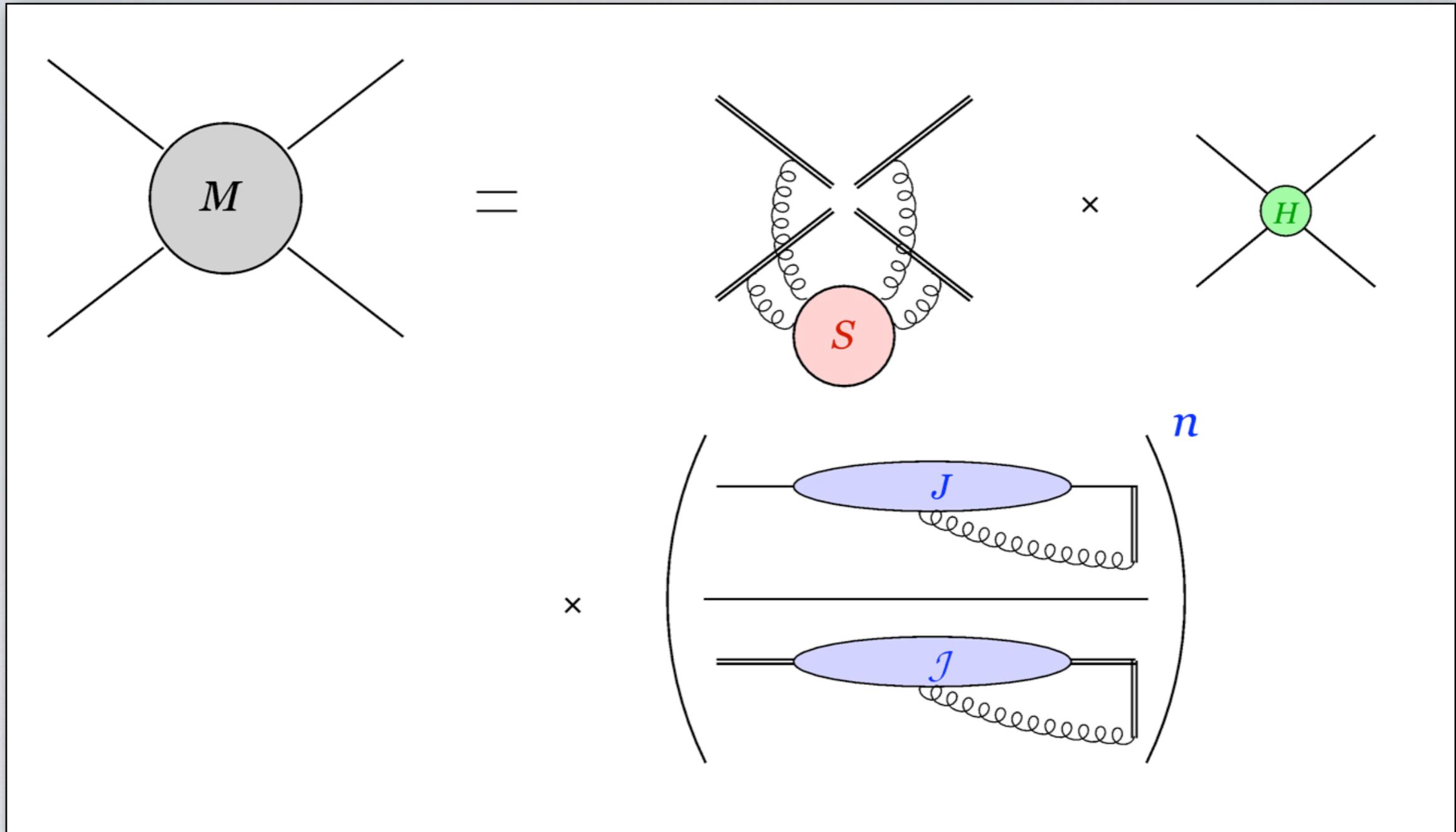
The **matrix element** is a **vector** in this space, and the Born cross section is

$$\mathcal{M}_{abcd} = \mathcal{M}_1 c_{abcd}^{(1)} + \mathcal{M}_2 c_{abcd}^{(2)} \longrightarrow \sum_{\text{color}} |\mathcal{M}|^2 = \sum_{J,L} \mathcal{M}_J \mathcal{M}_L^* \text{tr} \left[c_{abcd}^{(J)} \left(c_{abcd}^{(L)} \right)^\dagger \right] \equiv \text{Tr} [HS]_0$$

A virtual **soft gluon** will **reshuffle** color and mix the components of this vector

$$\text{QED} : \mathcal{M}_{\text{div}} = S_{\text{div}} \mathcal{M}_{\text{Born}} ; \quad \text{QCD} : [\mathcal{M}_{\text{div}}]_J = [S_{\text{div}}]_{JL} [\mathcal{M}_{\text{Born}}]_L$$

Sudakov factorization: pictorial



A pictorial representation of Sudakov factorization for fixed-angle scattering amplitudes

Soft Matrices

The **soft function** \mathcal{S} is a **matrix**, mixing the available color tensors. It is defined by a correlator of **Wilson lines**.

$$(c_L)_{\{\alpha_k\}} \mathcal{S}_{LK} (\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = \sum_{\{\eta_k\}} \langle 0 | \prod_{i=1}^n \left[\Phi_{\beta_i}(\infty, 0)_{\alpha_k, \eta_k} \right] | 0 \rangle (c_K)_{\{\eta_k\}} ,$$

The soft function \mathcal{S} obeys a **matrix** RG evolution equation

$$\mu \frac{d}{d\mu} \mathcal{S}_{IK} (\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = - \mathcal{S}_{IJ} (\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) \Gamma_{JK}^{\mathcal{S}} (\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon)$$

📌 $\Gamma^{\mathcal{S}}$ is **singular** due to overlapping **UV** and **collinear** poles.

