Quarkonium Binding and Dissociation: The Spectral Analysis of the QGP

Helmut Satz

Fakultät für Physik, Universität Bielefeld,
Postfach 100 131, D-33501 Bielefeld, Germany

Abstract

In statistical QCD, the thermal properties of the quark-gluon plasma can be determined by studying the in-medium behaviour of heavy quark bound states. The results can be applied to quarkonium production in high energy nuclear collisions, if these indeed form a fully equilibrated QGP. Modifications could arise if an initial charm excess persists in the collision evolution and causes quarkonium regeneration at hadronization.

1 Introduction

We know from statistical QCD that strongly interacting matter undergoes a deconfinement transition to a new state, the quark-gluon plasma. How can we study this state - which phenomena provide us with information about its thermal properties? The main probes considered so far are

- e-m signals (real or virtual photons)
- heavy flavours and quarkonia ($Q\bar{Q}$ pairs)
- jets (energetic partons)

The ultimate aim must be to carry out ab initio calculations of the in-medium behaviour of these probes in finite temperature QCD.

In high energy nuclear collisions, we want to study the deconfinement transition and the QGP in the laboratory. The ultimate aim here must be to show that experimental results confirm the predictions of statistical QCD, or that they disagree with them. If the latter should happen, we can unfortunately not conclude that statistical QCD is wrong; a more likely conclusion would be that nuclear collisions do not produce the medium studied in equilibrium QCD thermodynamics.
I want to consider here a specific case study for the program just outlined: the spectral analysis of quarkonia in a hot QGP and its application to nuclear collisions.

The theoretical basis for this analysis is:

- The QGP consists of deconfined colour charges, so that the binding of a $Q\bar{Q}$ pair is subject to the effects of colour screening.
- The screening radius $r_D(T)$ decreases with temperature $T$.
- When $r_D(T)$ falls below the binding radius $r_i$ of a $Q\bar{Q}$ state $i$, the $Q$ and the $\bar{Q}$ can no longer bind, so that quarkonium $i$ cannot exist [1].
- The quarkonium dissociation points $T_i$, specified through $r_D(T_i) \approx r_i$, thus determine the temperature of QGP, as schematically illustrated in Fig. 1.

Experimentally, quarkonium studies also provide a great tool:

- In $AA$ collisions, quarkonium production can be measured as function of collision energy, centrality, transverse momentum, and $A$.
- The onset of (anomalous) suppression for the different quarkonium states can be determined and correlated to thermodynamic variables, such as the temperature or the energy density.
- The resulting thresholds in the survival probabilities $S_i$ of states $i$ can then be compared to the relevant QCD predictions, as illustrated in Fig. 2.

In this way we can, at least in principle, obtain a direct comparison between experimental results and quantitative predictions from finite temperature QCD.
2 In-medium behavior of quarkonia: Theory

We consider as quarkonia those bound states of heavy quarks which are stable under strong decay; they are thus pairs of charm ($m_c \simeq 1.3$ GeV) or beauty quarks ($m_b \simeq 4.7$ GeV), whose overall masses fall below the open charm or beauty thresholds. The large quark mass allows spectroscopy based on non-relativistic potential theory [2]. Hence the Schrödinger equation

\[ \left\{ 2m_c - \frac{1}{m_c} \nabla^2 + V(r) \right\} \Phi_i(r) = M_i \Phi_i(r), \quad (1) \]

using the “Cornell” form for the confining potential [3],

\[ V(r) = \sigma r - \frac{\alpha}{r}, \quad (2) \]

in terms of the string tension $\sigma \simeq 0.2$ GeV$^2$ and the gauge coupling $\alpha \simeq \pi/12$, determines the masses $M_i$ and the radii $r_i$ of the different charmonium and bottomonium states. The results are summarized in Table 1 and are seen to give a good account of quarkonium spectroscopy, with an error of less than 1% in the mass determination $\Delta M$ for all (spin-averaged) states.

