Power corrections to jet distributions at hadron colliders

Lorenzo Magnea

Università di Torino – INFN, Sezione di Torino

Work in collaboration with: M. Cacciari, M. Dasgupta, G. Salam.

DIS 2007 – Munich – 18/04/07
Outline

Introduction
Nonperturbative effects at TeV colliders
Jet energy scale studies
Discriminating power corrections

Resummation
Factorization and resummation
Resummed jet $p_T$ distribution

Power corrections
Dipole resummation
Jet size dependence

MonteCarlo results
Comparing algorithms
Comparing parton channels

Perspective
Nonperturbative effects at TeV colliders

Why bother?

- Do power corrections matter for TeV jets?
  \[ \Lambda/Q \sim 10^{-3} \rightarrow \text{true asymptotics?} \]

- Precision measurements require precise jet energy scale
  \[ 1\% \text{ uncertainty} \leftrightarrow \Delta M_{\text{top}} \sim 1\text{GeV}/c^2 \]

- Steeply falling distributions magnify power corrections
  power corrections necessary to fit Tevatron data

- Hadronization and underlying event: different physics
  \[ \rightarrow \text{disentangle computable effects} \]

- QCD dynamics in full colors.
  \[ \rightarrow \text{beauty from gauge invariance} \]
Determining the jet energy scale

CDF, hep-ex/0510047

- **Precision** for the jet energy scale $E_T$ is important

  \[ \frac{\Delta E_T}{E_T} = 10^{-2} \quad \longrightarrow \quad \Delta \sigma_{\text{jet}}/\sigma_{\text{jet}} \big|_{500\text{GeV}} = 10^{-1} \]

- **Determining** the jet energy scale is experimentally **difficult**

  \[ p_T^{\text{parton}} = \left( p_T^{\text{jet}} \times C_{\eta} - C_{\text{MI}} \right) \times C_{\text{ABS}} - C_{\text{UE}} + C_{\text{OOC}} \]

- Experimental **issues**: $C_{\eta}$, $C_{\text{MI}}$, $C_{\text{ABS}}$
  - Calorimeter and detector efficiencies
  - Multiple interactions

- Theoretical **input**: $C_{\text{UE}}$, $C_{\text{OOC}}$
  - Underlying event, hadronization, out-of-cone radiation
  - Models, Monte-Carlo, analytic results?
The *ratio* of single-inclusive jet $E_T$ distributions at different $\sqrt{S}$ should *scale* up to logarithms.

- Cross section ratio should *scale* up to PDF ad $\alpha_S$ effects.
- Data can be fitted with *shift* in distribution.
- *Small* $\Lambda$ has impact at *high* $E_T$.
- $\sigma(E_T) \sim E_T^{-n} \rightarrow \frac{\delta \sigma}{\sigma} \sim -n \delta E_T$
- *Several* sources of energy flow *in* and *out* of jets.

Fit of CDF data with NLO QCD assuming $E_T$-independent shift $\Lambda$ in jet energy.
**Discriminating power corrections**

- **Sources** of power corrections at hadron colliders
  - Soft radiation from *hard antenna* $\Rightarrow$ *hadronization*.
    - *Calculable* in perturbative QCD.
    - Partially *localized* in phase space.
    - Tools: *resummations, dispersive techniques*.
  - *Background* soft radiation $\Rightarrow$ *underlying event*.
    - *Not calculable* in perturbative QCD.
    - *Fills* phase space (*minijets?*).
    - Tools: *models, Monte-Carlo*.

- Experimental *issues* impact on theory.
  - Detector *coverage* and phase space *cuts*.
Factorization and resummation

Consider inclusive production of a jet with momentum \( p^\mu_J \) in hadron-hadron collisions, near partonic threshold.

- **Partonic threshold:** \( s_4 \equiv s + t + u \rightarrow 0 \)
  \[ \rightarrow \alpha_s^n \left[ \log^{2n-1}(s_4)/s_4 \right]_+ \text{ in the distribution.} \]
- Sudakov logs arise from collinear and soft gluons, which factorize, with nontrivial color mixing.

