W boson and Top quark mass measurement

M. Musich
XXIII PhD cycle – Torino Gruaduate School in Physics

- Particle Physics Course report -

Outline

- Electroweak symmetry breaking
- Tree-level mass of W
- W production at hadron colliders
- Top production
- Top decays
- Challenges in top measurement
- Methods of top mass determination
  - Templates
  - Matrix element
- Results on top mass

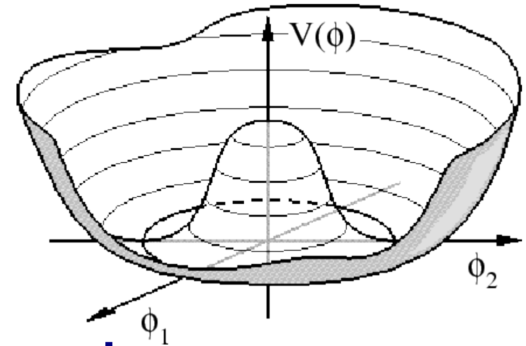
EWSB role in Vector Bosons masses

Higgs sector for Electroweak Lagrangian:

$$L_{higgs}=\left|\left(i\,\partial_\mu - g\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - g'\frac{Y}{2}B_\mu\right)\phi\right|^2 - V(\phi)$$

EWSB

$$\langle\phi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$



Let's consider the mass part of the Lagrangian:

$$L_{mass}=\left|\left(-g\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - \frac{g'}{2}B_\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}\right|^2=$$

$$=\frac{1}{8}\left|\begin{pmatrix} gW^3_\mu+g'B_\mu & g\left(W^1_\mu-iW^2_\mu\right)\\ g\left(W^1_\mu+iW^2_\mu\right) & -gW^3_\mu+g'B_\mu\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix}\right|^2=$$

Weak mixing angle:

$$\begin{cases} W^+_\mu=\frac{1}{2}\left(W^1_\mu-iW^2_\mu\right)\\ W^-_\mu=\frac{1}{2}\left(W^1_\mu+iW^2_\mu\right)\end{cases}\qquad \begin{cases} A_\mu=\frac{\left(g'W^3_\mu+gB_\mu\right)}{\sqrt{g'^2+g^2}}\\ Z_\mu=\frac{\left(gW^3_\mu-g'B_\mu\right)}{\sqrt{g'^2+g^2}}\end{cases}$$

$$=\frac{1}{8}v^2g^2\left[\left(W^1_\mu\right)^2+\left(W^2_\mu\right)^2\right]+\frac{1}{8}v^2\left[g'B^2_\mu-2gg'B_\mu W^{3\mu}+g^2\left(W^3_\mu\right)^2\right]=$$

$$=\frac{1}{4}v^2g^2\left(W^+_\mu W^{-\mu}\right)+\frac{1}{8}v^2\left(g^2+g'^2\right)Z_\mu Z^\mu=M_W^2\left(W^+_\mu W^{-\mu}\right)+\frac{1}{2}M_Z^2\left(Z_\mu Z^\mu\right)$$

$$\begin{cases} M_W=\frac{1}{2}g\,v\\ M_Z=\frac{1}{2}v\sqrt{g'^2+g^2}=\frac{1}{2}v\,g\,\frac{1}{\cos(\theta_W)}\end{cases}$$

SM prediction:

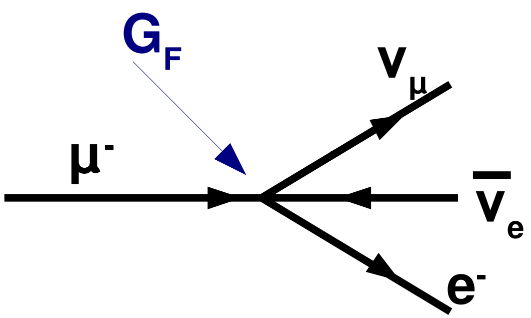
$$\frac{M_W}{M_Z}=\cos(\theta_W)\ \Rightarrow \rho=\frac{M_W}{M_Z\cos(\theta_W)}=1$$

Tree level mass of the W

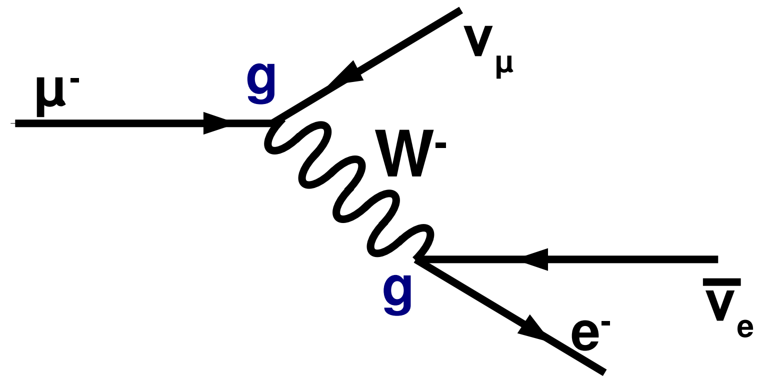
- From Fermi 4 fermion contact theory to standard model:

$\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu$

FERMI MODEL



STANDARD MODEL



$$M^F_{fi} = \frac{4G_F}{\sqrt{2}}\left[\overline{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)v_{\bar{\nu}_e}\overline{u}_{\nu_\mu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)u_\mu\right]$$

$$M^{SM}_{fi} = \left(i\frac{g}{\sqrt{2}}\right)^2\left[\overline{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)v_{\bar{\nu}_e}\left(\frac{-ig_{\mu\nu}}{M_W^2}\right)\overline{u}_{\nu_\mu}\gamma^\nu\left(\frac{1-\gamma^5}{2}\right)u_\mu\right]$$

- **4 fermion matrix element**

W propagator: $S_W^{\mu\nu}(q) = i\frac{-g^{\mu\nu}+q^\mu q^\nu / M_W^2}{q^2-M_W^2}$

gauge invariant term

$$4\frac{G_F}{\sqrt{2}} = \frac{g^2}{2M_W^2} \Rightarrow M_W = \left(\frac{\sqrt{2}g^2}{8G_F}\right)^{\frac{1}{2}}$$

- But since we have: $\begin{cases} g^2 = \frac{e^2}{\sin^2(\theta_W)} \\ \alpha_{EM} = \frac{e^2}{4\pi} \end{cases}$

$$M_W^{LO} = \left(\frac{\pi\alpha_{EM}}{\sqrt{2}G_F\sin^2(\theta_W)}\right)^{\frac{1}{2}}$$

Motivation for W mass measurement

- Standard model prediction for Z mass:

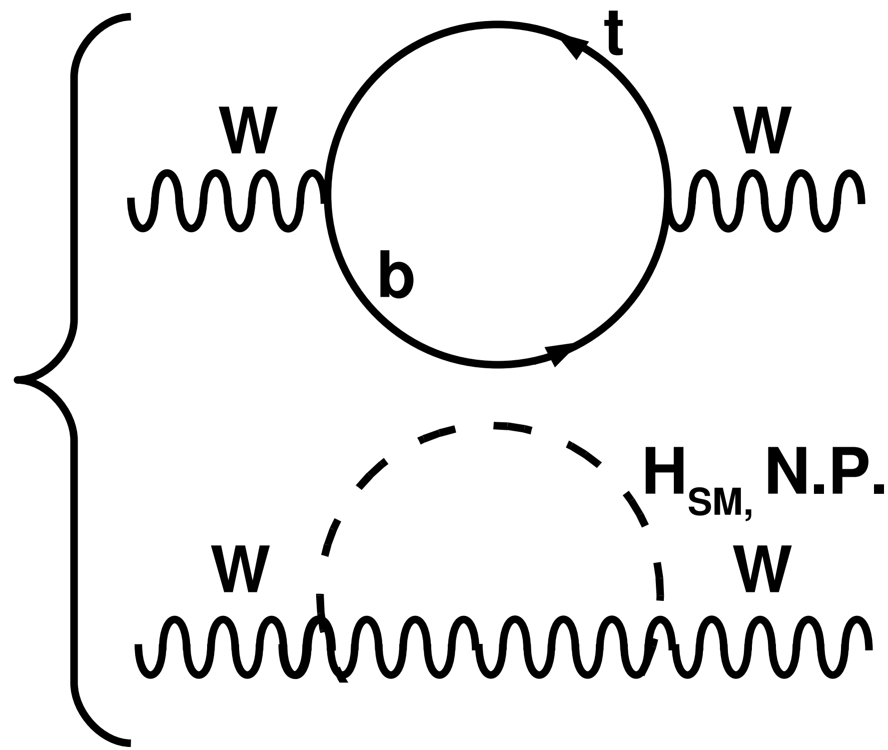
$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F} \right)^{\frac{1}{2}} \frac{1}{\sin(\theta_W) \sqrt{1 - \Delta r}}$$

- In the SM prediction appear:
  - The Fermi Constant **G_F**
  - The weak mixing angle **$\sin(\theta_W)$**
  - Beyond tree-level corrections **Δr** **radiative corrections**

$$\Delta r = \Delta \alpha \oplus \Delta r\ (top) \oplus \Delta r (H)$$

$$\Delta M_W \propto \Delta r (top) \propto m_{top}^2$$

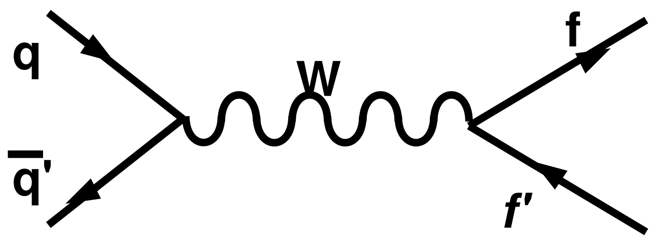
$$\Delta M_W \propto \Delta r (H) \propto \ln\left({}^{M_H}\!/_{M_Z} \right)$$



- **Sensitivity to top, higgs physics (Improvement in EW global fit) + new physics**

W boson production X-section

- W production at hadron colliders has the following LO elementary matrix element:



- definition of center of mass energy in the qq' reference system:

$$s' = \left(p_q^\mu + p_{\bar{q}'}^\mu\right)^2 = \left(p_f^\mu + p_f^\mu{}'\right)^2$$

$$-i M_{fi}(q\bar{q}' \to W \to f\bar{f}') = \underbrace{-i\frac{g}{\sqrt{2}} V_{q\bar{q}'}\left(\bar{q}\gamma_\mu \frac{1}{2}\left(1-\gamma^5\right)q'\right)}_{V-A\ \text{quark current}} \overbrace{\frac{-ig_{\mu\nu}}{s' - M_W^2 + iM_W\Gamma_W}}^{W\ \text{propagator}} \underbrace{\frac{-ig}{\sqrt{2}}\left(\bar{f}\gamma_\mu \frac{1}{2}\left(1-\gamma^5\right)f'\right)}_{V-A\ \text{fermion current}}$$

- with s' energy in the qq' center of mass **(NOT LAB System)**:
- differential cross section in that system is:

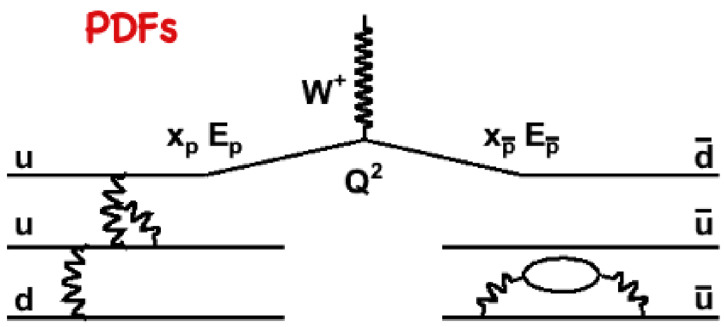
$$\frac{d\sigma}{d\cos(\theta^*)} = \frac{1}{32\pi s'}\overline{|M_{fi}|}^2 = \frac{|V_{q\bar{q}'}|^2}{8\pi}\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \frac{s'\left(1-\cos(\theta^*)\right)^2}{\left(s'-M_W^2\right)^2 + \left(M_W\Gamma_W\right)^2}$$

- and in the narrow resonance approximation: ($\Gamma_W << M_W$) the total cross section is:

$$\sigma(q\bar{q}' \to W + X) = |V_{qq'}| 2\frac{\pi \cdot G_F M_W^2}{\sqrt{2}}\delta(s' - M_W^2)$$

- Not the end of the story, PDF's still to be taken into account!