\mathcal{S} is a **pure counterterm**. In dimensional regularization, using $\alpha_s(\mu^2 = 0, \epsilon < 0) = 0$,

$$\mathcal{S} (\beta_i \cdot \beta_j, \alpha_s(\mu^2), \epsilon) = P \exp \left[-\frac{1}{2} \int_0^{\mu^2} \frac{d\xi^2}{\xi^2} \Gamma^{\mathcal{S}} (\beta_i \cdot \beta_j, \alpha_s(\xi^2, \epsilon), \epsilon) \right] .$$

The determination of the **soft anomalous dimension matrix** $\Gamma^{\mathcal{S}}$ is the **keystone** of the resummation program for multiparton **amplitudes** and **cross sections**.

- 📌 It **governs** the interplay of **color** exchange with **kinematics** in multiparton processes.
- 📌 It is the only **source** of multiparton **correlations** for singular contributions.
- 📌 **Collinear** effects are '**color singlet**' and can be extracted from **two-parton** scatterings.

The Dipole Formula

For massless partons, the soft anomalous dimension matrix obeys an **exact equation** based on a **'conformal anomaly'**, which correlates color exchange with kinematics.

The **simplest solution** to this equation is a **sum over color dipoles** (Becher, Neubert; Gardi, LM, 09). It gives an **ansatz** for the all-order singularity structure of **all** multiparton fixed-angle **massless** scattering amplitudes: the **dipole formula**.

📌 All **soft** and **collinear** singularities can be **collected** in a multiplicative operator **Z**

$$\mathcal{M} \left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) = Z \left(\frac{p_i}{\mu_f}, \alpha_s(\mu_f^2), \epsilon \right) \mathcal{H} \left(\frac{p_i}{\mu}, \frac{\mu_f}{\mu}, \alpha_s(\mu^2), \epsilon \right),$$

📌 **Z** contains both soft singularities from **S**, and collinear ones from the jet functions. It must **satisfy** its own matrix **RG equation**

$$\frac{d}{d \ln \mu} Z \left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) = - Z \left(\frac{p_i}{\mu}, \alpha_s(\mu^2), \epsilon \right) \Gamma \left(\frac{p_i}{\mu}, \alpha_s(\mu^2) \right).$$

The matrix **Γ** has a surprisingly simple **dipole structure**. It reads

$$\Gamma_{\text{dip}} \left(\frac{p_i}{\mu}, \alpha_s(\mu^2) \right) = -\frac{1}{4} \hat{\gamma}_K(\alpha_s(\mu^2)) \sum_{j \neq i} \ln \left(\frac{-2 p_i \cdot p_j}{\mu^2} \right) \mathbf{T}_i \cdot \mathbf{T}_j + \sum_{i=1}^n \gamma_{J_i}(\alpha_s(\mu^2)).$$

Note that **all singularities** are **generated by integration** over the scale of the coupling.

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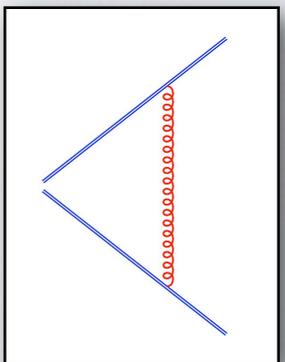
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Features of the dipole formula

- All known results for IR divergences of massless gauge theory amplitudes are recovered.
- The absence of multiparton correlations implies remarkable diagrammatic cancellations.
- The color matrix structure is fixed at one loop: path-ordering is not needed.
- All divergences are determined by a handful of anomalous dimensions.
- The cusp anomalous dimension plays a very special role: a universal IR coupling.

Can this be the definitive answer for IR divergences in massless non-abelian gauge theories?

► There are precisely two sources of possible corrections.

- Quadrupole correlations may enter starting at three loops: they must be tightly constrained functions of conformal cross ratios of parton momenta.

$$\Gamma\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) = \Gamma_{\text{dip}}\left(\frac{p_i}{\mu}, \alpha_s(\mu^2)\right) + \Delta(\rho_{ijkl}, \alpha_s(\mu^2)) \quad , \quad \rho_{ijkl} = \frac{p_i \cdot p_j p_k \cdot p_l}{p_i \cdot p_k p_j \cdot p_l}$$

- The cusp anomalous dimension may violate Casimir scaling beyond three loops.

$$\gamma_K^{(i)}(\alpha_s) = C_i \hat{\gamma}_K(\alpha_s) + \tilde{\gamma}_K^{(i)}(\alpha_s)$$