![Diagram of jet production](image_url)
**NLL jet $E_T$ distribution**

G. Sterman, N. Kidonakis, J. Owens ...

**Factorization** leads to resummation. For $q\bar{q}$ collisions

\[ E_J \frac{d^3 \sigma}{d^3 p_J} = \frac{1}{s} \exp \left[ \mathcal{E}_F + \mathcal{E}_{IN} + \mathcal{E}_{OUT} \right] \cdot \text{Tr} \left[ HS \right]. \]

**Incoming** partons build up a *Drell-Yan* structure

\[ \mathcal{E}_{IN} = - \sum_{i=1}^{2} \int_0^1 dz \frac{z N_i^{N_i-1}}{1-z} \left\{ \frac{1}{2} \nu_q \left[ \alpha_s \left( \left(1-z\right) Q_i^2 \right) \right] + \int_{\left(1-z\right)^2}^{1} \frac{d\xi}{\xi} A_q \left[ \alpha_s \left( \xi Q_i^2 \right) \right] \right\}. \]

**Note:** $N_1 = N(-u/s), N_2 = N(-t/s), Q_1 = -u/\sqrt{s}, Q_2 = -t/\sqrt{s}$

**Outgoing** partons near threshold cluster in *two jets*

\[ \mathcal{E}_{OUT} = - \sum_{i=J,R} \int_0^1 dz \frac{z^{N_i-1}}{1-z} \left\{ B_i \left[ \alpha_s \left( \left(1-z\right) p_T^2 \right) \right] + C_i \left[ \alpha_s \left( \left(1-z\right)^2 p_T^2 \right) \right] \right\} + \int_{\left(1-z\right)^2}^{1-z} \frac{d\xi}{\xi} A_i \left[ \alpha_s \left( \xi p_T^2 \right) \right]. \]
Color exchange near threshold

Soft gluons change the color structure of the hard scattering.

- **Choose a basis** in color configuration space

\[ c^{(1)}_{\{r_i\}} = \delta_{r_1 r_3} \delta_{r_2 r_4}, \quad c^{(2)}_{\{r_i\}} = (T_A)_{r_3 r_1} (T_A)_{r_2 r_4} = \frac{1}{2} (\delta_{r_1 r_2} \delta_{r_3 r_4} - \frac{1}{N_c} \delta_{r_1 r_3} \delta_{r_2 r_4}) \]

- **At tree level**, for \( q\bar{q} \) collisions

\[ \mathcal{M}_{\{r_i\}} = \mathcal{M}_1 c^{(1)}_{\{r_i\}} + \mathcal{M}_2 c^{(2)}_{\{r_i\}} \rightarrow |\mathcal{M}|^2 = \mathcal{M}_I \mathcal{M}_J^* \text{tr} \left[ c^{(I)}_{\{r_i\}} (c^{(J)}_{\{r_i\}})^\dagger \right] \equiv \text{Tr} [HS]_0 \]

- **Renormalization group** resums soft logarithms

\[
\text{Tr} [HS] \equiv H_{AB} \left( \alpha_s(\mu^2) \right) S^{AB} \left( \frac{p_T}{N\mu}, \alpha_s(\mu^2) \right) = \\
H \left( \alpha_s(p_T^2) \right) \cdot \overline{P} \exp \left( \int_{p_T}^{p_T^N} \frac{d\mu}{\mu} \Gamma_S^\dagger \left( \alpha_s(\mu^2) \right) \right) \cdot S \left( 1, \alpha_s \left( \frac{p_T^2}{N^2} \right) \right) \cdot P \exp \left( \int_{p_T}^{p_T^N} \frac{d\mu}{\mu} \Gamma_S \left( \alpha_s(\mu^2) \right) \right) \]

- **Note:** \[ \left[ \Gamma_{S}^{q\bar{q}} \right]^{(1)}_{11} = 2C_F \log (-t/s) + i\pi \ldots \]
Issues of globalness and jet algorithms

Resummations in hadron-hadron collisions require a precise definition of the observable.