W production cross section



- Cross-section for the process $p\bar{p} \to WX$, should be written in terms of **parton distribution functions**

$$d\sigma(p\bar{p}\to W+X)=dx_1 dx_2\frac{1}{N_c}\sum_{qq'}\left[q(x_1)\overline{q'}(x_2)\right]\sigma(q\overline{q'}\to W;s')$$

$$=dx_1 dx_2\frac{2\pi G_F M_W^2}{\sqrt{2}}\frac{1}{N_C}\sum_{q\bar{q}'}|V_{q\bar{q}'}|^2\left[q(x_1)\overline{q(x_2)}\right]\delta(s'-M_W^2)$$

with x_1 and x_2 fraction of momentum carried by the partons involved in the scattering:

- One can write the differential cross-section for the process $p\bar{p} \to WX$ in terms of the W rapidity which is correlated to x_1 and x_2:

$$y_W=\frac{1}{2}\ln\frac{E_w+p_{Wz}}{E_w-p_{zW}}=\frac{1}{2}\ln\frac{x_1}{x_2}\Leftrightarrow x_{1,2}=\frac{M_W^2}{\sqrt{s}}e^{\pm y_w}$$

$$x_1 x_2 = s'/s = M_W^2/s$$

- So that:

$$\frac{d\sigma(p\bar{p}\to W+X)}{dy_W}=\frac{2\pi G_F}{\sqrt{2}}\frac{1}{N_C}\sum_{q\bar{q}'}x_1 x_2\left[q(x_1)\overline{q(x_2)}\right]$$

- Now one should parametrize $q(x_1)$ and $q(x_2)$ with the known (from other experiments) PDFs of proton and antiproton

W production cross section

- In a proton antiproton interaction the relevant pdf are the ones for u,d,s quarks, coming either from the proton or the anitproton:

$$\frac{d\sigma(p\bar{p}\to W+X)}{dy_W}=\frac{2\pi G_F}{N_c\sqrt{2}}x_1x_2\left[\cos^2\theta_C\left(u(x_1)\overline{d(x_2)}+d(x_1)\overline{u(x_2)}\right)+\sin^2\theta_C\left(u(x_1)\overline{s(x_2)}+s(x_1)\overline{u(x_2)}\right)\right]$$

- Assuming xq(x) barely constant over integration variable y_W we get the total cross-section:

$$\sigma(p\bar{p}\to W+X)\simeq\frac{2\pi G_F}{N_C\sqrt{2}}\int_{-\ln(\sqrt{s}/M_W^2)}^{+\ln\sqrt{s}/M_W^2}dY_W\sum_{q\bar{q}}q(x_1)q(x_2)$$

- So that:

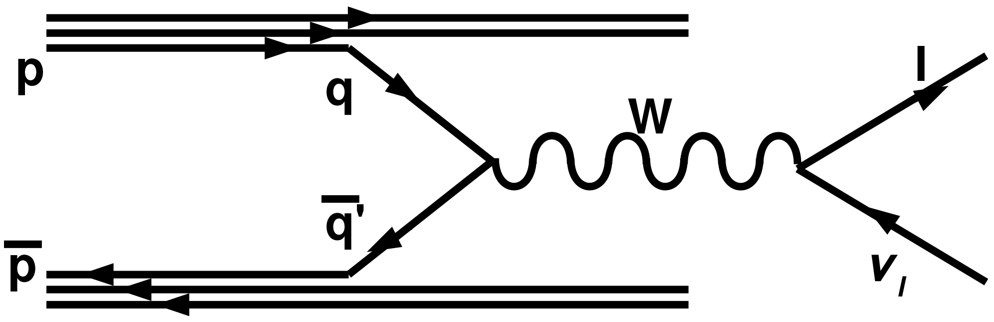
$$\sigma(p\bar{p}\to W+X)\simeq\frac{2\pi G_F}{N_C\sqrt{2}}\ln\frac{s}{M_W^2}$$

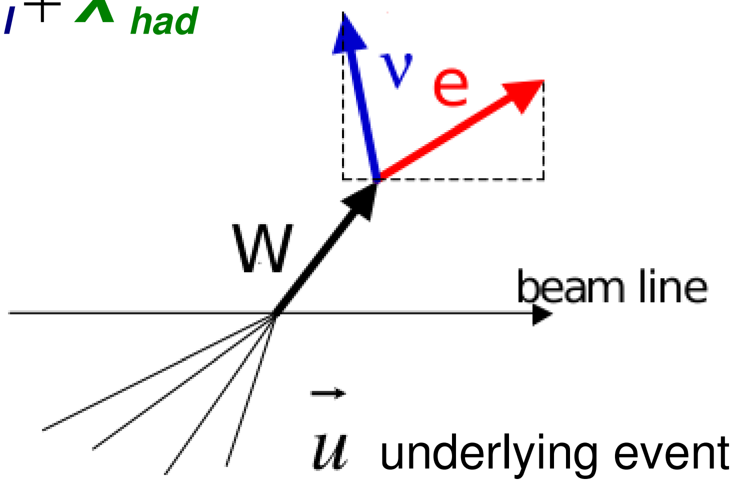
- The total cross section for W production at the proton anti-proton collider is increasing logarithmically in the proton anti-pron center of mass energy because of the PDFs

W production at hadron colliders

- W decay at $p\bar{p}$ colliders is searched via the golden mode (leptonic):

$$p\,\bar{p} \rightarrow W^{\pm} + X_{had} \rightarrow l^{\pm}\,\nu_l + X_{had}$$





- 4-momentum conservation at W decay vertex yields:

$$p\,\bar{p} \rightarrow W^{\pm} + X_{had} \rightarrow l^{\pm}\,\nu_l + X_{had}$$

$$M_W^2 \gg m_l^2 \rightarrow m_l^2 \simeq 0 \rightarrow E_l \simeq |\vec{p}_l|$$

$$M_W^2 = E_l^2 + E_\nu^2 + 2\,E_l\,E_\nu - |\vec{p}_l|^2 - |\vec{p}_\nu|^2 - 2\,\vec{p}_l \cdot \vec{p}_\nu$$

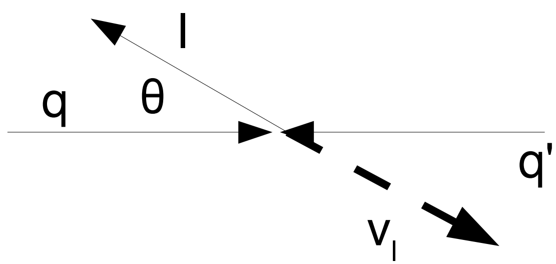
$$= m_l^2 + m_\nu^2 + 2\,E_l\,E_\nu - 2\,\vec{p}_l \cdot \vec{p}_\nu \simeq 2\,E_l\,E_\nu\,(1 - \cos(\Delta\theta_{l\nu}))$$

- In the transverse (rφ) plane:

$$M_W^T \simeq \sqrt{E_l^T\,E_\nu^T\,(1 - \cos(\Delta\phi_{l\nu}))}$$

W mass measurement kinematics

The lepton prduced by the W decay has in the qq' center of mass fram, transverse momentum:



$$p_t^e \simeq \frac{M_W}{2} \sin\theta$$

Cross-section can be written in terms of the p_T of the lepton:

$$\frac{d\sigma}{dp_t} = \frac{d\sigma}{d\cos\theta^*} \cdot \frac{d\cos\theta^*}{dp_t} \quad \Rightarrow \quad \frac{d\sigma}{dp_t} = \frac{d\sigma}{d\cos\theta^*} \cdot \frac{2p_t}{M_w} \boxed{\frac{1}{\sqrt{(M_W/2)^2 - p_t^2}}}$$

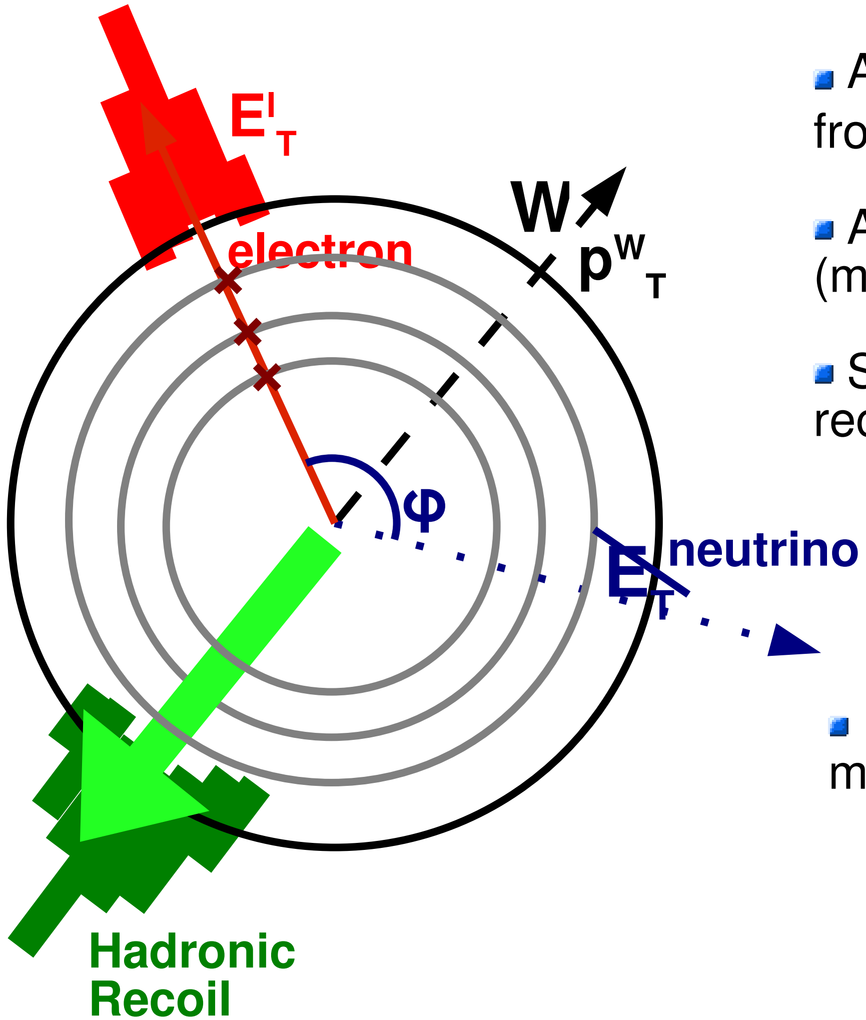
Term giving rise to the jacobian peak

It's convenient to re-express the differential cross section in terms of a the transverse mass of the W:

$$\frac{d\sigma}{dM_{WT}^2} = \frac{dp_T^2}{dM_{TW}^2} \frac{d\cos\theta^*}{dp_T^2} \frac{d\sigma}{d\cos\theta^*} \quad \Rightarrow \quad \boxed{\frac{d\sigma}{dM_{TW}^2} = \underbrace{\frac{2 - M_{TW}^2}{\sqrt{1 - M_{TW}^2}}}_{Jacobian} \frac{\pi G_F}{24\sqrt{2} N_C} \sum_{q\bar{q}'} |V_{q\bar{q}'}|^2 \int dx_1 dx_2 q(x_1) q(x_2)}$$

This is the quantity measured at the hadron collider

W Boson signature

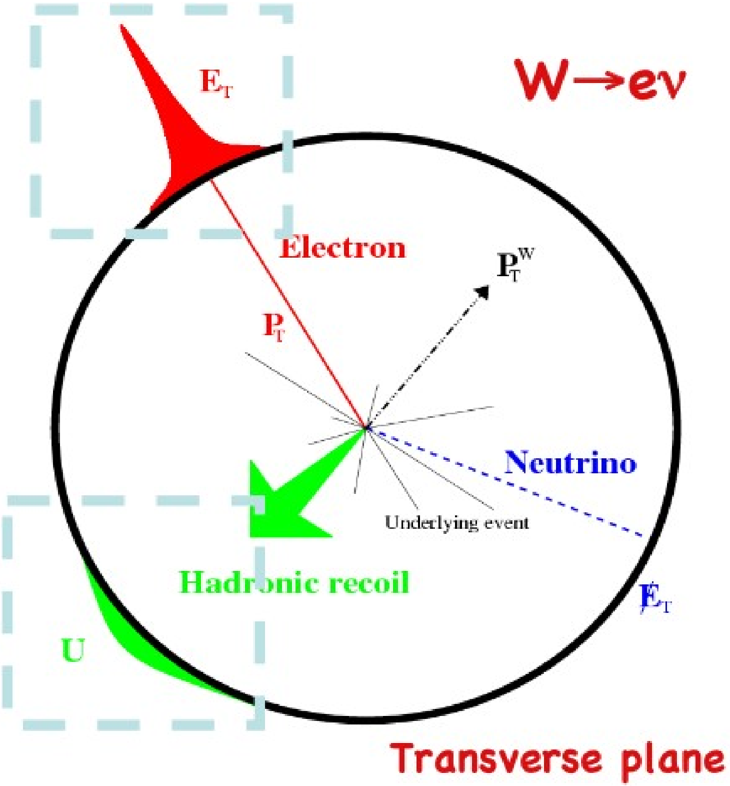


- Experimentally in a detector the W decay is seen as:
- A high p_T lepton (either a muon or an electron) coming from primary vertex
- A high missing energy in the transverse plane (meaning a neutrino escaping the detector)
- Some activity in the hadron calorimeter, due the recoiling hadronic mass from the primary scattering

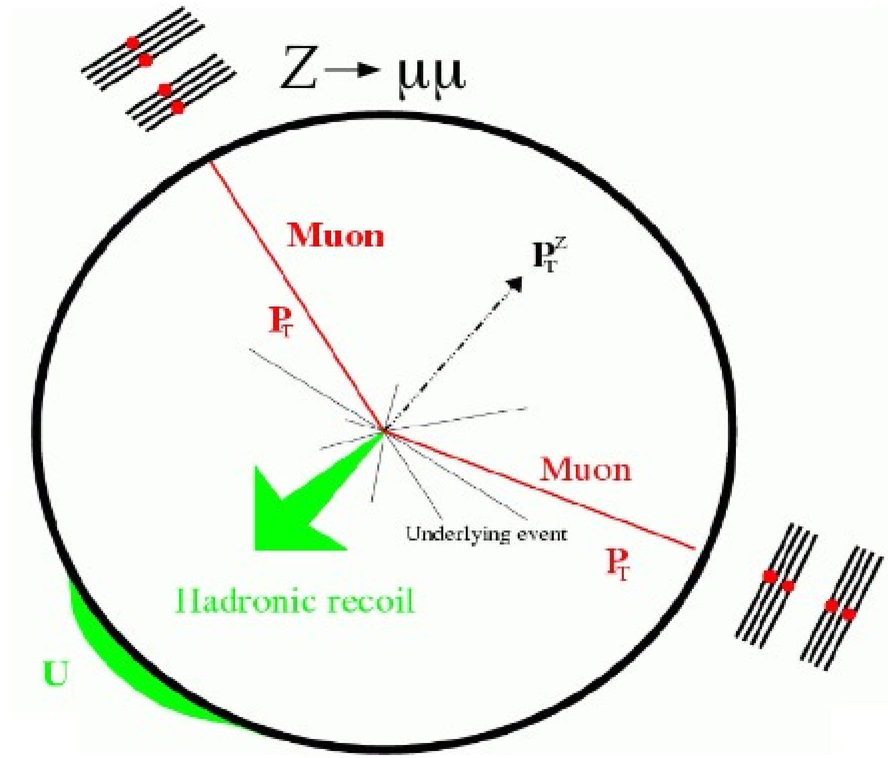
- The transverse missing energy due to energy-momentum conservation can be written as:

$$MET = -E_T^l - \sum_{i \in HAD} E_{T,i} = -E_T^l - u$$

Measurement strategy



- Calibrate $l^{\pm}$ track momentum with mass measurements of J/Ψ and Y decays into μ
- Calibrate calorimeter energy using track momentum of *e* from W decays
- Cross check with Z mass measurement, then add Z's as a calibration point