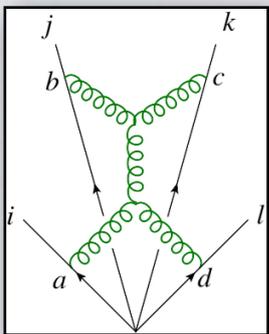
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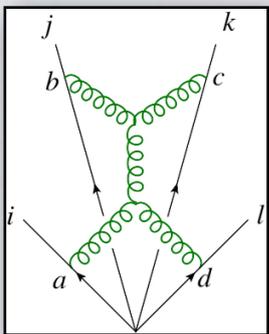
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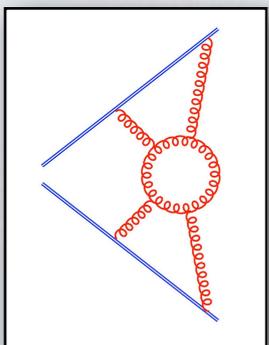
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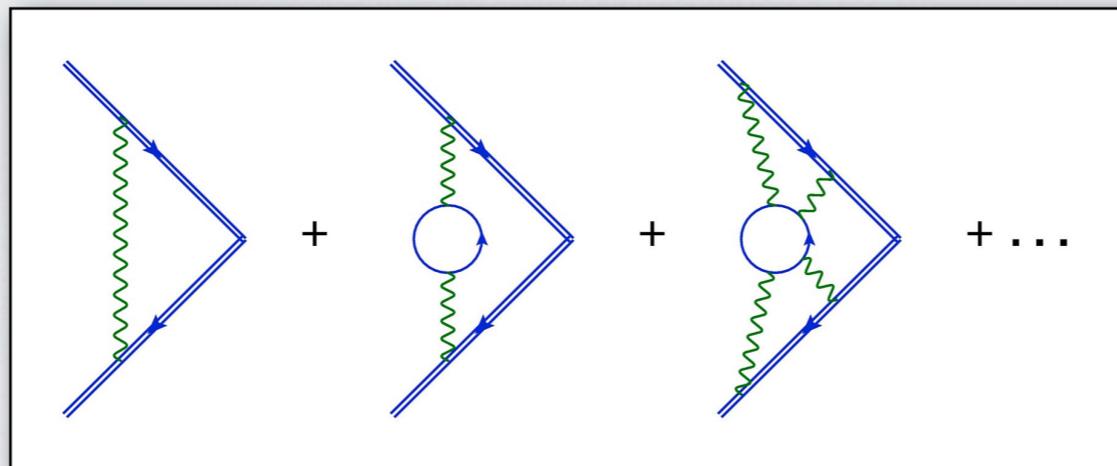
Infrared exponentiation

All correlators of Wilson lines, regardless of shape, resum in **exponential form**.

$$S_n \equiv \langle 0 | \Phi_1 \otimes \dots \otimes \Phi_n | 0 \rangle = \exp(\omega_n)$$

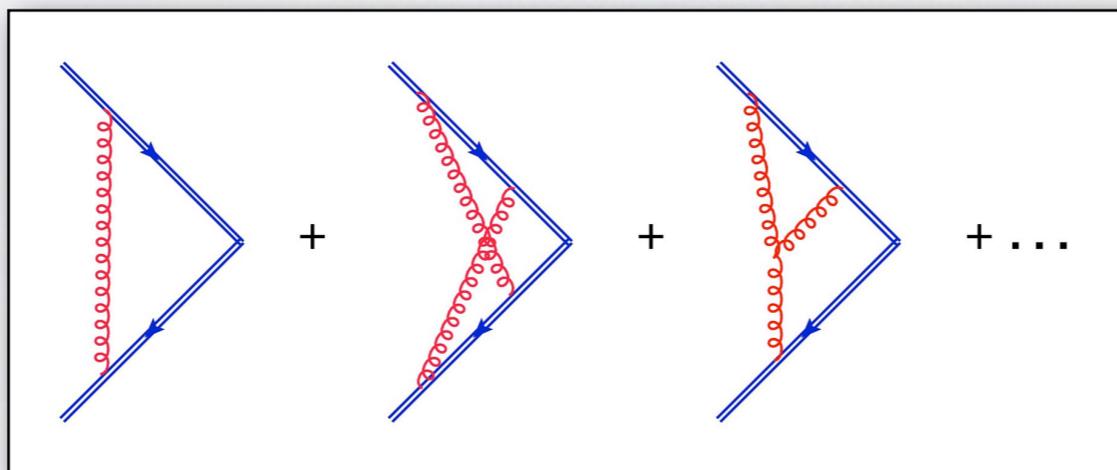
Diagrammatic rules exist to compute **directly the logarithm** of the correlators.

$$\omega_{2,\text{QED}} =$$



Only **connected** photon **subdiagrams** contribute to the logarithm.

$$\omega_{2,\text{QCD}} =$$



Only gluon **subdiagrams** which are **two-eikonal irreducible** contribute to the logarithm. They have **modified color factors**.

For **eikonal form factors**, these diagrams are called **webs** (Gatheral; Frenkel, Taylor; Sterman).

Multiparticle webs

The concept of **web** generalizes non-trivially to the case of **multiple Wilson lines**.
(Gardi, Smillie, White, et al).

A **web** is a **set of diagrams** which **differ** only by the **order** of the **gluon attachments** on each Wilson line. They are **weighted** by **modified color factors**.

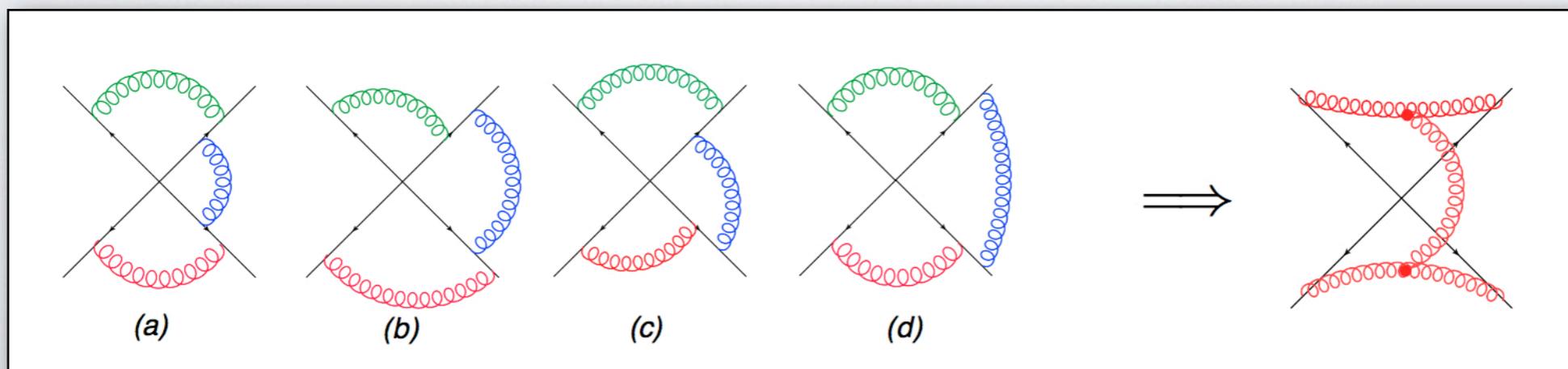
Writing each diagram as the product of its natural **color** factor and a **kinematic** factor

