Yrast structure of the neutron-rich $N = 31$ isotones $^{51}$Ca and $^{52}$Sc

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The low-lying yrast states in the $^{51}$Ca and $^{52}$Sc nuclei were investigated to obtain information on the evolution of the $p_{1/2}$ and $f_{5/2}$ neutron single-particle orbitals in neutron-rich nuclei near proton number $Z = 20$. Level structures associated with neutron excitations into these two orbitals and with proton excitations across the $Z = 20$ shell gap were identified. Shell-model calculations with the recently proposed GXPF1A interaction account reasonably well for the $fp$-shell states. The energy separation between the $vp_{1/2}$ and $vf_{5/2}$ orbitals in the Ca isotopes appears to be overestimated by the GXPF1A Hamiltonian.

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I. INTRODUCTION

The properties of neutron-rich nuclei above doubly magic $^{48}$Ca have recently attracted much attention due to the presence of an $N = 32$ subshell closure inferred from a series of experimental findings in the isotones $^{52}$Ca, $^{54}$Ti, and $^{56}$Cr. In $^{56}$Cr, a rise in energy of the first excited $2^+$ state with respect to the location of this level in the neighboring even Cr isotopes was observed by Prisciandaro et al. [1]. In addition, the analysis of the energy ratios for the first $4^+$ and $2^+$ states along the even Cr isotopic chain exhibited a pronounced minimum for $^{56}$Cr [2], as might be expected in the presence of a significant shell gap. In $^{54}$Ti, the energy spacing between the first three yrast excitations, i.e., the $2^+$, $4^+$, and $6^+$ members of the two valence proton $f_{7/2}$ multiplet, as well as the magnitude of the energy gap between the $6^+$ state and the higher-lying yrast levels, were found to be similar to those observed in the semimagic nucleus $^{50}$Ti [3]. The $B(E2; 0^+ \rightarrow 2^+)$ transition probability in $^{54}$Ti was subsequently measured to be of the order of only a few single-particle units, as is also the case for $^{50}$Ti. This value was also shown to be significantly smaller than the same quantity in the even neighbors $^{52}$Ti and $^{56}$Ti [4]. Finally, a two-proton knock-out study [5] and a recent $^{50}$K $\beta$-decay measurement [6] confirmed the location of the first excited $2^+$ state in $^{52}$Ca at the relatively high energy of 2564 keV, i.e., at the energy tentatively proposed by Huck et al. in 1985 [7].
p_{1/2} orbital, i.e., at N = 34. However, the issue did not receive much attention at the time, as the structure of very neutron-rich nuclei with $Z \sim 20$ appeared to be out of reach.

In recent years, the possible presence of this $N = 34$ magic gap in Ca isotopes and its rapid erosion at larger Z values, arising from the already mentioned strong proton $\pi f_{5/2}$-neutron $\nu f_{5/2}$ monopole interaction, have been pointed out in Ref. [10]. This picture got strong support when shell-model calculations with the newly developed GXPF1 interaction [11] clearly indicated that an $N = 34$ energy gap should appear in nuclei with $Z \sim 22$, i.e., already in the Ti isotopes. At about the same time, experimental progress was such that first data on the yrast structure of $^{56}{\text{Ti}}$ became available [12–14]: these simultaneously, experimental progress was such that first data on the yrast structure of $^{56}{\text{Ti}}$ became available [12–14]: these were found to be inconsistent with the presence of the predicted subshell closure. These findings in turn led to speculations that the effect of adding two protons to calcium around $N = 34$ might have been underestimated by the GXPF1 interaction and that, in reality, the two $f_{7/2}$ protons cause a significant lowering of the $f_{5/2}$ orbital, bringing it rather close in energy to the $p_{1/2}$ state. In Ca nuclei, however, the $(v p_{1/2}, v p_{1/2})-v f_{5/2}$ splitting may still be sufficient to produce a subshell closure at $N = 34$. To verify this hypothesis, the magnitude of the energy separation between the $f_{5/2}$ and $p_{1/2}$ orbitals in neutron-rich Ca isotopes needs to be derived from experimental data, although one should be aware that this task represents a significant challenge as the states involving the $f_{5/2}$ neutron in isotopes such as $^{31,33}{\text{Ca}}$ are rather difficult to reach.

Although the discussion presented above is couched in terms of the shell model and the associated effective interactions, it is worth pointing out that the underlying issues are of broader importance. Recently, changes in shell structure in this region have also been investigated theoretically in the context of density functional theory. Dobaczewski [15], on the one hand, and Rodrigues and Egidio [16], on the other, have examined possible $N = 32$ and $N = 34$ shell closures in the very Ca, Ti, and Cr nuclei under consideration here by exploring the role of tensor forces in the context of the shell model and the associated effective interactions. It is worth pointing out that the underlying issues are of broader importance.