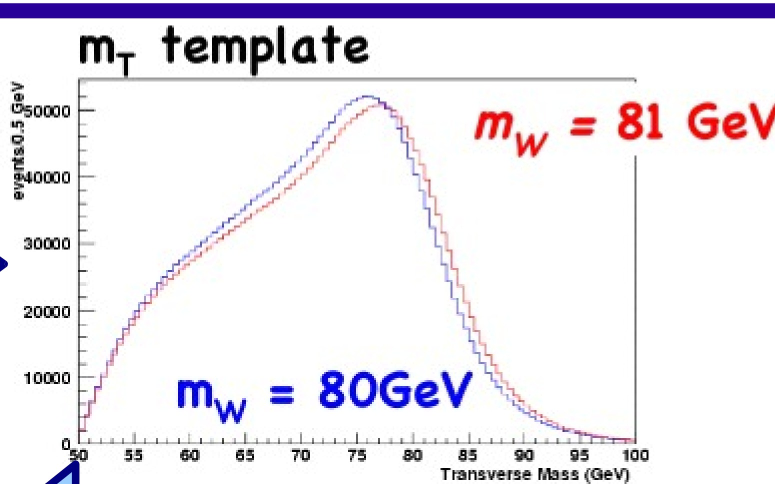


- Calibrate recoil measurement with Z decays into e,μ
- Cross-check with W recoil distributions
- Combine information into transverse mass:

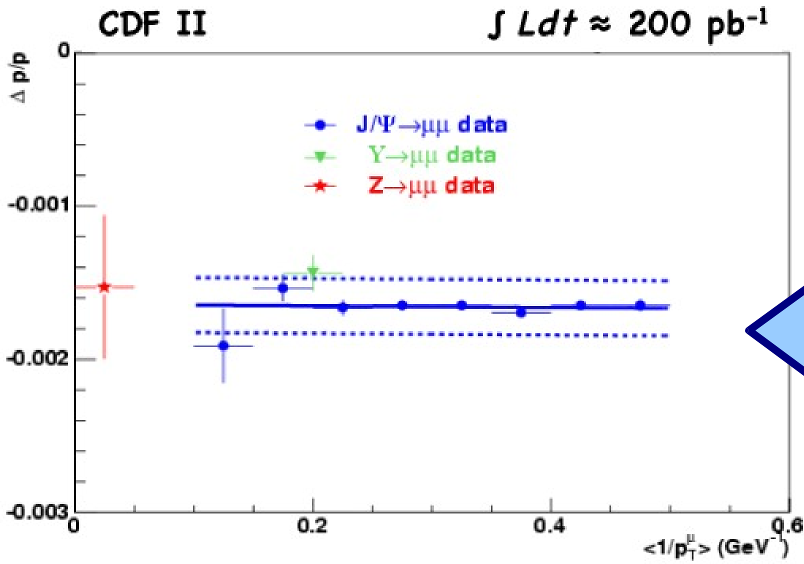
$$m_T^W = \sqrt{E_T MET \cdot (1 - \cos(\Delta \phi))}$$

Measurement strategy

W mass template fits to m^W_T, transverse lepton momentum/energy and $\not{E}_T$



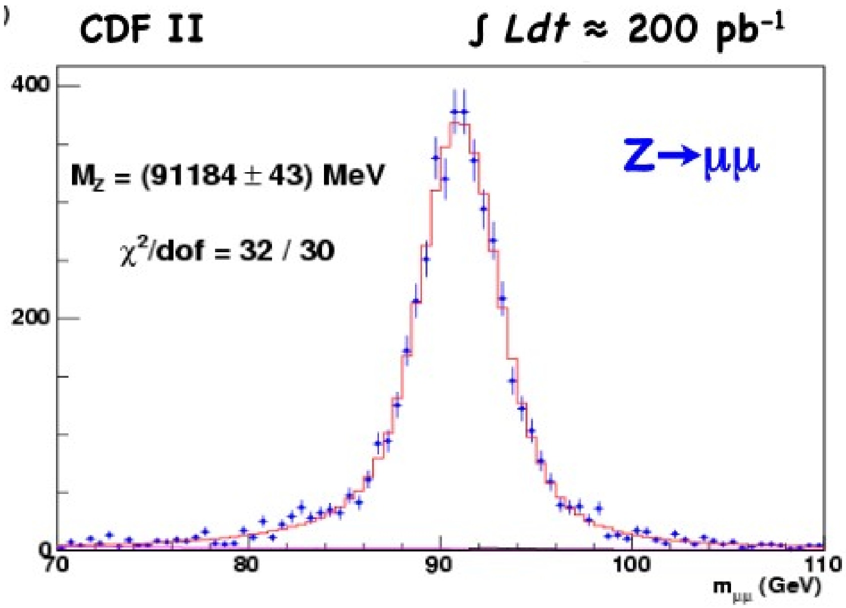
The Template fit needs:
- A **fast simulator of W/Z** production/decays
- With **calibrated detector simulation**
- contribution of **backgrounds** added to the templates

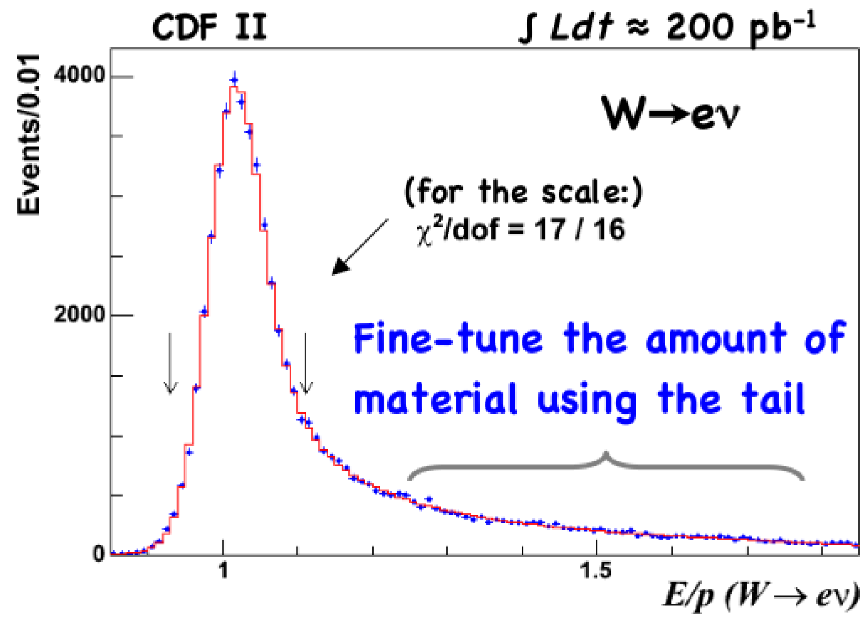


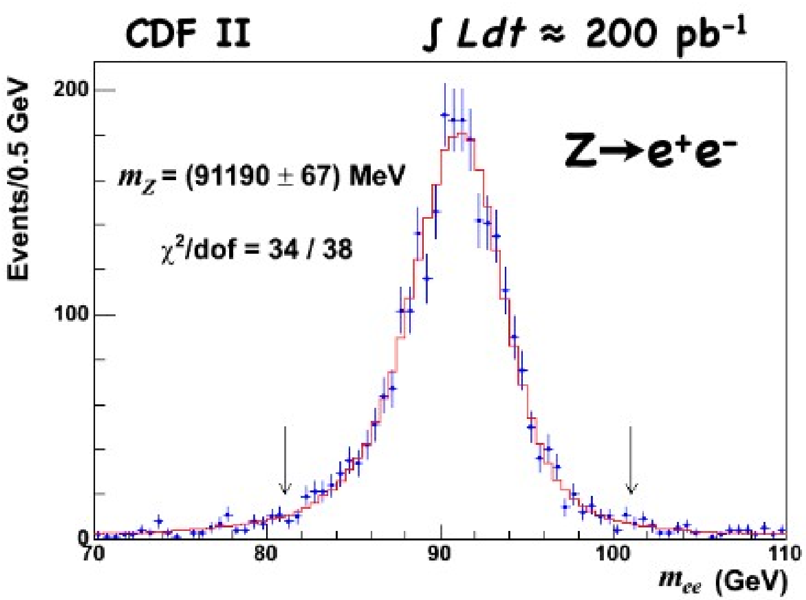
- PDFs, boson p_T, EWK corrections

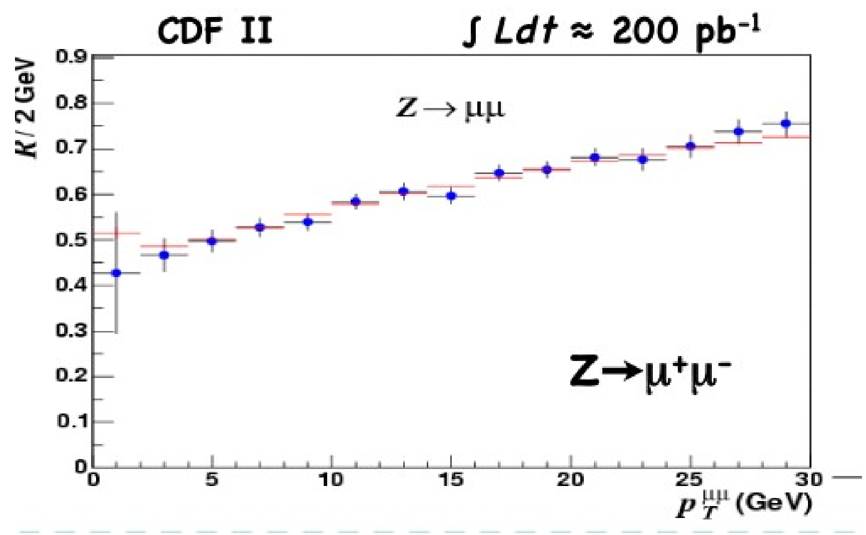
- Calibrate lepton track momentum with mass of J/Ψ and Υ(1S)
- Calibrate calorimeter energy using track momentum of electrons from W decay
- Calibrate recoil simulation with Z decays