$$D = C(D)\mathcal{F}(D)$$

a **web W** can be expressed as a **sum of diagrams** in terms of a **web mixing matrix R**

$$W = \sum_D \tilde{C}(D)\mathcal{F}(D) = \sum_{D,D'} C(D')R(D',D)\mathcal{F}(D)$$

The **non-abelian exponentiation theorem** holds: each web has the color factor of a **fully connected** gluon subdiagram (Gardi, Smillie, White).



Computing webs

Bare Wilson-line correlators **vanish** beyond tree level in **dimensional regularization**: they are given by **scale-less integrals**. We require **renormalized** correlators, which depend on the **Minkowsky angles** between the Wilson lines.

$$S_{\text{ren}}(\gamma_{ij}, \alpha_s, \epsilon) = S_{\text{bare}}(\gamma_{ij}, \alpha_s, \epsilon) Z(\gamma_{ij}, \alpha_s, \epsilon) = Z(\gamma_{ij}, \alpha_s, \epsilon), \quad \gamma_{ij} = \frac{2\beta_i \cdot \beta_j}{\sqrt{\beta_i^2 \beta_j^2}}$$

To compute the **counterterm** **Z** we make use of an **auxiliary, IR-regularized** correlator

$$\begin{aligned} \hat{S}_{\text{ren}}(\gamma_{ij}, \alpha_s, \epsilon, m) &= \hat{S}_{\text{bare}}(\gamma_{ij}, \alpha_s, \epsilon, m) Z(\gamma_{ij}, \alpha_s, \epsilon) \\ &\equiv \exp(\omega) \exp(\zeta) = \exp\left\{\omega + \zeta + \frac{1}{2}[\omega, \zeta] + \dots\right\} \end{aligned}$$

The expression of **Z** in terms of the **anomalous dimension** **Γ** follows from **RG** arguments

$$Z = \exp\left[\frac{\alpha_s}{\pi} \frac{1}{2\epsilon} \Gamma^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{1}{4\epsilon} \Gamma^{(2)} - \frac{b_0}{4\epsilon^2} \Gamma^{(1)}\right) + \left(\frac{\alpha_s}{\pi}\right)^3 \left(\frac{1}{6\epsilon} \Gamma^{(3)} + \frac{1}{48\epsilon^2} [\Gamma^{(1)}, \Gamma^{(2)}] + \dots\right)\right]$$

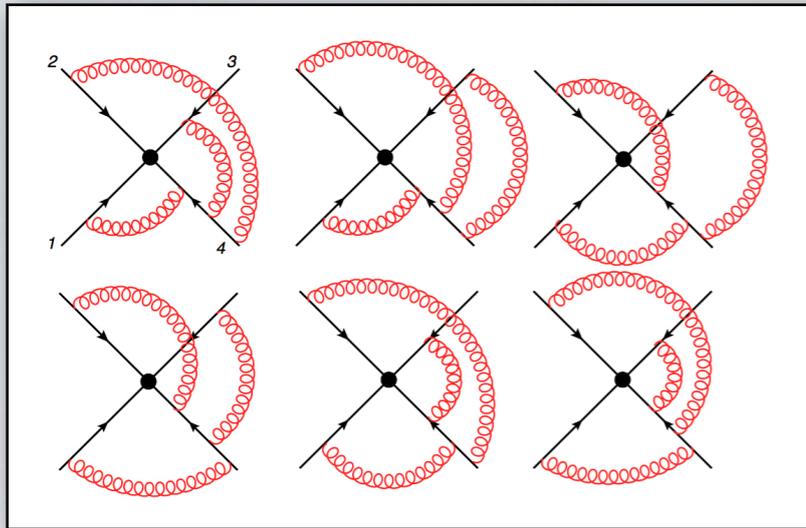
Combining informations one can **get** **Γ** directly from the **logarithm** of the **regularized S**

$$\begin{aligned} \Gamma^{(1)} &= -2\omega^{(1,-1)} \\ \Gamma^{(2)} &= -4\omega^{(2,-1)} - 2\left[\omega^{(1,-1)}, \omega^{(1,0)}\right] \end{aligned} \quad \omega = \sum_{n=1}^{\infty} \sum_{k=-n}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \epsilon^k \omega^{(n,k)}$$

Computing **regularized webs** is a game of **combinatorics** and **renormalization** theory.

Three-loop progress

The computation of the **three-loop** multi-particle **soft anomalous dimension** is **under way**.