<table>
<thead>
<tr>
<th>state</th>
<th>$J/\psi$</th>
<th>$\chi_c$</th>
<th>$\psi'$</th>
<th>$\Upsilon$</th>
<th>$\chi_b$</th>
<th>$\Upsilon'$</th>
<th>$\chi_b'$</th>
<th>$\Upsilon''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E$ [GeV]</td>
<td>0.64</td>
<td>0.20</td>
<td>0.05</td>
<td>1.10</td>
<td>0.67</td>
<td>0.54</td>
<td>0.31</td>
<td>0.20</td>
</tr>
<tr>
<td>$\Delta M$ [GeV]</td>
<td>0.02</td>
<td>-0.03</td>
<td>0.03</td>
<td>0.06</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.07</td>
</tr>
<tr>
<td>radius [fm]</td>
<td>0.25</td>
<td>0.36</td>
<td>0.45</td>
<td>0.14</td>
<td>0.22</td>
<td>0.28</td>
<td>0.34</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 1: Quarkonium spectroscopy in non-relativistic potential theory

The charmonium and bottomonium ground states are thus tightly bound, with a binding energy $\Delta E = 2M_{D,B} - M_0 \gg \Lambda_{QCD} \simeq 0.2$ GeV, and very small, with $r_0 \ll r_h \simeq 1$ fm. What happens to them in a QGP?

The effect of colour screening is that the binding becomes weaker and of shorter range. When the force range or the screening radius fall below the binding radius, the $Q$ and $\bar{Q}$ can no longer “see” each other, and hence the bound state becomes dissociated. As already noted, the quarkonium dissociation points therefore determine the temperature and thus also the energy density of the QGP. The basic question thus is how to calculate the quarkonium dissociation temperatures.
Early attempts were based on models of the heavy quark potential, essentially obtained from $d = 1$ electrodynamics. Using these in the Schrödinger equation, together with a crude lattice form for the screening mass, led to first predictions\cite{4,5}

\[ T_{J/\psi} \gtrsim T_c, \quad T_{\chi} \& T_{\psi'} \lesssim T_c, \quad (3) \]

where $T_c$ is the deconfinement temperature.

When lattice results for the heavy quark potential became available from finite temperature lattice studies, these were employed in the Schrödinger equation in various forms\cite{6} - \cite{11}. The results eventually converged fairly well and the present status is schematically summarized in Table 2.

<table>
<thead>
<tr>
<th>state</th>
<th>$J/\psi(1S)$</th>
<th>$\chi_c(1P)$</th>
<th>$\psi'(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d/T_c$</td>
<td>2.1</td>
<td>1.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 2: Charmonium dissociation temperatures in lattice-based potential theory

Both previous approaches assume the validity of a two-body potential treatment at finite temperature near a critical point. This assumption is no longer necessary if the quarkonium spectrum can be calculated directly in finite temperature lattice QCD. Such calculations have become possible in recent years and results were presented by several groups, first in quenched QCD\cite{12} - \cite{15} and now also in full (two-flavour) QCD\cite{16,17}. The present state of these results is summarized in Table 3.

<table>
<thead>
<tr>
<th>state</th>
<th>$J/\psi(1S)$</th>
<th>$\chi_c(1P)$</th>
<th>$\psi'(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d/T_c$</td>
<td>&gt; 2.0</td>
<td>&lt; 1.1</td>
<td>?</td>
</tr>
</tbody>
</table>

Table 3: Charmonium dissociation temperatures from finite temperature Lattice QCD calculations

Very recently also first lattice results have been presented for bottomonium dissociation in quenched QCD\cite{18,19}; one finds there

\[ T_{\Upsilon} \gtrsim 2 \, T_c, \quad T_{\chi_b} \lesssim 1.15 \, T_c. \quad (4) \]

The low value reported for the $\chi_b$, which has approximately the same binding energy as the $J/\psi$, remains at present quite puzzling.

We thus find from direct finite temperature lattice studies, both in quenched and in full QCD, as well as in lattice-based potential work, that the $J/\psi$ and the $\Upsilon$ survive up to $T \geq 2 \, T_c$, which means up to energy densities of 25
GeV/fm$^3$ or more. In contrast, the $\chi_c$ and (so far only from potential studies) the $\psi'$ melt near $T_c$, i.e., for energy densities in the range 0.5 - 2.0 GeV/fm$^3$. It should be noted that “survival” here means that the corresponding signal is seen up to the temperature in question. So far, lattice QCD results do not yet allow a determination of the widths as function of temperature, and hence it is not known if even the ground states acquire a very large width with increasing $T$. Moreover, the comparison of lattice and potential theory can be carried out on a more detailed level than given by just the dissociation temperatures. A study of correlators in both approaches can thus certainly provide more insight [11].

In closing this section, we summarize the modifications in theory which have led to the present new theoretical view, in particular of $J/\psi$ survival in a hot QGP.