Calibrations

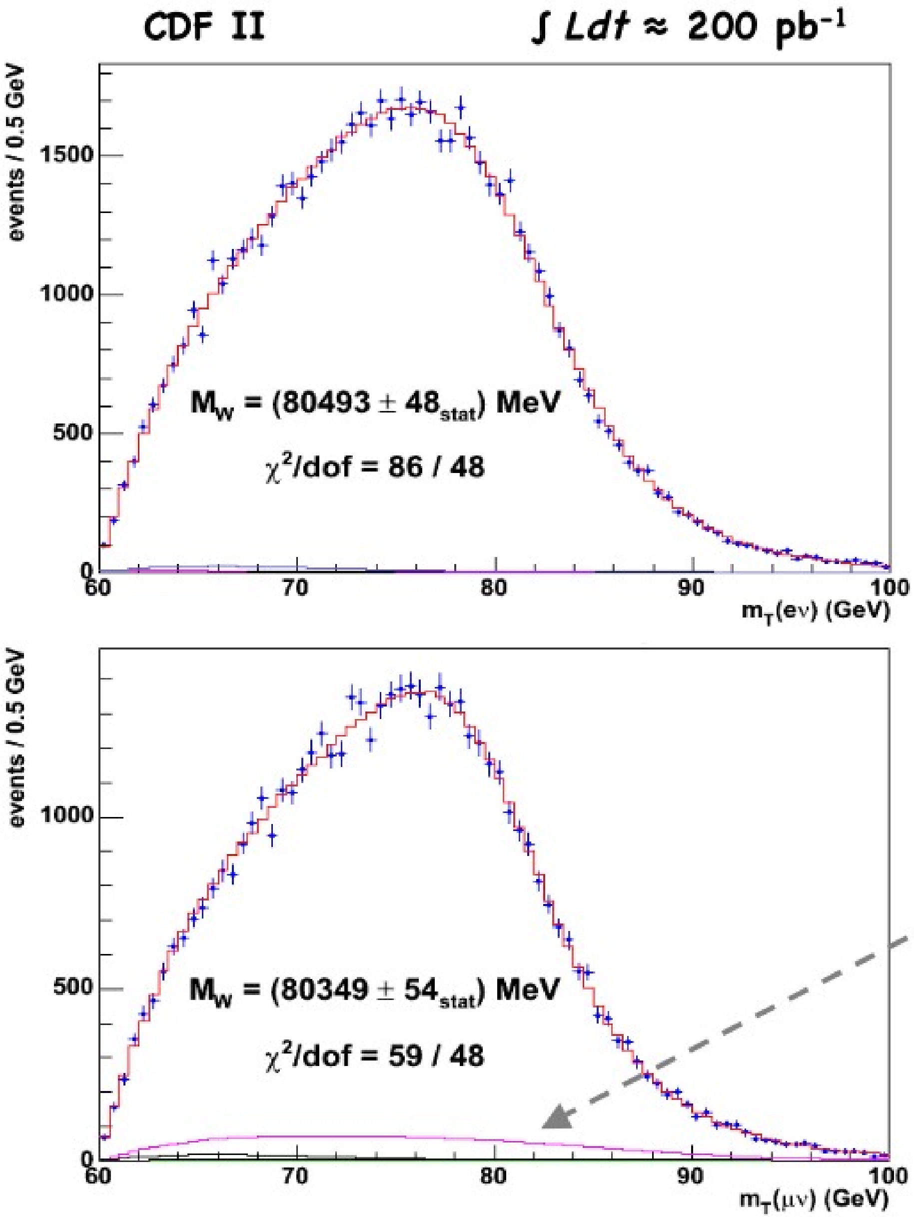








Results

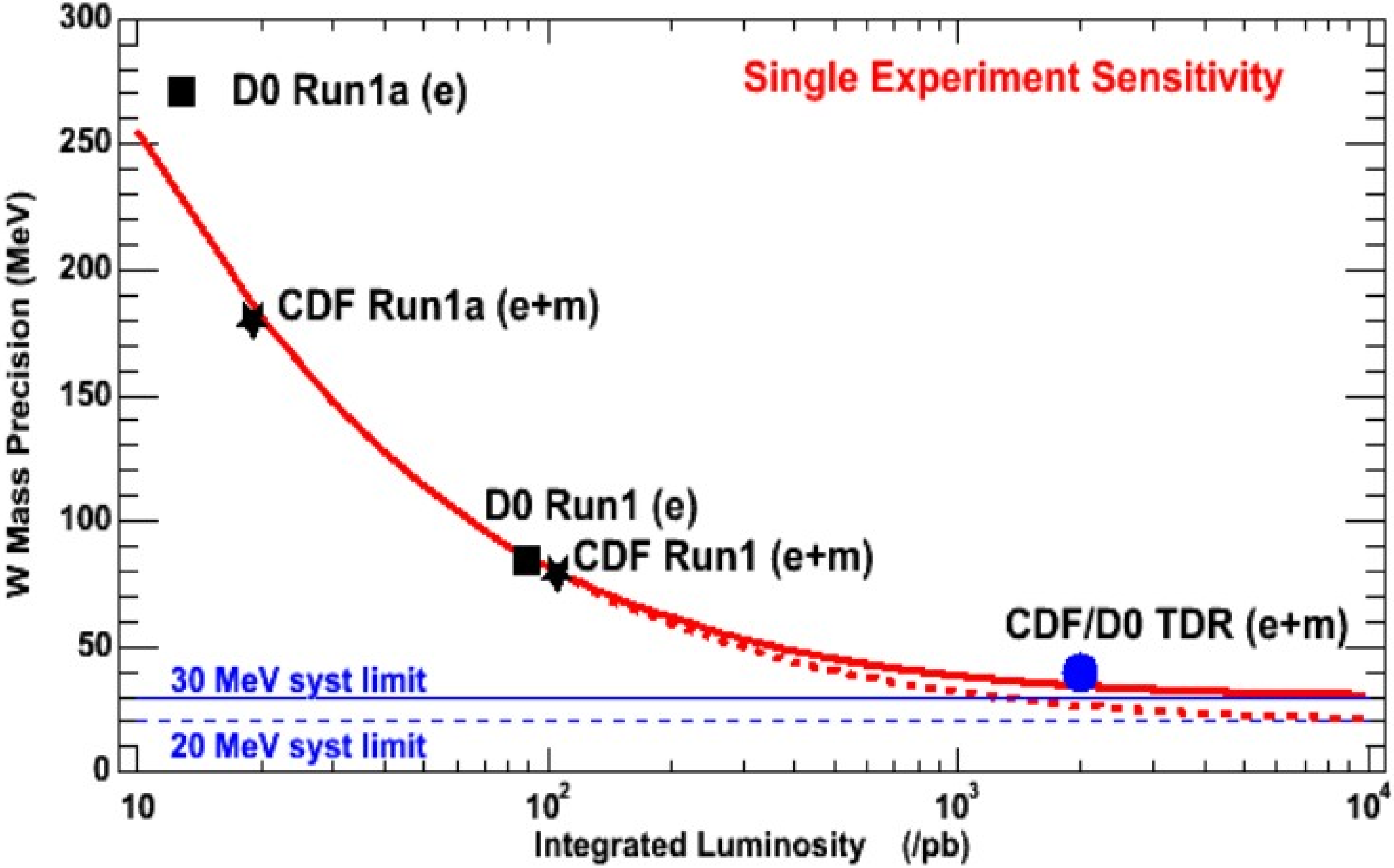


CDF II $\int Ldt \approx 200\ pb^{-1}$

m_T Uncertainty [MeV]	Electrons	Muons	Common
Lepton Scale	30	17	17
Lepton Resolution	9	3	0
Recoil Scale	9	9	9
Recoil Resolution	7	7	7
$u_{\parallel}$ Efficiency	3	1	0
Lepton Removal	8	5	5
Backgrounds	8	9	0
$p_T(W)$	3	3	3
PDF	11	11	11
QED	11	12	11
Total Systematic	39	27	26
Statistical	48	54	0
Total	62	60	26

Background contributions:
- simulated using MC W EWK backgrounds (Z, τ decays)

W mass precision



Quark Top

- All top quark properties (except its mass) are fixed in the Standard model:

Family	**3**
Charge	**+2/3 e**
Spin	**½**
Isospin	**½**

- just another Isospin + ½ quark (up tipe quark)

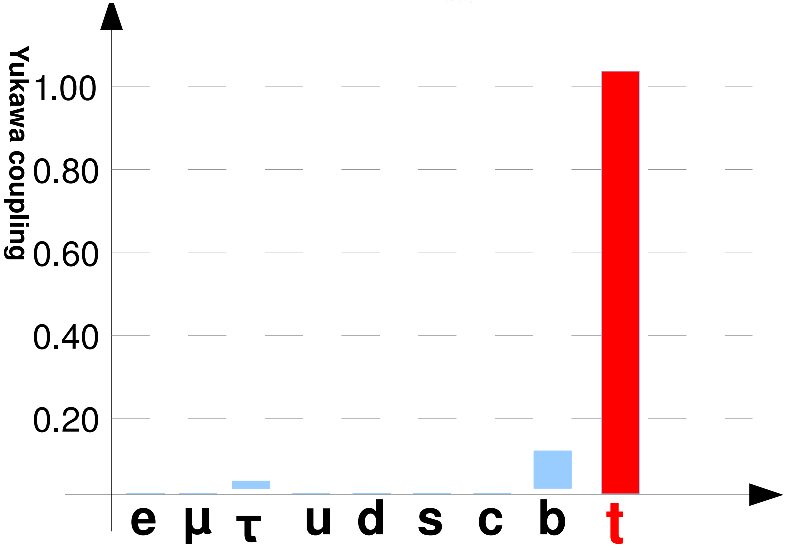
- In addiction Standard Model predicts: $|V_{tb}| \sim 1$ so top has a dominant decay $t \rightarrow W\, b$

$$L_{t\rightarrow Wb} = -\frac{g}{\sqrt{2}} W_{\mu} V_{tb} \bar{b} \gamma^{\mu} \left(\frac{1-\gamma^5}{2} \right) t + h.c.$$

- Most of the interest in quark top comes from the potential to find non standard effects

- Is the Yukawa coupling G_{top} to Higgs field a hint?

$$L_{mass}^{top} = -\frac{G_{top} v}{\sqrt{2}} \left(\overline{t_L} t_R + \overline{t_R} t_L \right)$$

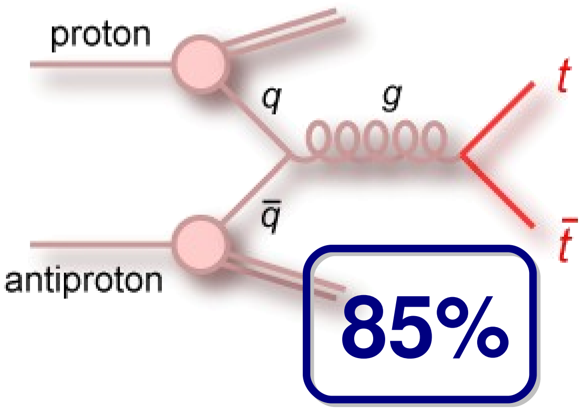


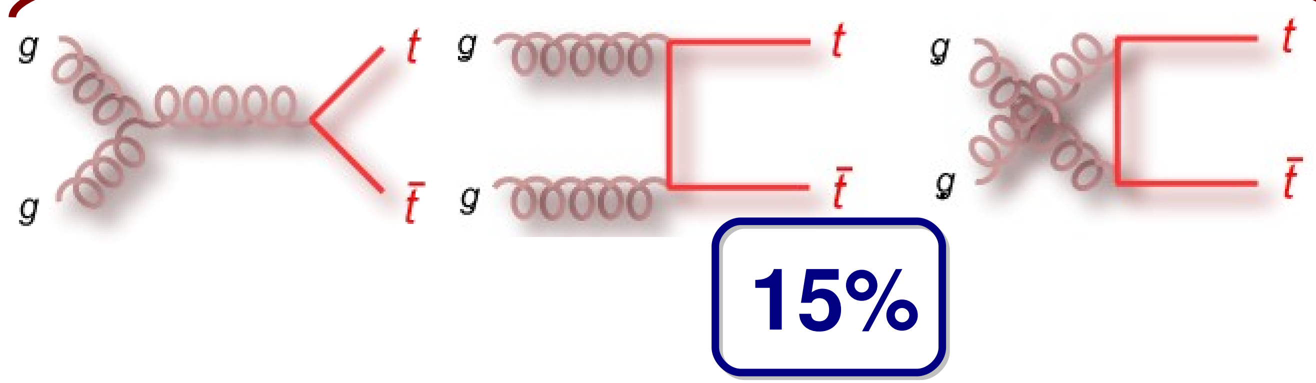
Top quark production / backgrounds

- Production mechanisms of top pairs at the hadron collider:

$q\bar{q}\rightarrow t\bar{t}$ $\qquad$ $gg\rightarrow t\bar{t}$

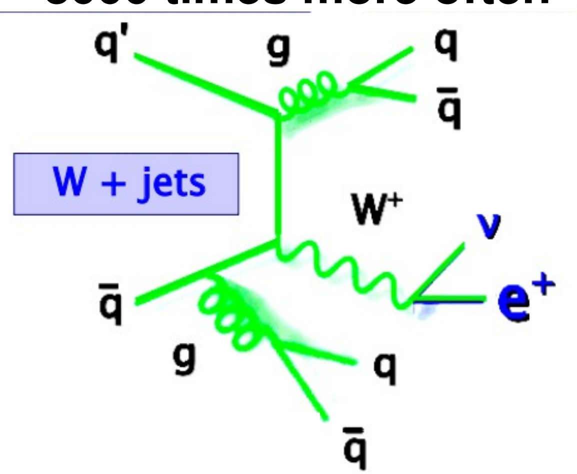
- quark-antiquark annihilation
- gluon gluon fusion (3 LO diagrams)



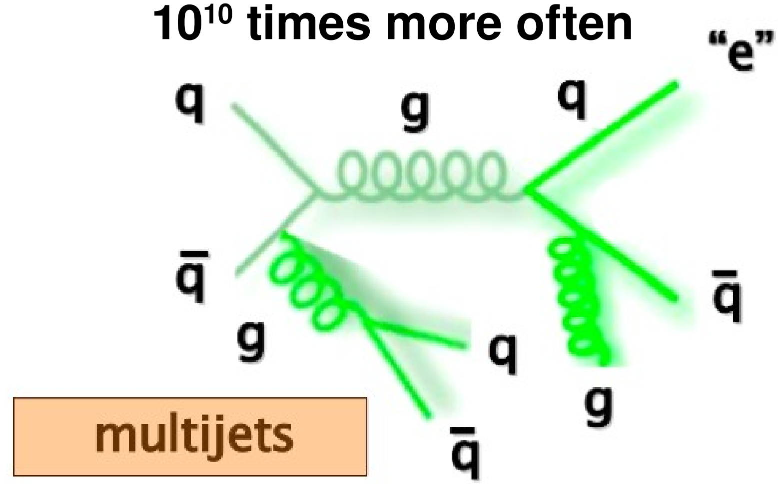


- Main backgrounds:
  - W+ jets
  - multijets (fake leptons)

3000 times more often

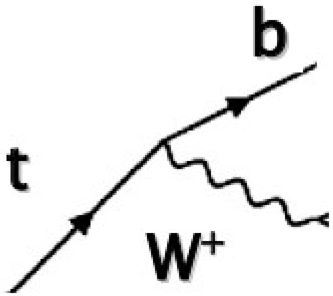


10^{10} times more often



Top quark decay

- Top quark decays weakly ~ 100% into a b quark and a W boson, since the CKM matrix element V_{tb} is ~ 1. Matrix element of the process is:



$$M_{fi} = \underbrace{\frac{g}{2\sqrt{(2)}}}_{\text{weak coupling}} \overbrace{V_{tb}}^{CKM} \underbrace{e_\mu^{W*}}_{W\ polarization} \overbrace{\overline{u}_b \gamma^\mu (1-\gamma_5) u_t}^{\text{quark left current}}$$

- So neglecting b quark mass we get:

- Using gauge invariance:

$$\sum_\lambda e_\mu^W(\lambda) e_\nu^{W*}(\lambda) = -g_{\mu\nu} + \frac{p_\mu^W p_\nu^W}{M_W^2}$$

$$\frac{1}{2}\sum_{spin} |M_{fi}|^2 = \frac{g^2}{16}\left[\sum_{\lambda=1}^{3} e_\mu^W(\lambda) e_\nu^{W*}(\lambda)\right] Tr\left[\not{p}_b \gamma^\mu \not{p}_t \gamma^\nu (1-\gamma^5)\right]$$