- Precisely defining threshold
  - For dijet distributions: $M_{12} = (p_1 + p_2)^2$ differs from $M_{12} = 2p_1 \cdot p_2$ at LL level.
  - For single inclusive distributions: fixed and integrated rapidity differ ($N_i \rightarrow N$).

- Precisely defining the observable
  - Jet algorithm: IR safety a must.
  - Jet momentum: four-momentum recombination
    $$p_\perp = \sum_i E_i \sin \theta_i \text{ vs. } p_\perp = \sum_i E_i \cdot \sin \theta_{\text{eff}}.$$ 

- Beware of nonglobal logarithms
  - Pick global observable: satisfied by $x_T$ distribution.
  - Minimize impact of nonglobal logs: $k_\perp$ algorithm for energy flows; joint distributions.
Soft gluons in dipoles

Y. Dokshitzer, G. Marchesini

• Define *eikonal* soft gluon *current*.

\[ j^{\mu,b}(k) = \sum_{i=1}^{N_p} \frac{\omega p_i^\mu}{(k \cdot p_i)} T_i^b ; \quad \sum_{i=1}^{N_p} T_i^b = 0. \]

• Eikonal *cross section* is built by *dipoles*.

\[ j^2(k) = 2 \sum_{i>j} T_i \cdot T_j \frac{\omega^2 (p_i \cdot p_j)}{(k \cdot p_i)(k \cdot p_j)} \equiv 2 \sum_{i>j} T_i \cdot T_j \, w_{ij}(k), \]

• By *color conservation*, up to *three* hard emitters have *no color mixing* (*unique representation content*).

  • \(-2T_1 \cdot T_2 = T_1^2 + T_2^2 = 2C_F ; \quad -2T_1 \cdot T_2 = T_1^2 + T_2^2 - T_3^2 , \)
  
  • \(-j^2(k) = T_1^2 \cdot W_{23}^{(1)}(k) + T_2^2 \cdot W_{13}^{(2)}(k) + T_3^2 \cdot W_{12}^{(3)}(k) , \)
  
  • \( W_{23}^{(1)} = w_{12} + w_{13} - w_{23} . \)

• Note: \( W_{jk}^{(i)} \) isolates *collinear* singularity along \( i \).
**Soft gluons in dipoles**

*Beyond three emitters different color representations contribute.*

- The *eikonal cross section* acquires *noncommuting* dipole combinations

\[-j^2(k) = T_1^2 W_{34}^{(1)}(k) + T_2^2 W_{34}^{(2)}(k) + T_3^2 W_{12}^{(3)}(k) + T_4^2 W_{12}^{(4)}(k) + T_t^2 \cdot A_t(k) + T_u^2 \cdot A_u(k).\]

with *nonCasimir* color factors

\[T_t^2 = (T_3 + T_1)^2 = (T_2 + T_4)^2, \quad T_u^2 = (T_4 + T_1)^2 = (T_2 + T_3)^2.\]

- The resulting *distributions* are *collinear safe*

\[A_t = w_{12} + w_{34} - w_{13} - w_{24}, \quad A_u = w_{12} + w_{34} - w_{14} - w_{23},\]

- *Angular integrals* yield *momentum dependence* of *radiators*

\[
\int \frac{d\Omega}{4\pi} A_t(k) = -2 \ln \frac{-t}{s}; \quad \int \frac{d\Omega}{4\pi} A_u(k) = -2 \ln \frac{-u}{s}.
\]

- Dipole approach *practical* for power corrections.
Power corrections by dipoles