- Earlier lattice studies had provided only the colour average of the in-medium $Q\bar{Q}$ free energy; now it is possible to separate out the colour singlet contribution.
- Earlier potential models had used the free energy as potential in the Schrödinger equation; today we can specify the colour singlet internal energy, which is more realistic as the relevant potential and leads to a stronger binding.
- There now exist direct finite temperature lattice QCD studies of the in-medium behaviour of charmonia, allowing $ab$ $initio$ conclusions not based on any potential model, and they support a higher $J/\psi$ dissociation temperature.

What then does this imply for quarkonium production as QGP probe in nuclear collisions?

3 In-medium behavior of quarkonia: Phenomenology

The modifications observed when comparing $J/\psi$ production in $AA$ collisions to that in $pp$ interactions have two distinct origins. Of primary interest is obviously the effect of the secondary medium produced in the collision - this is the candidate for the QGP we want to study. In addition, however, the presence of cold nuclear matter in target and projectile can also affect the production process and final rates. This ambiguity in the origin of any observed $J/\psi$ suppression thus has to be resolved.

A second empirical feature to be noted is that the measured $J/\psi$ production consists of directly produced $1S$ states as well as of feed-down from $\chi_{c}(1P)$ and $\psi'(2S)$ decay. In the previous section, we had seen that a hot QGP affects the higher excited quarkonium states much sooner (at lower temperatures) than
the ground states. This results in another ambiguity in observed $J/\psi$ production - are only the higher excited states affected, or do all states suffer?

An ideal solution of these problems would be to measure separately $J/\psi$, $\chi_c$ and $\psi'$ production first in $pA$ (or $dA$) collisions, to determine the effects of cold nuclear matter, and then measure, again separately, the production of the different states in $AA$ collisions as function of centrality at different collision energies.

Until such studies become available, we resort to a more operational approach, whose basic features are:

- We assume that the $J/\psi$ feed-down rates in $pA$ and $AA$ are the same as same as in $pp$, i.e., 60% direct $J/\psi(1S)$, 30% decay of $\chi(1P)$, and 10% decay of $\psi'(2S)$.
- We specify the effects due to cold nuclear matter by a Glauber analysis of $pA$ or $dA$ experiments in terms of $\sigma_{\text{abs}}^i$ for $i=J/\psi$, $\chi_c$, $\psi'$. This $\sigma_{\text{abs}}^i$ is not meant as a real cross-section for charmonium absorption by nucleons in the nucleus; it is rather used to parametrize all initial and final state nuclear effects, including shadowing/antishadowing, parton energy loss as well as pre-resonance and resonance absorption.
- In the analysis of $AA$ collisions, we then use $\sigma_{\text{abs}}^i$ in a Glauber analysis to obtain the predicted form of normal $J/\psi$ suppression. This allows us to identify anomalous $J/\psi$ suppression as the difference between the observed production distribution and that expected from only normal suppression. We parametrize the anomalous suppression through the survival probability

$$S_i = \frac{(dN_i/dy)_{\text{exp}}}{(dN_i/dy)_{\text{Glauber}}} \quad (5)$$

for each quarkonium state $i$.

With the effects of cold nuclear matter thus accounted for, what form do we expect for anomalous $J/\psi$ suppression? If $AA$ collisions indeed produce a fully equilibrated QGP, we should observe a sequential suppression pattern for $J/\psi$ and $\Upsilon$, with thresholds predicted (in terms of temperature or energy density) by finite temperature QCD [4,5], [20]-[22]. The resulting pattern for the $J/\psi$ is illustrated in Fig. 3.

Its consequences are quite clear. If, as present statistical QCD studies indicate, the direct $J/\psi(1S)$ survives up to about $2T_c$ and hence to $\epsilon \geq 25 \text{ GeV}/\text{fm}^3$, then all anomalous suppression observed at SPS and RHIC must be due to the dissociation of the higher excited states $\chi_c$ and $\psi'$. The suppression onset for these is predicted to lie around $\epsilon \simeq 1 \text{ GeV}/\text{fm}^3$, and once they are gone, only the unaffected $J/\psi(1S)$ production remains. Hence the $J/\psi$ survival probability (under anomalous suppression) should be the same for central $Au-Au$
collisions at RHIC as for central $Pb - Pb$ collisions at the SPS.