- And going into top rest frame:

$$\frac{1}{2}\sum_{spin} |M_{fi}|^2 = \frac{g^2}{4}(m_t^2 - M_W^2)\frac{2M_W^2 + m_t^2}{M_W^2}$$

Introducing phase space: $\Gamma = \frac{p^*}{8\pi M^2}|\overline{M_{fi}}|^2$

So that: $$\Gamma_{top} = \frac{g^2}{4\pi}\frac{(m_t^2 - M_W^2)(m_t^2 + 2M_W^2)}{16 M_W^2 m_t^3}$$

⇒

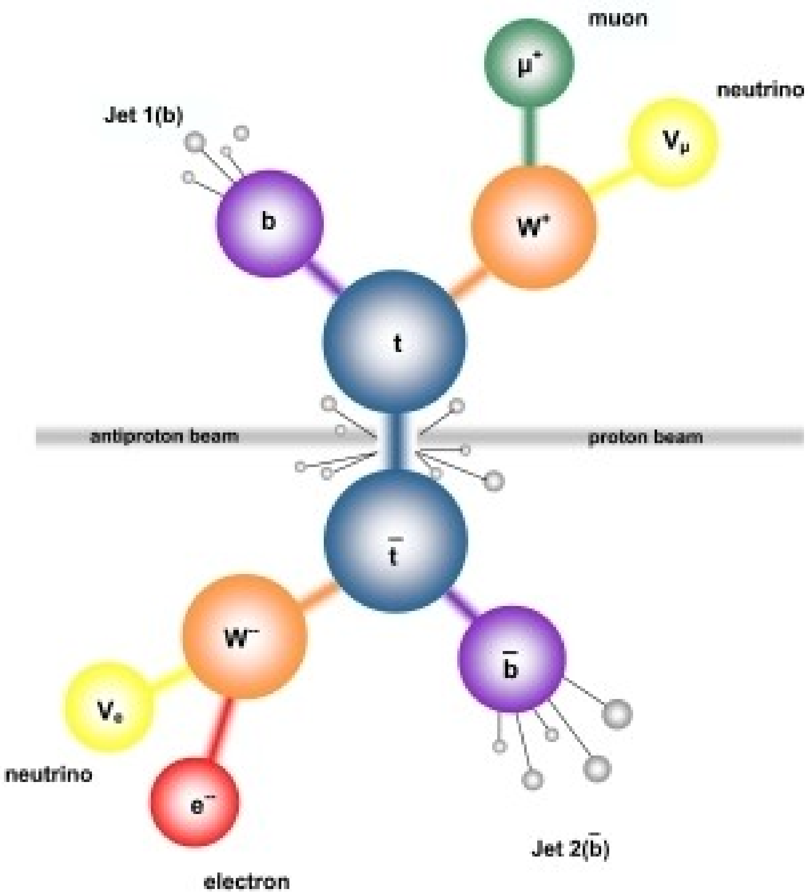
Top width is ~1.5 GeV, i.e. a large width

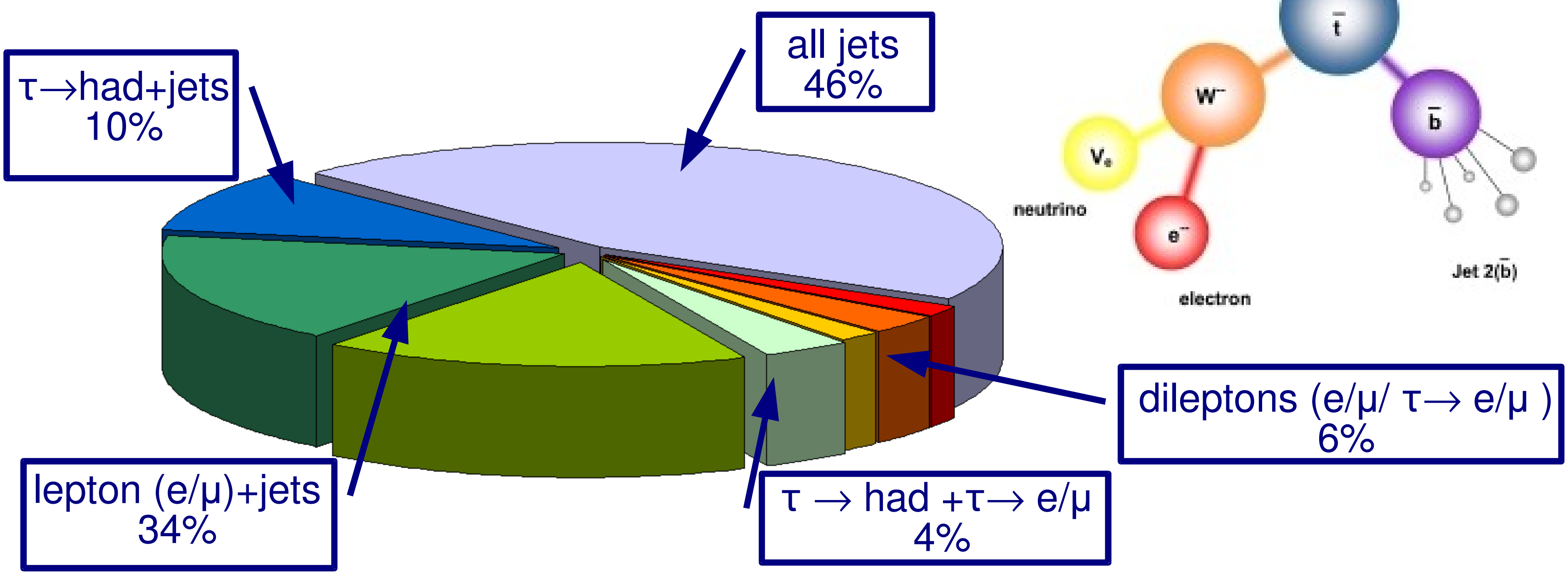
Top quark decays weakly via t → Wb before it can hadronize

➔ possibility to study a bare quark decay

Standard model top quark decay

- **t→Wb** with *B.R.* ~ 100%
  - **W→ qq'** with *B.R.* ~ 67%
  - **W→ $\ell\nu_\ell$** with *B.R.* ~ 11%
    - **τ → e/μ** with *B.R.* ~ 17 %
- Final state signatures for top-antitop pairs:
  - 2 b-tagged jets
  - leptons + missing energy
  - ...





Final state and event selection

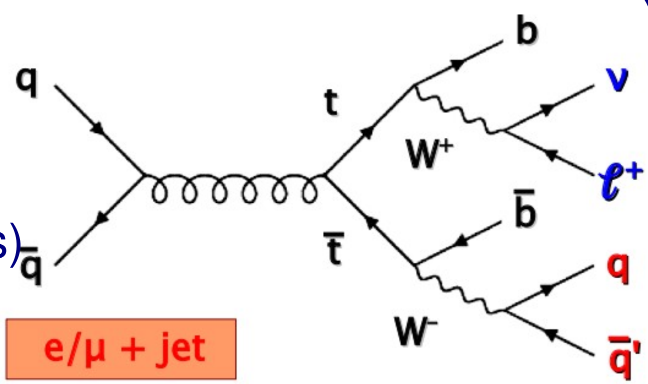
- Possible final states:

Mode	Br.(%)	
dilepton	5%	Clean but few signal. **Two ν's** in final state.
lepton+jets	30%	One ν in final state. Manageable bkgd.
all hadronic	44%	**Large background**.
τ + X	21%	τ-ID is challenging.

Event signature and Selection:

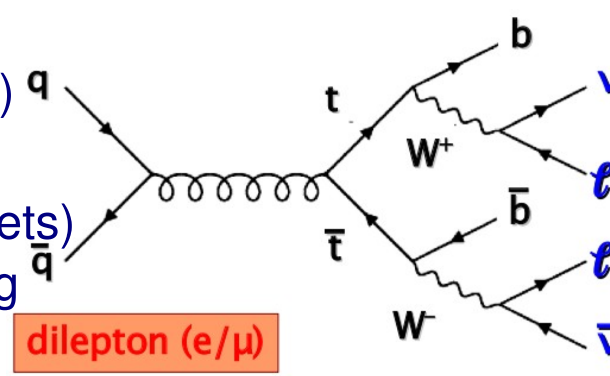
L+jets

- 1 lepton (e/μ)
- $\not{E}_T$
- 4 jets (2 *b*-jets)
- *b*-tagging.



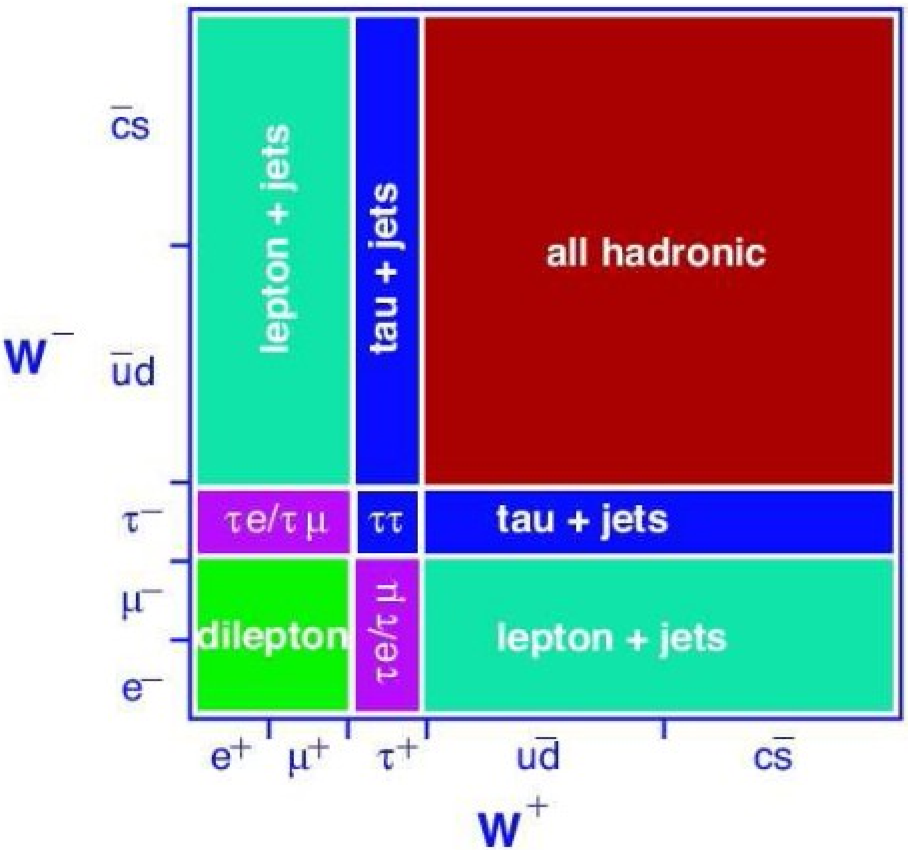
Dilepton

- 2lepton (e/μ)
- $\not{E}_T$
- 2 jets (2 *b*-jets)
- No *b*-tagging



Challenges in top measurements

Table of tt decay modes

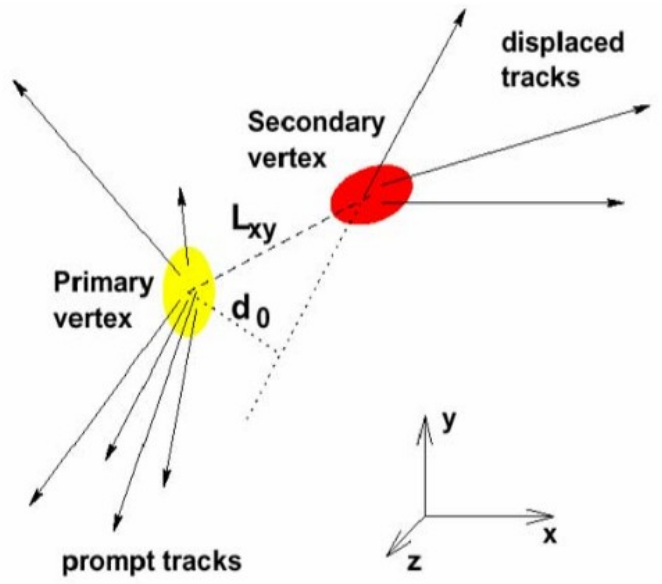


- **Combinatorics**: leading 4 jets combinations
  - 12 possible jet-parton assignments
  - 6 with 1 b-tag (b-tag helps)
  - 2 with 2 b-tags
- **Jet energy scale (JES)** and resolution
  - Note that two jets come from a decay of a particle with well measured mass – W-boson – built-in thermometer for jet energies
- **Gluon radiation**
  - Can lead to jet misassignment and gluon radiation changes kinematics of the final state partons
- **Backgrounds due to W+jets production**
  - many diagrams, especially for high jet multiplicities → uncertainties in modeling, especially for heavy flavor jets

NEEDS:

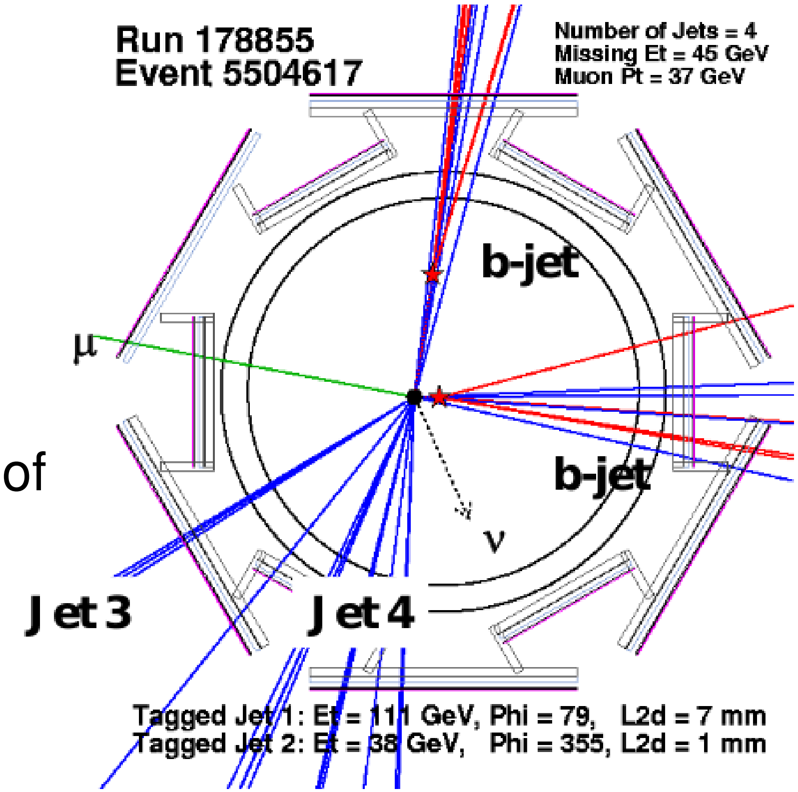
Good *b-tagging* and *jet energy scale* and resolution and *good algorithm to reconstruct M_{top}*

b-tagging



- b-quark has a long life-time: **<τ >=1.67 ps**
- for a 50 GeV jet this means it decays **L = βγc <τ> ~ few mm** far away from primary vertex
- This means one can tag the flavour of the jet.

lepton + jet:



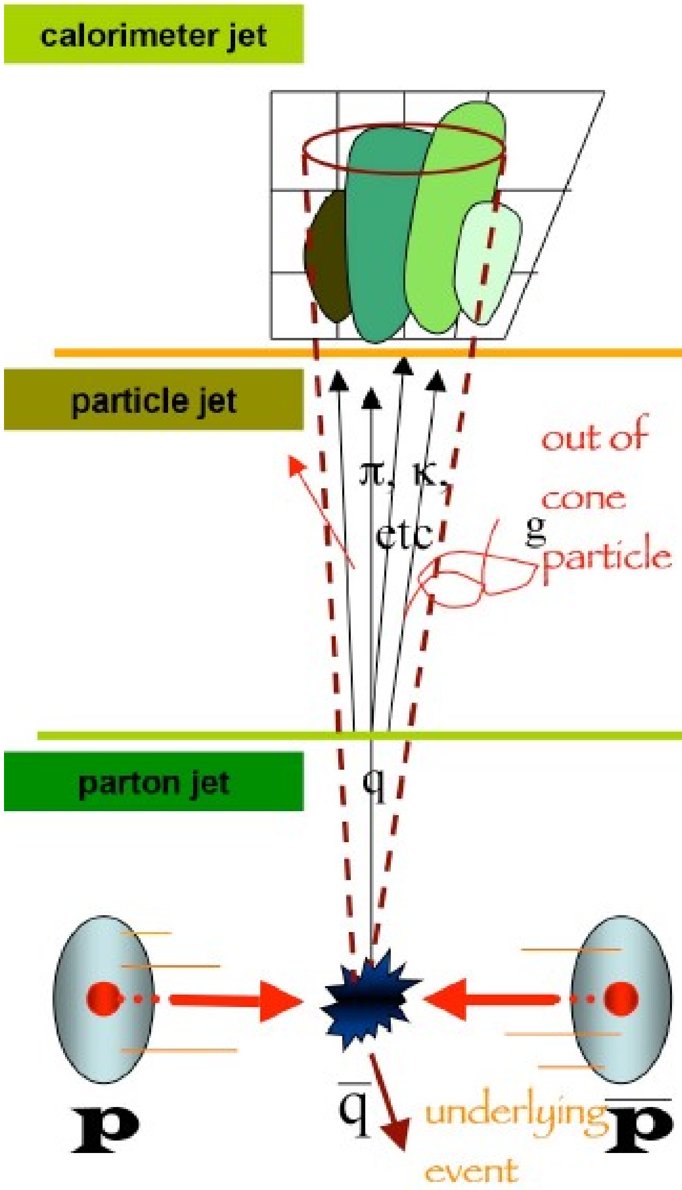
- Cut variables:
  - impact parameter = distance of closest approach from primary vertex = d_0
  - Decay length in transverse plane L_{xy}
  - Invariant mass of the tracks coming out of secondary vertex

$$\left(\sum_i^{n_{tracks}} E_i^2 - \sum_i^{n_{tracks}} \mathbf{p}_i^2\right) = m_b^2$$

  - Number of tracks associated to the secondary vertex

Jet Energy Scale

- Partons (quarks produced as a result of hard collision) realize themselves as jets seen by detectors
  - Due to strong interaction partons turn into parton jets
  - Each quark hardonizes into particles (mostly π's and K's)
  - Energy of these particles is absorbed by calorimeter
  - Clustered into calorimeter jet using cone algorithm
- Jet energy is not exactly equal to parton energy
  - Particles can get out of cone
  - Some energy due to underlying event (and detector noise) can get added
  - Detector response has its resolution



Top mass reconstruction methods

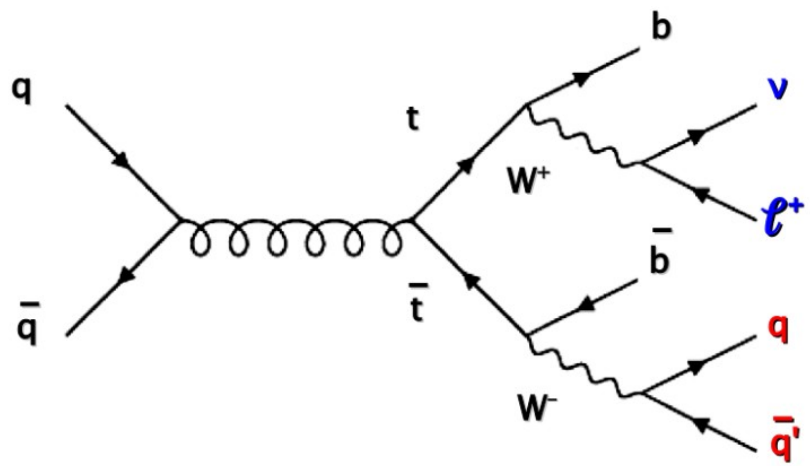
Template Method

- Reconstruct event-by-event a kinematic quantity M_{top} (***JES***).
- Describe dependence of M_{top} distribution on true top mass m_{top} using MC — Templates.
- Likelihood fit looks for m_{top} that describes data M_{top} distribution best (template fit).
- Pros:
  - less assumptions / robust measurement (takes care of detector resolution via Geant4 simulation)
  - simple algorithms
- Cons:
  - all events have the same weight

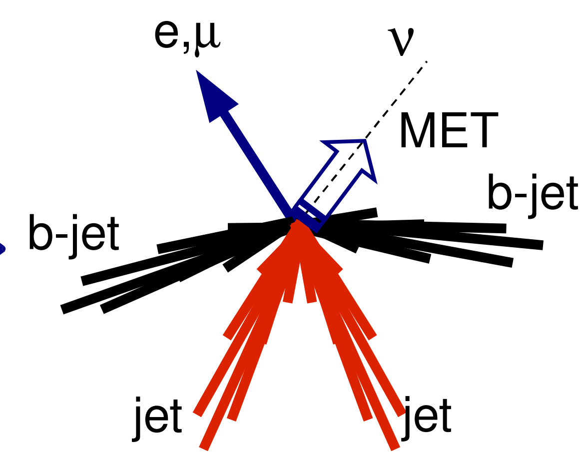
Matrix Element Method

- Calculate likelihood (probability) for m_{top} in each event by Matrix Element calculation.
- Multiply the likelihood over the candidate events.
- m_{top} determination by the joint likelihood maximum.
- Pros:
  - Better statistical power (event by event weighting)
- Cons:
  - needs long computing time and accurate modeling of theoretical input
  - less accurate resoultion parametrization

Template – top mass reconstruction



The actual process



What we expect in the detector

- **High p_T lepton**
- **MET**
- **2 b-tagged jets**
- **2 other jets**

- Constraint on Jet Energy scale (JES)
  - done via calibration of:

$$m_{jj}^2(JES) = (\sum_j E_j^2 - \sum_j \mathbf{p}_j^2) \simeq M_W^2$$

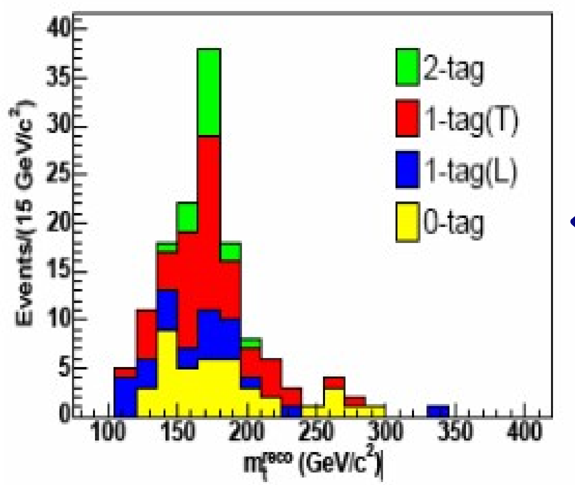
  - a systematical error is transformed into a statistical one

- **Combinatorics on parton-jet assignment**
- **Subdivide sample in to 1-tag, 2-tag, 0-tag events (b-tagging)**

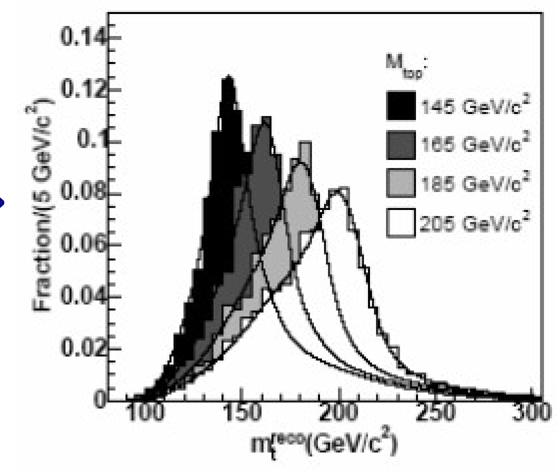
Template Method

Comparison between:

Distribution of reconstructed top mass M^{reco} as extracted from data



Distribution of top mass m_{top} from simulation (**templates**). **Background too is simulated**



- First extract event by event M^{reco} by means of a kinematical fit (input measured quantities) minimizing a χ^2:

$$\chi^2 = \chi^2 (p_T^{leptons}, p_T^{jets}, E_T^{U.E.}, M_{jj}, M_{l\nu})$$

- Then fit the reconstructed mass distribution with a template by means of a likelihood fit.
- The template which maximizes the likelihood $L(M_{Top})$ gives M_{Top}

Template method

- Minimize χ^2 to reconstruct event-by-event top mass.

$$\chi^2 = \sum_{i=l,jets} \frac{(\hat{p}^i_T - p^i_T)^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(\hat{UE}^j_T - UE^j_T)^2}{\sigma_i^2} + \frac{(m_{l\nu} - M_W)^2}{\Gamma_W^2} + \frac{(m_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(m_{bl\nu} - M_{TOP})^2}{\Gamma_T^2} + \frac{(m_{bjj} - M_{TOP})^2}{\Gamma_T^2}$$

variate lepton/jets momenta accordingly to detector resolution

Top mass is a free parameter

constrain to W mass

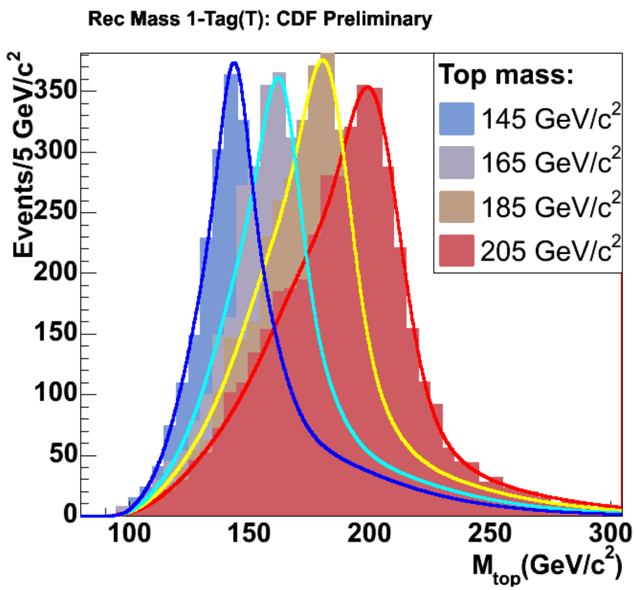
t and $\bar{t}$ have the same mass.