- **Consider** the single inclusive distribution for a jet observable \( O(y, p_T, R) \), with an effective jet radius \( R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2} \).
- **Measure** effect of **single soft gluon** emission on the distribution **dipole by dipole** at power accuracy.
- **Define** \( R \)-dependent power correction
  \[
  \Delta O_{ij}^\pm (R) \equiv \int_{\pm} d\eta \frac{d\phi}{2\pi} \int_{\mu_c}^{\mu_f} d\kappa_T^{(ij)} \alpha_s \left( \frac{\kappa_T^{(ij)}}{k_T} \right) k_T \left| \frac{\partial k_T}{\partial \kappa_T^{(ij)}} \right| \frac{p_i \cdot p_j}{p_i \cdot k p_j \cdot k} \delta O^\pm (k_T, \eta, \phi) .
  \]
- **Compute** in-cone and out-of-cone contributions
  \[
  \Delta O_{ij} (R) = \Delta O_{ij}^+ (R) + \Delta O_{ij}^- (R) = \Delta O_{ij}^+ (R) + \Delta O_{ij}^{\text{all}} (R) - \Delta O_{ij}^{\text{in}} (R) .
  \]
- **Express** leading power \( R \) dependence in terms of (universal?) moment of coupling \( \mathcal{A} \)
  \[
  \mathcal{A} (\mu_f) = \int_{\mu_f}^{\mu_f} \frac{dk_{\perp}}{k_{\perp}} \alpha_s (k_{\perp}) \cdot k_{\perp}
  \]
Radius dependence: $p_{T}$ distribution

Let $O = \xi_T \equiv 1 - 2p_T/\sqrt{S}$. In this case

- **In-In dipole**
  \[
  \Delta \xi_{T,12}(R) = -\frac{4}{\sqrt{S}} \int_{+} d\eta \frac{d\phi}{2\pi} \alpha_s(k_t) \frac{dk_t}{k_t} k_t \cos \phi = -\frac{4}{\sqrt{S}} A(\mu_f) \left( \frac{R^2}{2} - \frac{R^4}{16} + \frac{R^6}{384} + \ldots \right).
  \]

- **In-Jet dipoles**
  \[
  \Delta \xi_{T,1j}(R) = -\sqrt{\frac{2}{S}} \int_{\eta^2 + \phi^2 < R^2} d\eta \frac{d\phi}{2\pi} \alpha_s(\kappa_t) \frac{d\kappa_t}{\kappa_t} \cos \phi e^{\frac{3\eta}{2}} \frac{\cos \phi}{(\cosh \eta - \cos \phi)^{\frac{3}{2}}} \\
  = \frac{2}{\sqrt{S}} A(\mu_f) \left( \frac{2}{R} - \frac{5}{8} R + \frac{23}{1536} R^3 + \ldots \right).
  \]

- **Jet-Recoil dipole**
  \[
  \Delta \xi_{T,jr}(R) = \frac{2}{\sqrt{S}} A(\mu_f) \left( \frac{2}{R} + \frac{1}{2} R + \frac{1}{96} R^3 + \ldots \right)
  \]

- **In-Recoil dipoles**
  \[
  \Delta \xi_{T,1r}(R) = -\frac{2}{\sqrt{S}} A(\mu_f) \left( \frac{1}{8} R^2 - \frac{9}{512} R^4 - \frac{73}{24576} R^6 + \ldots \right)
  \]
Radius dependence: mass distribution

For comparison, let $O = \nu_J \equiv M_J^2 / S$. Now only gluons recombined with the jet contribute, and one finds nonsingular $R$ dependence.

- **In-In dipole**
  \[
  \Delta \nu_{J,12}(R) = \frac{1}{\sqrt{S}} A(\mu_f) \left( \frac{1}{4} R^4 + \frac{1}{4608} R^8 + \mathcal{O}(R^{12}) \right),
  \]

- **In-Jet dipoles**
  \[
  \Delta \nu_{J,1j}(R) = \frac{1}{\sqrt{S}} A(\mu_f) \left( R + \frac{3}{16} R^3 + \frac{125}{9216} R^5 + \frac{7}{16384} R^7 + \mathcal{O}(R^9) \right),
  \]