A further check to verify that the observed $J/\psi$ production in central collisions is indeed due to the unmodified survival of the directly produced $1S$ state is provided by its transverse momentum behaviour. Initial state parton scattering causes a broadening of the $p_T$ distributions of charmonia [23]-[26]: the gluon from the proton projectile in $pA$ collisions can scatter a number of times in the target nucleus before fusing with a target gluon to produce a $c\bar{c}$. Assuming the protonic gluon to undergo a random walk through the target leads to

$$\langle p_T^2 \rangle_{pA} = \langle p_T^2 \rangle_{pp} + N_c^A \delta_0$$

for the average squared transverse momentum of the observed $J/\psi$. Here $N_c^A$ specifies the number of collisions of the gluon before the parton fusion to $c\bar{c}$, and $\delta_0$ the kick it receives at each collision. The collision number $N_c^A$ can be calculated in the Glauber formalism; here $\sigma_{abs}$ has to be included to take into account the presence of cold nuclear matter, which through a reduction of $J/\psi$ production shifts the effective fusion point further “down-stream” [27].

In $AA$ collisions, initial state parton scattering occurs in both target and projectile, and the corresponding random walk form becomes

$$\langle p_T^2 \rangle_{AA} = \langle p_T^2 \rangle_{pp} + N_c^{AA} \delta_0;$$

here $N_c^{AA}$ denotes the sum of the number of collisions in the target and in the projectile, prior to parton fusion. It can again be calculated in the Glauber scheme including $\sigma_{abs}$. The crucial point now is that if the observed $J/\psi$’s in central $AA$ collisions are due to undisturbed $1S$ production, then the centrality dependence of the $p_T$ broadening is fully predicted by such initial state parton scattering [22]. In contrast, any onset of anomalous suppression of the $J/\psi(1S)$ would lead to a modification of the random walk form [27].

In Fig. 4 we summarize the predictions for $J/\psi$ survival and transverse momentum behaviour in $AA$ collisions at SPS and RHIC, as they emerge from our present state of knowledge of statistical QCD. Included are some prelimi-
nary and some final data; for a discussion of the data analysis and selection, see ref. [22].

![Graph showing J/ψ survival and transverse momentum at SPS and RHIC](image)

Fig. 4. $J/\psi$ survival and transverse momentum at SPS and RHIC

We conclude that the present experimental results are compatible with the present information from statistical QCD. This was not the case previously, and such a conclusion can be drawn today because of several changes in our theoretical and experimental understanding.

- As already noted, statistical QCD presently puts the onset of direct $J/\psi$ suppression at energy densities beyond the RHIC range; previous onset values were much lower (see, e.g., ref. [6]).
- SPS $In − In$ data [28] suggest an onset of anomalous suppression at $\epsilon \simeq 1$ GeV/fm$^3$; previous onset values from $Pb − Pb$ and $S − U$ interactions were considerably higher, with $\epsilon \simeq 2 − 2.5$ GeV/fm$^3$ (see, e.g., ref. [29]).
- within statistics, there is no further drop of the $J/\psi$ survival rate below 50 - 60%, neither at RHIC nor at the SPS; a second drop in very central SPS $Pb − Pb$ data (see, e.g., ref. [29]) is no longer maintained.

4 $J/\psi$ enhancement by regeneration

In this section we want to consider the possibility that the medium produced in high energy nuclear collisions differs from the deconfined state of matter studied in finite temperature QCD. The basic idea here is that nuclear collisions initially produce more than the thermally expected charm, and that this excess, if it survives, may lead to a new form of combinatorial charmonium production at hadronization.

A crucial aspect in the QGP argumentation of the previous sections was that charmonia, once dissociated, cannot be recreated at the hadronization stage, since the abundance of charm quarks in an equilibrium QGP is far too low to allow this. The thermal charm quark production rate, relative to that of light quarks, is

$$c\bar{c}/q\bar{q} \simeq \exp\{-m_c/T_c\} \simeq 6 \times 10^{-4},$$

(8)
with $m_c = 1.3$ GeV for the charm quark mass and $T_c = 0.175$ GeV for the transition temperature. The initial charm production in high energy nuclear interactions, however, is a hard non-thermal process, and the resulting $c/\bar{c}$ production rates grow with the number $N_{\text{coll}}$ of nucleon-nucleon collisions. In contrast, the light quark production rate grows (at least in the present energy regime) essentially as the number $N_{\text{part}}$ of participant nucleons, i.e., much slower. When the charm production cross-section becomes sizeable, the initial charm abundance in $AA$ collisions is then much higher than the thermal value; we illustrate this in Fig. 5 for $A = 200$ as function of the collision energy $\sqrt{s}$. What happens to this excess in the course of the collision evolution?