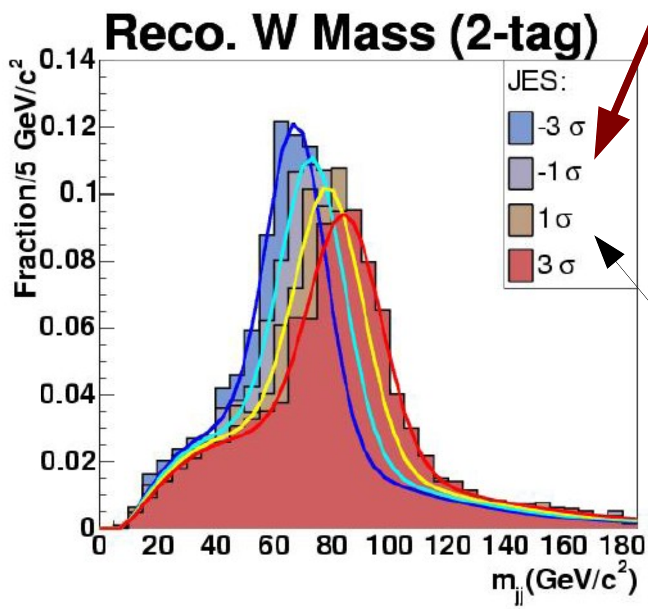
- 2 jets from W decay / 2 *b*-jets. $\rightarrow$1,2 jet-parton assignments.
- *B*-tagging helps reject wrong assignments besides reduces background.
- Subdivide candidate events into 0, 1, 2 tag.
- Choose assignment with smallest χ^2.
- Only events with $\chi^2 < 9$ are accepted

Likelihood fit and calibration

- After having detemined on event by event basis the reconstructed mass of top minimizing χ^2 a likelihood fit is performed to fit the distribution of M^{reco} with a template determined by Monte Carlo simulation

$$L(M_{top}) = \underbrace{L_{shape}^{m_{Top}^{reco}}(m_{top}^{reco}, m_{top}^{templ}; \Delta JES)}_{\text{likelihood of } m_{top} \text{ shape}} \times \underbrace{L_{shape}^{m_{jj}^{reco}}(m_{jj}^{reco}, m_{jj}^{templ}; \Delta JES)}_{\text{likelihood of } m_w \text{ shape}} \times \underbrace{L_{N_{evt}} \times L_{bkg}}_{\text{likelihood for background and yield}}$$





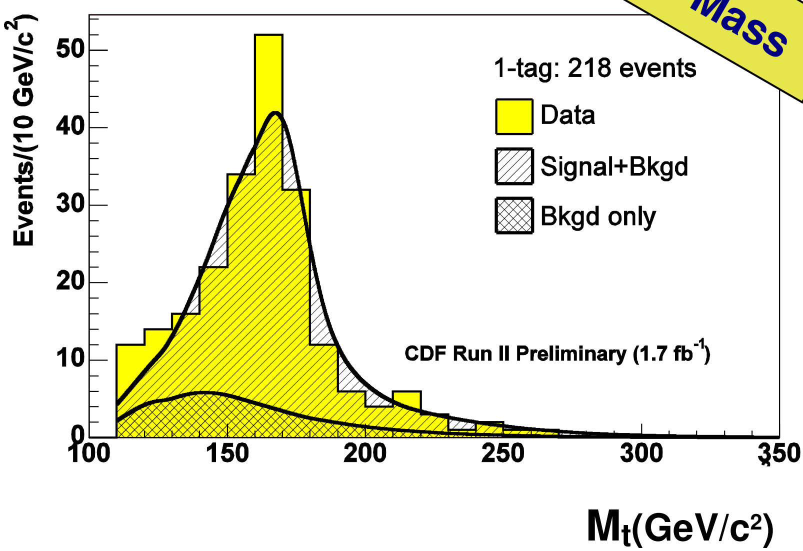
JES shifted by –3σ,-1σ,… of generic jet calibration

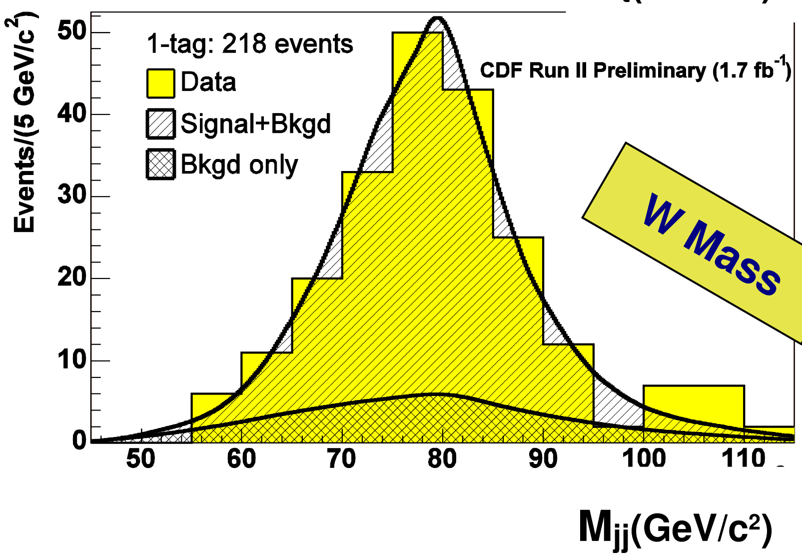
- The largest systematic uncertainty is the **Jet Energy Scale**. In order to minimize it from the kinematic variables is extracted $\mathbf{m_{jj}}$ (invariant mass of the couple of jets coming out of a W) and then fitted by a $\mathbf{m_{jj}}$ template
  - This distribution is constrained to peak at M_W which is a very well known quantity, and miscalibration is parametrized via **ΔJES**

Template method results

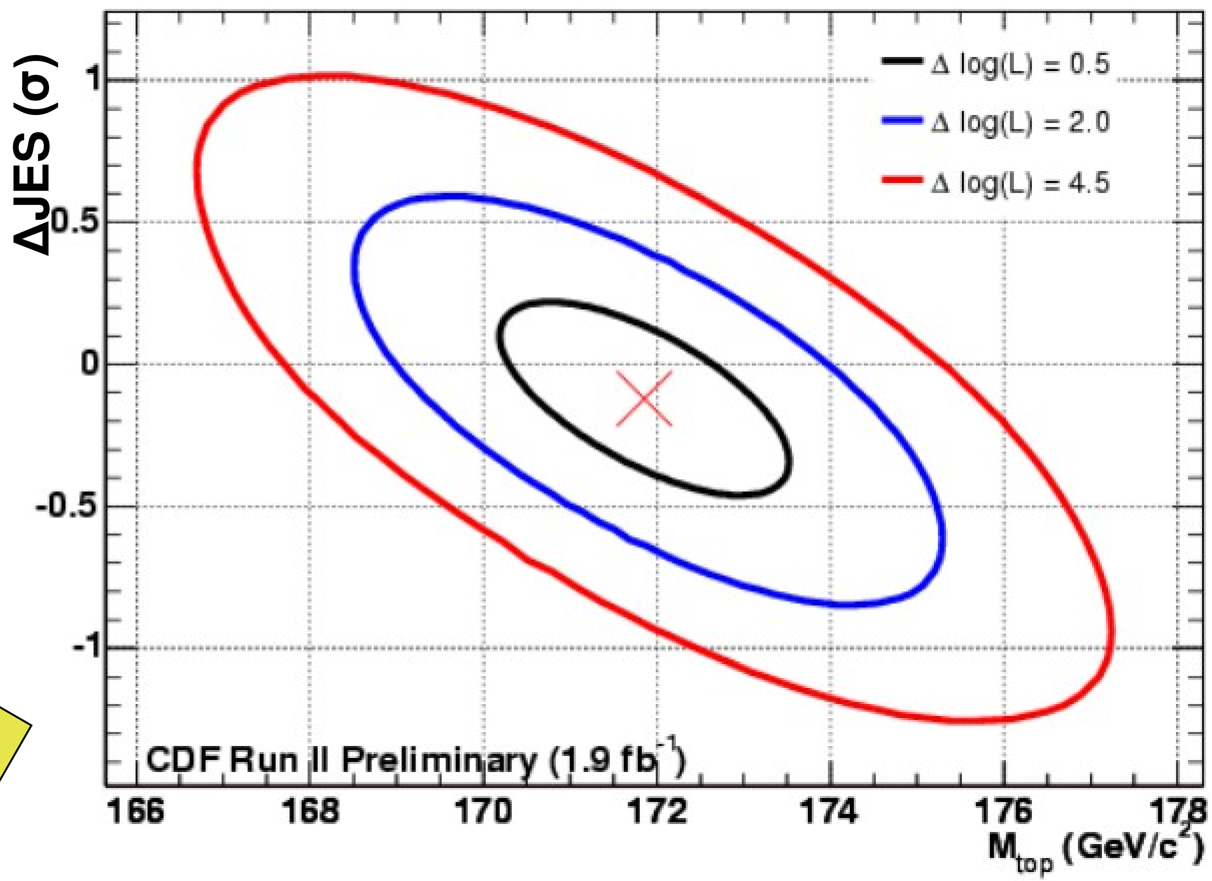
One gets this kind of distriutions for both m_{TOP} and m_{jj} fitted with templates:

Templates





Log likelihood contour plot in the m_{TOP} ΔJES space **(CDF L+jets channel)**



m_{top} = 171.9 ± 1.7 (stat+JES) ± 1.0 (syst) GeV

Matrix Element Method

- Calculate likelihood as a function of m_{top} according to Matrix Element for each event.

Sum over jet-parton combination.

Probability for P_T of tt system

$$P_{sig}(m_{top}, \mathbf{y}) = \sum \int dq_a \, dq_b \frac{2\pi^4}{Flux} PDF_{a/p}(q_a) \cdot PDF_{a/\bar{b}}(q_b) \cdot f(P_T)$$

$$\times \left| M_{t\bar{t}}(\mathbf{a}, \mathbf{b} \rightarrow \mathbf{x}; \mathbf{m}_{top}) \right|^2 \cdot W(\mathbf{x}, \mathbf{y}) \, d\,\mathbf{x}$$

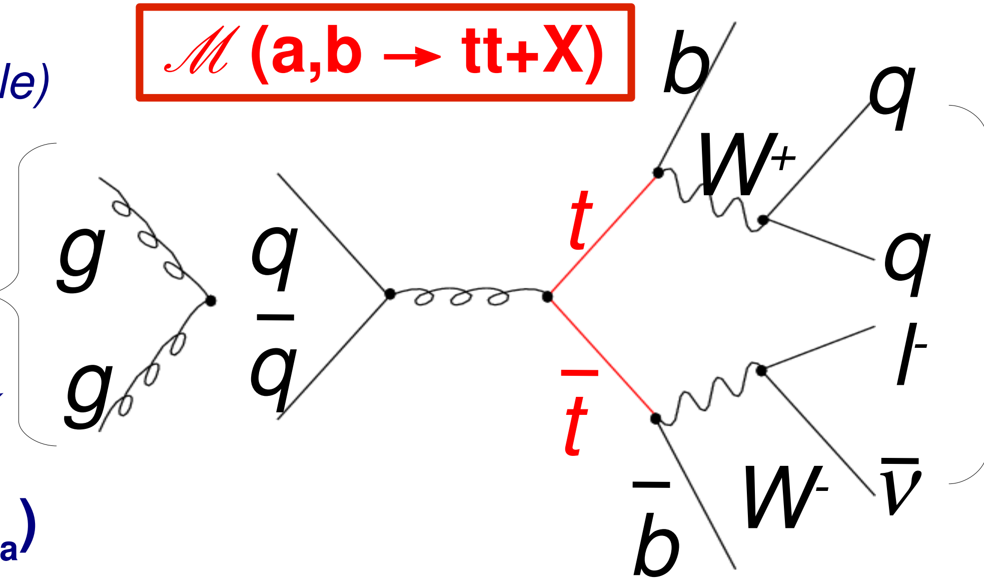
Signal LO Matrix element

Transfer Function:
e.g. parton $E_T \rightarrow$ jet E_T

- ***x***(Parton), ***y***(Observable)

$\mathscr{M}$ **(a,b → tt+X)**

Incoming partons:
a,b



- outgoing final state particles ***X***
- after hadronization and color reconnection → observable ***Y***

$PDF_{a/H}(z_a)$

W(x,y)

Matrix Element Method

- Calculate likelihood as a function of m_{top} according to Matrix Element for each event.

$$P_s(\mathbf{y}\,|\,M_{top}) = \frac{1}{\sigma(M_{top})}\frac{d\sigma(M_{top})}{d\,\mathbf{y}}$$

y = Measured quantities:
- lepton momenta
- jet angles
- ...

To be calculated with:
- LO Matrix element
  - lepton momenta, b quarks
  - neutrino momenta
- Measured quantities
  - lepton/jet momenta
  - lepton/jet angles
- Not measured quantities
  - Energy, momenta of neutinos

Transfer functions

Integrated out

Hypothesis:
- lepton moment are **well measured**
- the **4 jets all come from b quarks**
- jet angles are well measured
- total transverse momentum of system ~ 0

$$d\sigma = |\mathcal{M}|^2\, d\Phi(\mathbf{x})$$

$$P_s(\mathbf{y}\,|\,M_{top}) = \frac{1}{\sigma}\sum\int d^n\sigma(\mathbf{x},M_{top})\,dq_1\,dq_2\,f(q_1)\,f(q_2)\,W(\mathbf{x},\mathbf{y})$$

- Transfer function: probability for a measured variable x to come from a parton level variable y (e.g. parton $E_T \rightarrow$ jet E_T)

Matrix Element Method

Matrix Element

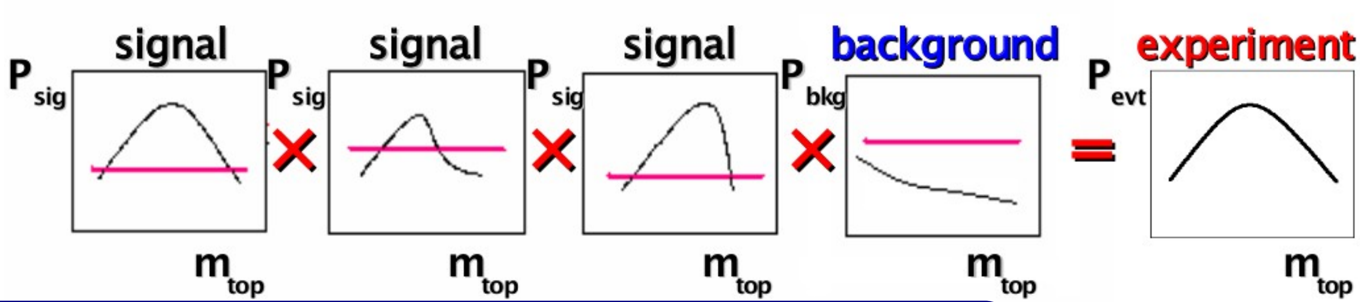
- Probability densities for every event as a function of top mass m_{top}
- Signal probability is built as:

$$P_{sig}(\mathbf{y}; M_{top}, JES) = \underbrace{Acc(\mathbf{y})}_{\substack{\text{Acceptance} \\ \text{Trigger}, \ldots}} \times \frac{1}{\sigma} \times \int \underbrace{d^n\sigma(\mathbf{x}, M_{top})}_{\substack{LO-\text{matrix element} \\ x\ \text{phase space}}} \underbrace{dq_1 dq_2 f(q_1) f(q_2)}_{PDF's} \underbrace{W(\mathbf{x}, \mathbf{y}, JES)}_{\substack{\text{Transfer function} \\ \text{probability to} \\ \text{measure } \mathbf{y} \text{ given } \mathbf{x}}}$$