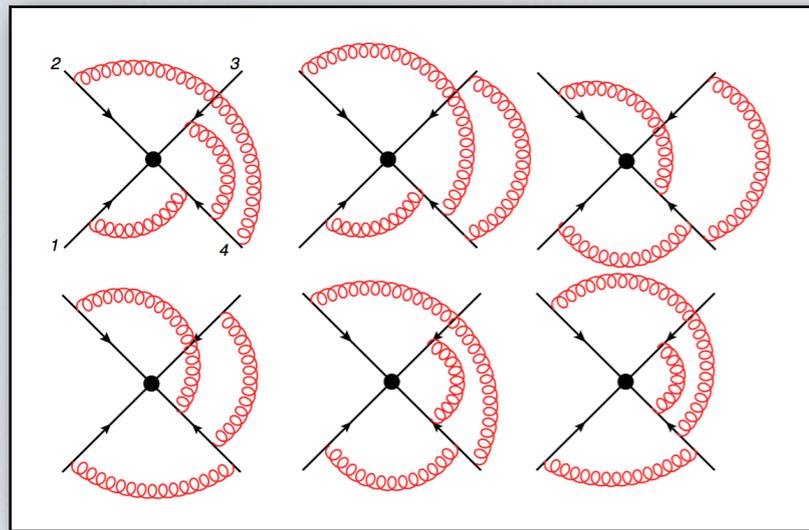


Gardi 1310.5268
Falcioni et al. 1407.3477.

A Multiple Gluon Exchange Web

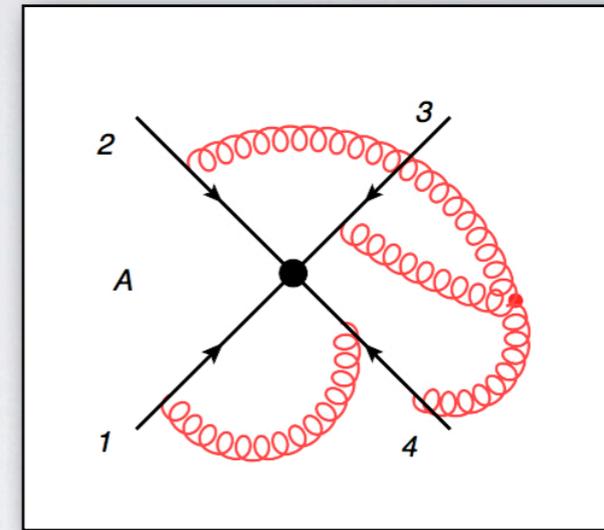
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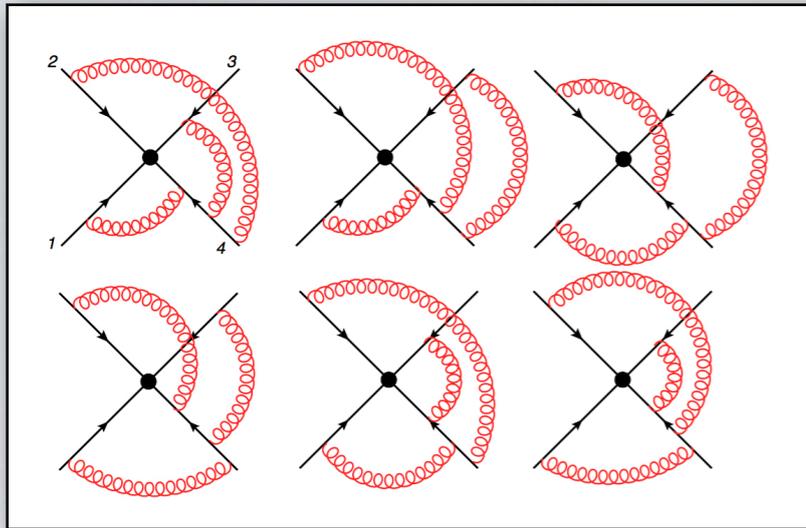


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Falcioni et al.,
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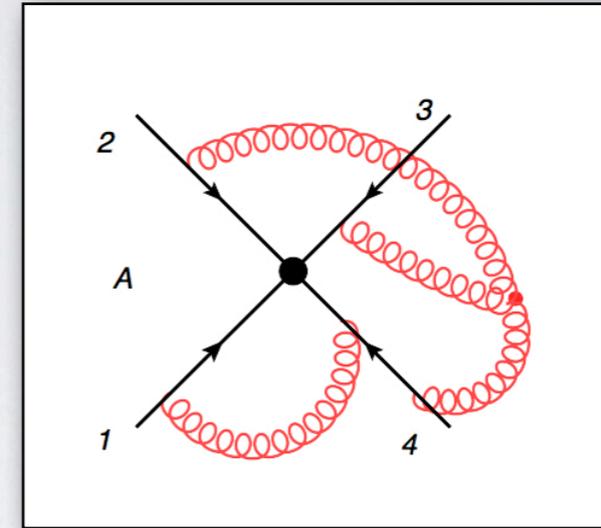
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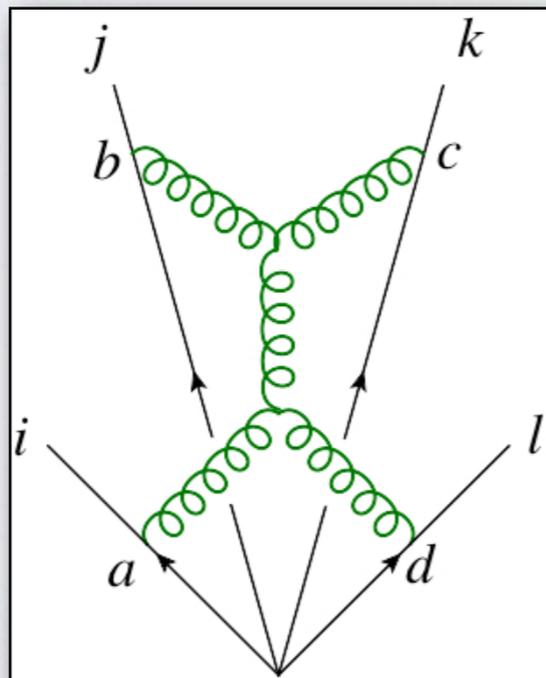
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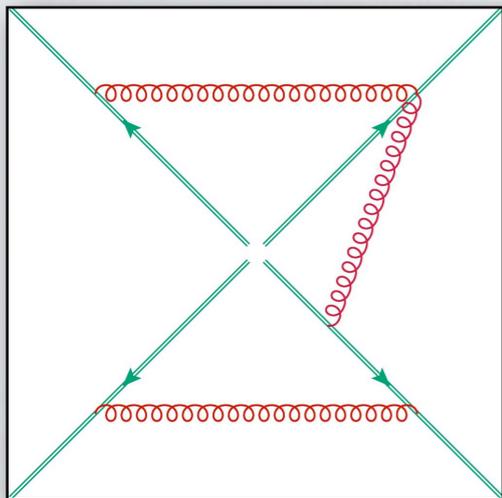
Almelid, Duhr and Gardi,
in progress.