In earlier work by some members of the present collaboration, deep-inelastic reactions occurring during collisions of a $^{48}{\text{Ca}}$ beam with $^{208}{\text{Pb}}$ and $^{238}{\text{U}}$ targets, at energies roughly 20% above the Coulomb barrier were successfully used to identify yrast sequences in $^{53–56}{\text{Ti}}$ [3,14,17]. The production of neutron-rich species in such processes is made possible by a tendency toward $N/Z$ equilibration of the dinuclear system formed during the collisions [18–21]. In the reactions under investigation, the light colliding partner, $^{48}{\text{Ca}}$, had lower $N/Z$ ratio than either of the two targets and, as a result, the production of nuclei with larger neutron excess than the projectile was favored. In the case of the titanium products, yields sufficient for detailed spectroscopic studies extended in neutron number up to $^{50}{\text{Ti}}$ for the $^{48}{\text{Ca}} + ^{238}{\text{U}}$ reaction, aided in no small measure by the detection sensitivity of the Gammasphere spectrometer [22].

From the observed production yields for the $^{48–56}{\text{Ti}}$ isotopes it became clear that, particularly in reactions on the $^{238}{\text{U}}$ target, Sc isotopes with masses up to at least 52 and Ca isotopes with masses up to 51 or higher had to be present in the data set with cross sections sufficient to examine in detail the coincidence relationships required to establish significant level schemes. This conclusion was supported further by the fact that coincidence events were observed between known $\gamma$ rays emitted in the $\beta$ decay of $^{51}{\text{Ca}}$ and $^{52}{\text{Sc}}$. Firm identification of $\gamma$ rays in the $^{51}{\text{Ca}}$ case remained, however, a challenge because the difficulty of a rather low production rate was compounded by the limited information on the location of the yrast states provided by the $^{51–52}{\text{K}}$ $\beta$-decay measurements. In the present work, a detailed level scheme of $^{51}{\text{Ca}}$ was firmly established by supplementing the high-fold coincidence data collected at Gammasphere with the $^{48}{\text{Ca}} + ^{238}{\text{U}}$ reaction on a thick target with the results of a second, independent experiment. In the latter, the same reaction was used, but with a thin target, and $\gamma$ rays were detected in coincidence with reaction products identified in a magnetic spectrometer. The same two data sets, together with further, complementary information from $\gamma$-ray spectroscopy following secondary fragmentation of $^{55}{\text{V}}$ and $^{57}{\text{Cr}}$ exotic beams [23], were also instrumental in unravelling the yrast structure of $^{52}{\text{Sc}}$, an isotope of $^{53}{\text{Ca}}$.

II. EXPERIMENTAL PROCEDURE AND ANALYSIS RESULTS

The first experiment was performed at Argonne National Laboratory with a 330-MeV $^{48}{\text{Ca}}$ beam from the ATLAS accelerator and the Gammasphere array [22], which consisted of 101 Compton-suppressed Ge detectors. The beam was focused on a 50-mg/cm$^2$-thick $^{238}{\text{U}}$ target. Gamma-ray coincidence data were collected with a trigger requiring three or more Compton-suppressed $\gamma$ rays to be present in prompt coincidence. Energy and timing information of all Ge detectors were recorded, as the decay of $^{51}{\text{Ca}}$ and $^{52}{\text{Sc}}$. Firm information from $\gamma$-ray spectroscopy following secondary fragmentation of $^{55}{\text{V}}$ and $^{57}{\text{Cr}}$ exotic beams [23], were also instrumental in unravelling the yrast structure of $^{52}{\text{Sc}}$, an isotope of $^{53}{\text{Ca}}$.

The spectrometer was set up for the detection of nuclei close in mass to the projectile. In this case, the event trigger required three- and higher-fold coincidence events was recorded. In the analysis, conditions were placed on the time parameters to obtain various versions of prompt and delayed $\gamma$-$\gamma$ coincidence matrices and cubes. The various $\gamma$-$\gamma$ histograms extended to a maximum energy of $\sim 4$ MeV.

In the second measurement the same projectile-target combination, $^{48}{\text{Ca}} + ^{238}{\text{U}}$, was investigated at the Laboratori Nazionali di Legnaro using the ALPI accelerator and the CLARA+PRISMA detection setup [24–26]. In this instance, the 330-MeV $^{48}{\text{Ca}}$ beam was impinging on a $^{238}{\text{U}}$ target of 600 $\mu$g/cm$^2$ thickness, placed in the center of the CLARA germanium detector array consisting of 24 Compton-suppressed clover detectors. The PRISMA magnetic spectrometer, used to identify product nuclei, was positioned at 53$^\circ$ with respect to beam direction, i.e., in the vicinity of the grazing angle.

The standard set of detectors described in Ref. [24] was used for tracking and identification of the reaction products. The spectrometer was set up for the detection of nuclei close in mass to the projectile. In this case, the event trigger required the detection in coincidence of a single $\gamma$ ray in CLARA and...
an ejectile at the PRISMA focal plane. A total of \(\sim 4 \times 10^6\) coincidence events was collected over a 5-day period.