- **Jet-Recoil dipole**
  \[
  \Delta \nu_{J,jr}(R) = \frac{1}{\sqrt{S}} A(\mu_f) \left( R + \frac{5}{576} R^5 + \mathcal{O}(R^9) \right),
  \]

- **In-Recoil dipoles**
  \[
  \Delta \nu_{J,1r}(R) = \frac{1}{\sqrt{S}} A(\mu_f) \left( \frac{1}{32} R^4 + \frac{3}{256} R^6 + \frac{169}{589824} R^8 + \mathcal{O}(R^{10}) \right).
  \]
Power corrections by MonteCarlo

The *analytical* estimate of power corrections provided by resummation is valid *near threshold*. It can be compared with *numerical* estimates from QCD-inspired *MonteCarlo models* of hadronization.

- **Run MC at** parton level \((p)\), hadron level without UE \((h)\) and finally with UE \((u)\)
- **Select** events with hardest jet in chosen \(p_T\) range, **identify** two hardest jets, **define** for each hadron level

\[
\Delta p_T^{(h/u)} = \frac{1}{2} \left( \Delta p_T^{(h/u)} = \frac{1}{2} \left(p_{T,1}^{(h/u)} + p_{T,2}^{(h/u)} - p_{T,1}^{(p)} - p_{T,2}^{(p)} \right) \right). \\
\Delta p_T^{(u-h)} = \Delta p_T^{(u)} - \Delta p_T^{(h)}.
\]

- **Compare** results for different jet algorithms, hadronization models, parton channels.
MC power corrections: comparing jet algorithms

Tevatron: $55 < p_t < 70$ GeV (bin 04)

Herwig $qq\rightarrow qq$

Herwig $gg\rightarrow gg$

Pythia $qq\rightarrow qq$

Pythia $gg\rightarrow gg$
**MC power corrections: quark channel**

Tevatron: $qq$ channel, $55 < p_t < 70$ GeV (bin 04)

- *kt*
  - Pythia
  - Herwig

- *cam*
  - UE
  - hadronisation

- *siscone*
- *midpoint*
MC power corrections: gluon channel

Tevatron: gg channel, $55 < p_t < 70$ GeV (bin 04)
**MC power corrections: quark channel, scaled**

Tevatron: $qq$ channel, $55 < p_t < 70$ GeV (bin 04), SCALED

- **kt**
  - Pythia
  - Herwig

- **cam**
  - UE: $f(R) = 2 \pi R J_1(R)$
  - hadronisation: $f(R) = 1/R$

- **siscone**

- **midpoint**
MC power corrections: gluon channel, scaled

Tevatron: gg channel, 55 < p_t < 70 GeV (bin 04), Scaled
Disentangling hadronization

- Single inclusive jet distributions have $\Lambda/p_T$ power corrections from hadronization.
- Hadronization corrections are distinguishable from underlying event effects because of singular $R$ dependence.
- In a “dispersive model” the size of leading power corrections can be related to parameters determined in $e^+e^-$ annihilation.
- Power corrections near partonic threshold are qualitatively compatible with Monte Carlo results.
- Work in progress.
  - Study rapidity dependence.
  - Investigate role of jet algorithms.
  - Combine with models of underlying event.
Perspective

- *Resummation* and *power correction* studies at hadron colliders are interesting: the full power of *nonabelian gauge invariance* is displayed.
- *Resummation* and *power correction* studies at hadron colliders are relevant: precision phenomenology requires reliable modeling of *hadronization* and *underlying event*.
- Refined *QCD tools* are available: nonabelian exponentiation, *dispersive* methods, MonteCarlo simulations.
- Disentangling *hadronization* and *underlying event* is useful: different *physics* requires different *tools*.
- Disentangling *hadronization* and *underlying event* is possible: jet-size dependence is a (first) *handle*.
- *Phenomenology* of power corrections at hadron colliders is a realistic prospect.

---

*Introduction*  
*Resummation*  
*Power corrections*  
*MonteCarlo results*  
*Perspective*