- Event probability is calculated as:

$$\underbrace{P_{evt}(\mathbf{y} | M_{top})}_{\text{event probability}} = \overbrace{P_S(\mathbf{y} | M_{top})}^{\text{signal probability}} \underbrace{p_s}_{\text{weight}} + \overbrace{\sum_{i=1}^{n_{bkg}} P_{bkg,i}(\mathbf{y}) p_{bkg,i}}^{\text{bkg probability}} = P_S(\mathbf{y} | M_{top}) p_s + P_{bkg1}(\mathbf{y}) p_{bkg1} + P_{bkg2}(\mathbf{y}) p_{bkg2} + \ldots$$

- Then fit top mass from a maximum likelihood fit:

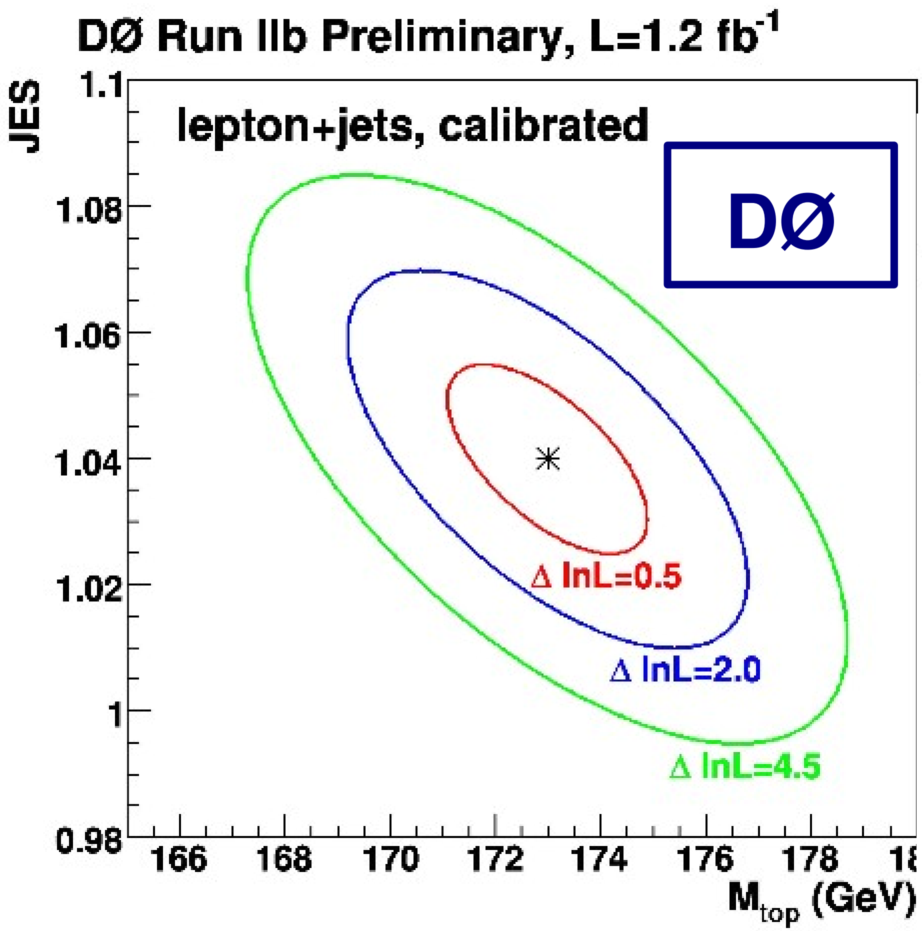


$$L(\mathbf{y}, M_{top}, JES, f_{top}) = \prod_{i=1}^{n_{evnets}} P_{evt,i}(\mathbf{y}, M_{top}, JES, f_{top})$$

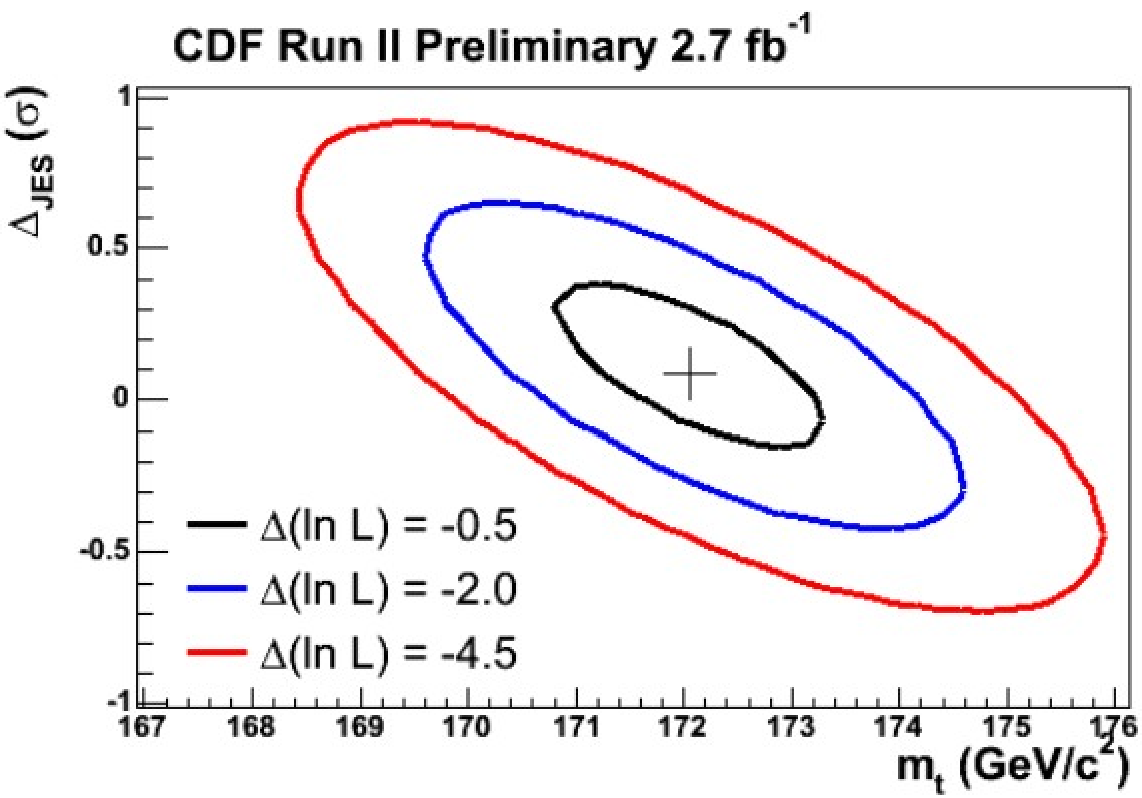
$\Rightarrow$ M_{top}, JES, f_{top}

Results for matrix element

Matrix Element



CDF

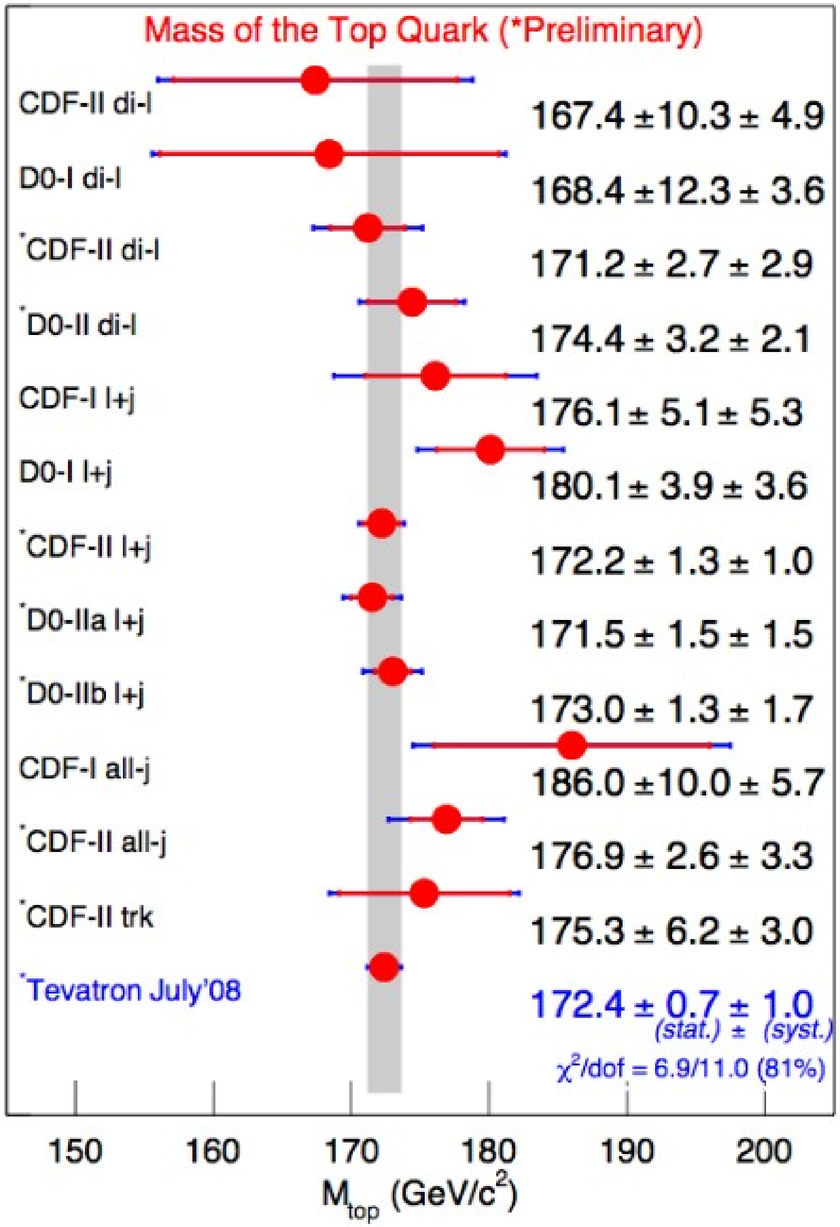


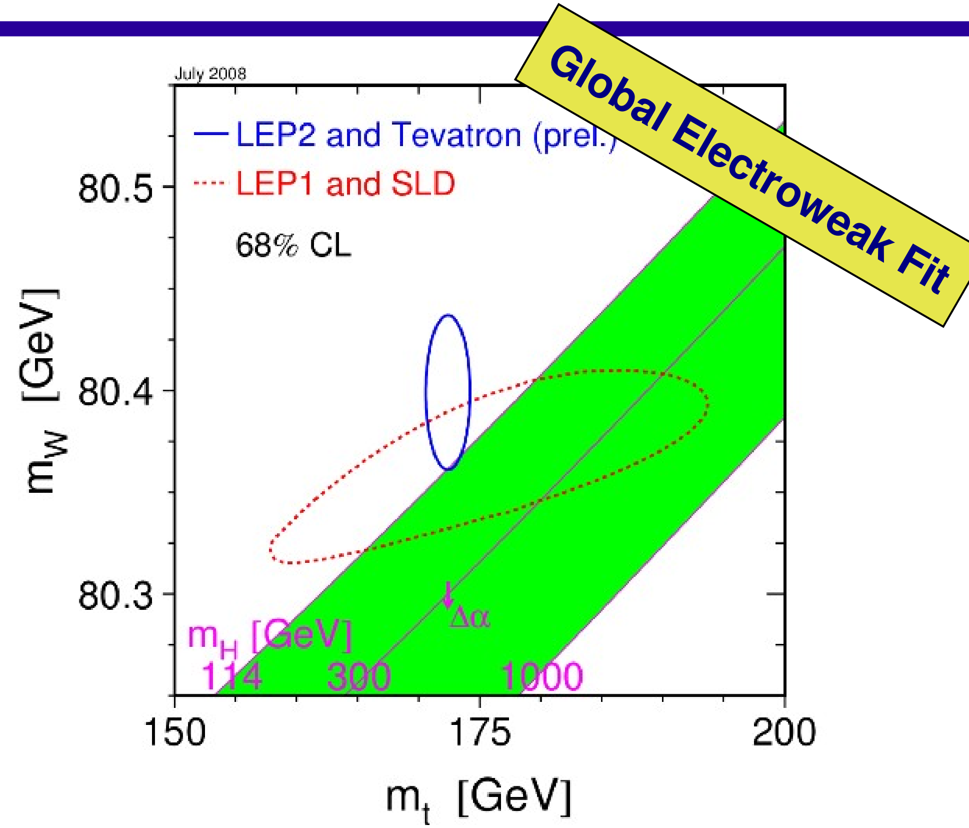
m_{top} = 172.2 ± 1.0 (stat) ± 1.4 (syst) GeV

m_{top} = 172.2 ± 1.0 (stat) ± 1.3 (syst) GeV

±1.0% **±1.0%**

Top quark mass results





- **M_{TOP}=172.7±1.2 GeV/c^2**
- Stat uncertainty: 0.7GeV/c^2
- Syst uncertainty: 1.0GeV/c^2

- Top Yukawa coupling to Higgs boson:
  $g_t = M_t\sqrt{2}$/v.e.v.= 0.993±0.017