Beyond the eikonal

Hadronic cross sections **near partonic threshold** receive **non-singular** logarithmic corrections $\alpha_s^p \log^k(1 - z)$, or $\alpha_s^p \log^k N/N$, which may be **relevant** for phenomenology. Can they also be organized and **resummed**? (Kraemer et al.; Vogt et al.; Grunberg, ...)

- For **two-parton** processes, $\mathcal{O}(N^0)$ contributions **exponentiate** (Laenen, LM, 03).
- **Phenomenological** evidence indicates that also ‘**sub-eikonal**’ logs partly **exponentiate**.
- An **ansatz** summarizes the resumable for **Drell-Yan** (and **DIS**) (Laenen et al., 06).

$$\ln \left[\hat{w}(N) \right] = \mathcal{F}_{\text{DY}}(\alpha_s(Q^2)) + \int_0^1 dz z^{N-1} \left\{ \frac{1}{1-z} D \left[\alpha_s \left(\frac{(1-z)^2 Q^2}{z} \right) \right] + 2 \int_{Q^2}^{(1-z)^2 Q^2 / z} \frac{dq^2}{q^2} P_s \left[z, \alpha_s(q^2) \right] \right\}_+$$



Is THIS a web?

A **systematic** study of soft-gluon dynamics **beyond the eikonal** approximation is under way (Laenen et al. 08, 10; Bonocore et al, in prep.).

- A class of **factorizable** contributions **exponentiate** via **NE webs**

$$\mathcal{M} = \mathcal{M}_0 \exp \left[\sum_{D_{\text{eik}}} \tilde{C}(D_{\text{eik}}) \mathcal{F}(D_{\text{eik}}) + \sum_{D_{\text{NE}}} \tilde{C}(D_{\text{NE}}) \mathcal{F}(D_{\text{NE}}) \right].$$

- “**Feynman rules**” for the NE exponent, including “**seagull**” vertices.
- **Non-factorizable** contribution can be studied using **Low’s theorem**.

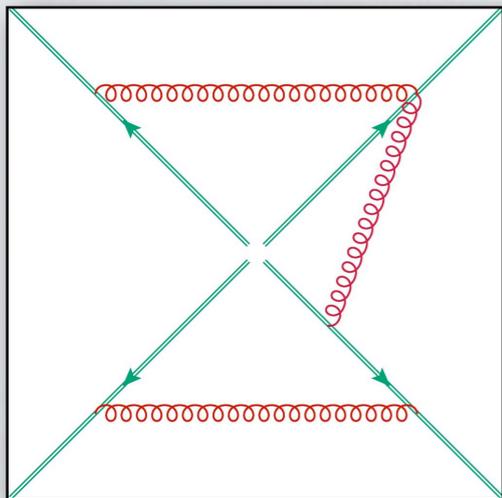
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$\frac{2}{1-z} \longrightarrow \frac{2z}{1-z}$



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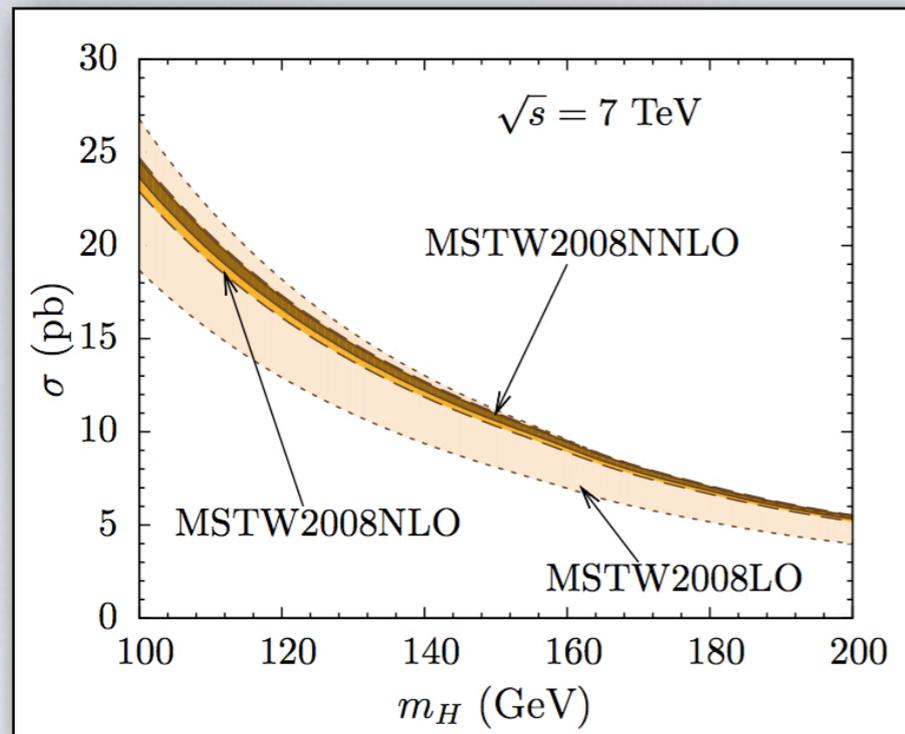
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USING LARGE LOGS