The basic assumption of the regeneration approach [30] - [32] is that the initial charm excess is maintained throughout the subsequent evolution, i.e., that the initial chemical non-equilibrium will persist up to the hadronization point. If that is the case, a $c$ from a given nucleon-nucleon collision can at hadronization combine with a $\bar{c}$ from a different collision (“off-diagonal” pairs) to create a $J/\psi$. This pairing provides a new exogamous charmonium production mechanism, in which the $c$ and the $\bar{c}$ in a charmonium state have different parents, in contrast to the endogamous production in a $pp$ collision. At sufficiently high energy, this mechanism will lead to enhanced $J/\psi$ production in $AA$ collisions in comparison to the scaled $pp$ rates. When should this enhancement set in?

In present work [30] - [32], it is first assumed that the direct $J/\psi$ production is strongly suppressed for $\epsilon \geq 3$ GeV/fm$^3$. This is evidently in contrast to the statistical QCD results discussed in the previous sections; however, we recall the caveat that the temperature dependence of the charmonium widths is so far not known. Moreover, it is of course always possible that the medium produced in nuclear collisions is quite different from the quark-gluon plasma of statistical QCD. Next, it is assumed that the regeneration rate is determined by statistical combination in a QGP in kinetic equilibrium, with or without wave function corrections. In other words, if a $c$ and a $\bar{c}$ meet under the right
kinematic conditions, they are taken to form a $J/\psi$. An evolution towards a QGP in chemical equilibrium would also allow annihilation at this point.

To account for the $J/\psi$ production rates observed at RHIC, it is assumed that the new exogamous production just compensates the proposed decrease of the direct endogamous $1S$ production, as illustrated in Fig. 6. At the LHC, with much higher energy densities, one should then observe a $J/\psi$ enhancement relative to the rates expected from scaled $pp$ results.

![Fig. 6. Regeneration of $J/\psi$ production](image)

We thus have to find ways of distinguishing between the two scenarios discussed here: the sequential suppression predicted by an equilibrium QGP or a stronger direct suppression followed by a $J/\psi$ regeneration in a medium with excess charm. Fortunately the basic production patterns in the two cases are very different, so that one may hope for an eventual resolution.

The overall $J/\psi$ survival probability in the two cases is illustrated in Fig. 7a. Sequential suppression provides a step-wise reduction: first the higher excited charmonium states are dissociated and thus their feed-down contribution disappears; at much higher temperature, the $J/\psi(1S)$ itself is suppressed. Both onsets are in principle predicted by lattice QCD calculations. In the regeneration scenario, the thermal dissociation of all “diagonal” $J/\psi$ production is obtained by extrapolating SPS data to higher energy densities. The main prediction of the approach is therefore the increase of $J/\psi$ production with increasing energy density. Ideally, the predictions for the LHC are opposite extremes [22,33,34], providing of course that here the feed-down from $B$-decay is properly accounted for.

The expected transverse momentum behaviour in the two cases is also quite different. In the region of full $J/\psi(1S)$ survival, sequential suppression predicts the normal random walk pattern specified through $pA$ studies; the eventual dissociation of direct $J/\psi(1S)$ states then leads to an anomalous suppression also in the average $p_T^2$ [27]. Regeneration alone basically removes the centrality dependence, since the different partners come from different collisions. It is possible to introduce some small centrality dependence [35], but the random
walk increase is essentially removed. The resulting behaviour is schematically illustrated in Fig. 7b. – More generally, the quarkonium momentum distributions, whether transverse or longitudinal, should in the regeneration scenario be simply a convolution of the corresponding open charm distributions; this provides a further check [35].

Fig. 7. Sequential suppression vs. regeneration: $J/\psi$ survival (a) and $\langle p_T \rangle$ (b)

5 Conclusions

- In statistical QCD, the spectral analysis of quarkonia provides a well-defined way to determine temperature and energy density of the QGP.
- If nuclear collisions produce a quark-gluon plasma in equilibrium, the study of quarkonium production can provide a direct way to connect experiment and statistical QCD.
- For a QGP with surviving charm excess, off-diagonal quarkonium formation by statistical combination may destroy this connection and instead result in enhanced $J/\psi$ production.

Acknowledgements


References

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[34] L. Yan, P. Zhuang and N. Xu, nucl-th/0608010.