Higgs production

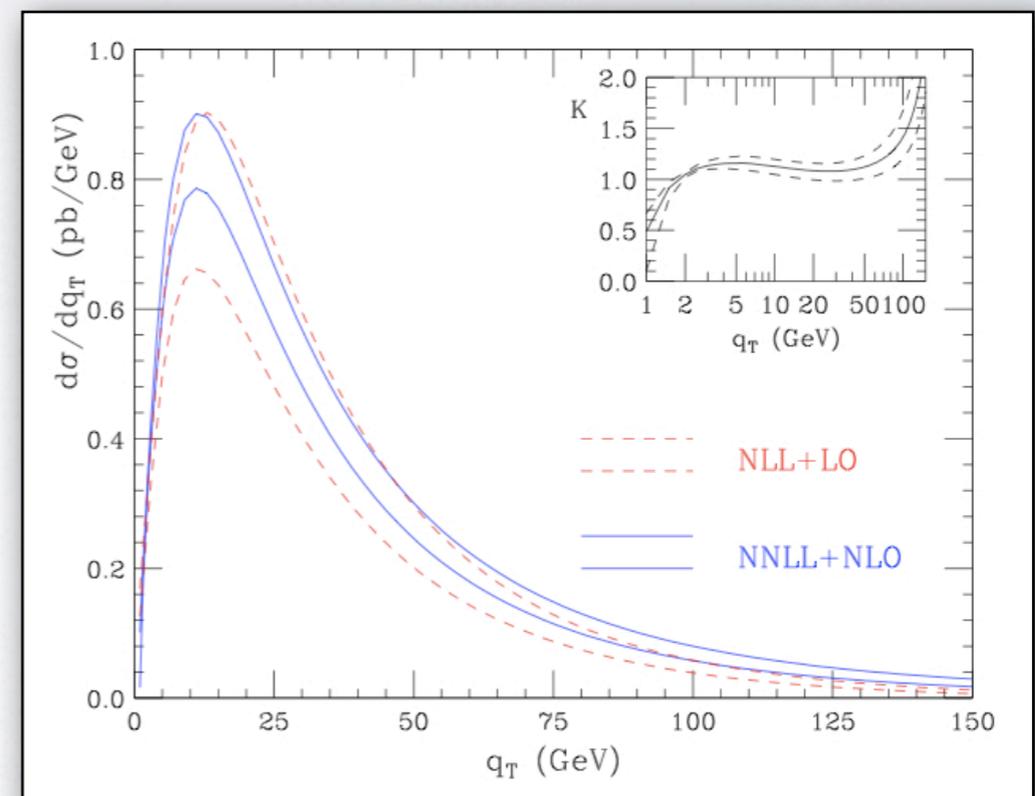


N^3LL resummed cross section for Higgs production via gluon fusion at LHC

The p_T distribution for $gg \rightarrow H$ is known to $NNLL$ and $NNLO$ (M. Grazzini et al. 07, 10 Ahrens et al. 11, Boughezal et al. 13).

- Resummation **reduces** scale uncertainty.
- Subtle **polarization effects** (Catani, Grazzini, 10).
- ‘Collinear **anomaly**’ in SCET (Becher, Neubert).
- Impact of **revised three-loop** coefficient likely very **small**.
- **Threshold** corrections at **large p_T** recently computed (Becher et al. 14).

- The **total cross section** for $gg \rightarrow H$ is known to N^3LL and $NNLO+$, with **NLO EW** corrections.
 - One of the **best-known** observables in the SM.
 - A combined analysis (Ahrens et al. 11) gives a **3%** (th) + **8%** (pdf) + **1%** (mq) **uncertainty**.
 - **Debate** on theoretical and pdf uncertainty, initiated in Baglio et al. 11. For **consensus** see <https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections>



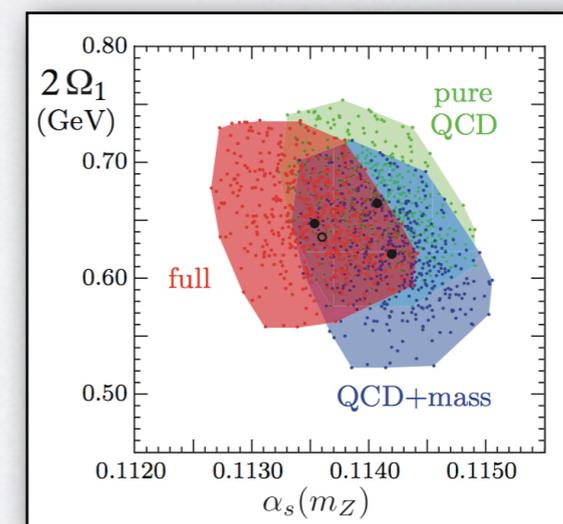
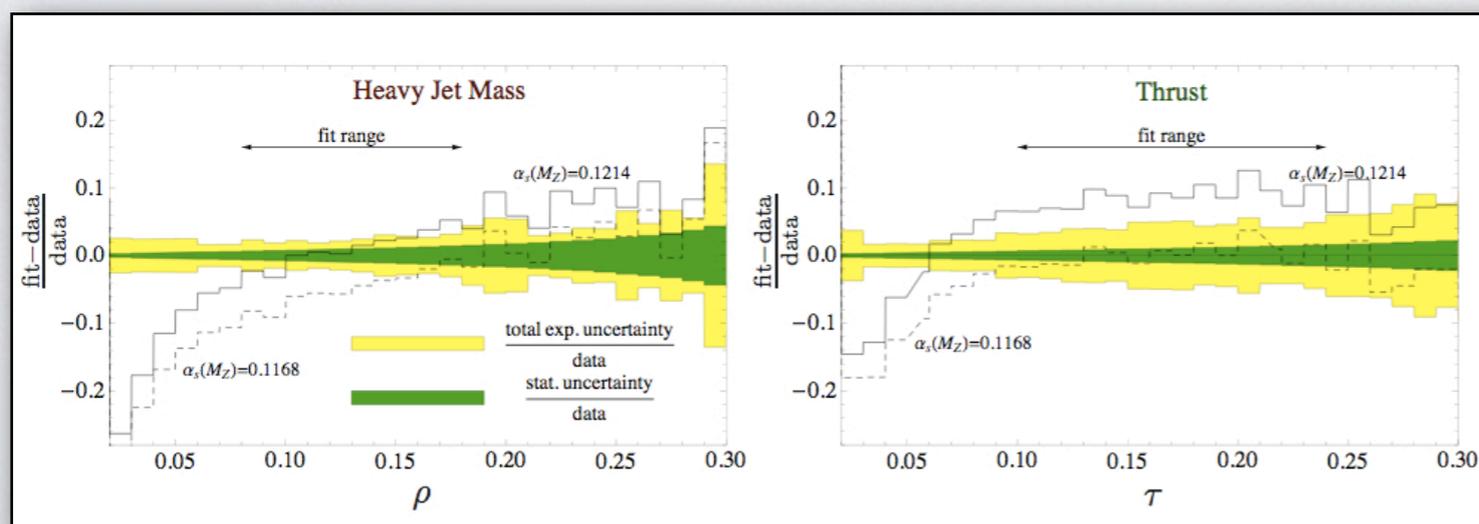
$NNLL$ resummed p_T distribution for Higgs production via gluon fusion at LHC

Event shapes

- First studies of **event shapes** with exact **NNLO** information and (well) approximated **N³LL** resummation have **appeared** (Becher, Schwartz, 08; Schwartz, Cien; Abbate et al. 10).
- The studies deploy **neat tricks** (Padé approximants, numerical determination of 2-loop soft coefficients) and **great care** (hadronization, b-mass, QED corrections).
- Perturbative **agreement** between SCET and standard resummation (Gehrmann et al., 11).
- Significant differences remain** in the final results for the **strong coupling**.

$$\begin{aligned} \alpha_s(M_Z^2) &= 0.1172 \pm 0.0022 && \text{thrust (BS)} \\ \alpha_s(M_Z^2) &= 0.1220 \pm 0.0031 && \text{jet mass (SC)} \\ \alpha_s(M_Z^2) &= 0.1135 \pm 0.0010 && \text{thrust (AFHMS)} \end{aligned}$$

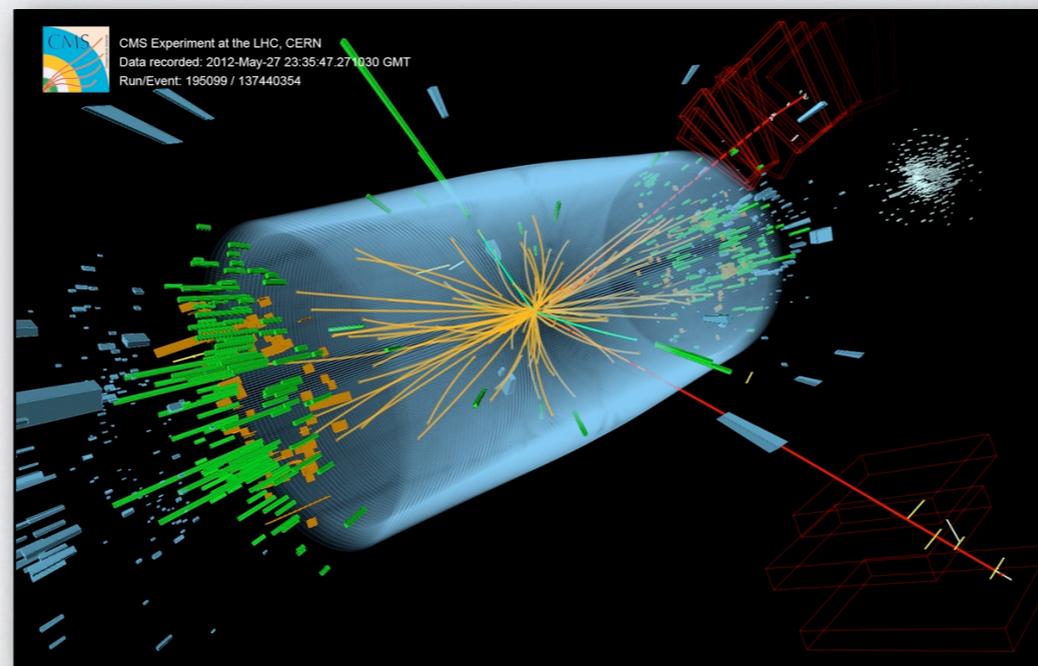
- Many** possible **sources** of discrepancy, the main suspect remains **hadronization/MC**.
- The problem is still **not fully understood**: do we really know α_s to **percent** accuracy?



Comparing the α_s fit quality for thrust and heavy jet mass at N³LL (SC)

Joint fit of α_s and hadronization parameter Ω_1 from N³LL thrust (AFHMS)

OUTLOOK



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We are ready for the challenges of LHC Run Two.

THANK YOU!