### A. \(^{51}\text{Ca}\)

Prior to the present investigations, the only information available on the \(^{51}\text{Ca}\) nucleus originated from the \(\beta\)-decay study of neutron-rich \(^{51}\text{K}\) and \(^{52}\text{K}\) by Perrot et al. [6]. Excited states at 1718, 2377, 2934, 3460, 3500, and 4493 keV were proposed, but only the 3460-keV level was tentatively assigned \(7/2^-\) spin and parity quantum numbers. The analysis reported here started from the \(\gamma\)-ray spectrum measured by the CLARA array in coincidence with \(^{51}\text{Ca}\) products; the latter is presented in Fig. 1(a). Among the \(\gamma\) lines found in the spectrum appear three transitions with respective energies of 2378, 2934, and 3462 keV that had been observed earlier in \(^{51}\text{Ca}\) decay. Other weak lines in the spectrum of Fig. 1(a) must also belong to \(^{51}\text{Ca}\), but their ordering and mutual coincidence relationships could not be established due to the low statistics of the \(\gamma\)-\(\gamma\)-ejectile coincidence data. In this situation, the set of \(\gamma\)-ray coincidence data obtained with the thick target at Gammasphere provided crucial complementary information.

In thick-target experiments, only \(\gamma\) rays emitted from stopped products appear in the spectra as sharp lines with a width equal to the intrinsic energy resolution of the germanium crystals. As a result, only those \(\gamma\) transitions for which the cumulative emission time is comparable to or longer than the stopping time in the thick target (typically of the order of 1 ps) can be thoroughly studied. This feature represents an intrinsic limitation of this experimental approach. Fortunately, because deep-inelastic processes populate yrast states preferentially, the associated half lives in nuclei of this mass region are often sufficiently long to explore level sequences up to moderate spin.

Using the Gammasphere data set, the coincidence spectrum gated on the most prominent \(\gamma\) line observed in coincidence with the \(^{51}\text{Ca}\) residues at 2378 keV was examined; two parts of this spectrum can be found in Figs. 1(b) and 1(c). It revealed a series of weak peaks, all potential candidates for transitions in \(^{51}\text{Ca}\). However, only two of those satisfied the condition of being present in the spectrum gated on \(^{51}\text{Ca}\) reaction products: these were the 1466- and 1942-keV lines. Subsequently, a double coincidence gate placed on the 1466–2378 keV pair in the prompt coincidence cube displayed the 476- and 311-keV \(\gamma\) rays; the latter two are also seen in coincidence with the \(^{51}\text{Ca}\) products. On the basis of those findings the following new states were placed in the \(^{51}\text{Ca}\) level scheme: a state at 3844 keV decaying by a 1466-keV transition, a level at 4320 keV de-exciting through the 476- and 1942-keV \(\gamma\) rays, and, tentatively, a 4155-keV state connected to the 3844-keV level by a 311-keV transition. Further support for the existence of this state at 4155 keV came from the inspection of the coincidence spectrum gated on the 3462-keV transition that displayed a weak line at 693 keV, possibly representing the connection between the 4155- and 3462-keV states. The resulting \(^{51}\text{Ca}\) level scheme is given in Fig. 3.

### B. \(^{52}\text{Sc}\)

Previous knowledge about excited states in \(^{52}\text{Sc}\) comes from the \(\beta\) decay of the \(^{52}\text{Ca}\) ground state [7] and from in-beam \(\gamma\)-ray spectroscopy following secondary fragmentation of \(^{55}\text{V}\) and \(^{57}\text{Cr}\) [23]. In the former study only low-spin states, with spin and parity assignments of \(1^+\) and \(2^+\), were proposed, as should be expected considering the selection rules pertaining to the \(\beta\)-decay mode. The investigation with fragmentation as a production method reported a new \(\gamma\) ray at 212(3) keV. It was assigned to the low-lying level scheme of \(^{52}\text{Sc}\) and proposed to correspond to a \(5^+ \rightarrow 4^+\) transition. Because deep-inelastic reactions strongly favor the population of yrast states in product nuclei, this \(5^+ \rightarrow 4^+\) transition was viewed as an ideal starting point for the analysis of the Gammasphere coincidence data. However, the spectrum gated on the 211–213 keV window appeared to be rather complex, presumably because transitions within this energy range occur in many product nuclei (they include, for example, a strong yrast \(\gamma\) ray at 211 keV in \(^{238}\text{U}\)). Fortunately, the PRISMA-CLARA data proved instrumental in clarifying the situation as the spectrum gated on the \(^{52}\text{Sc}\) recoils, presented in Fig. 2(a), exhibits a strong 212-keV peak, in agreement with the finding of Ref. [23], as well as a 2120-keV line. The latter was also seen in the 212-keV gated Gammasphere
data. These observations form the basis for a 212–2120 keV cascade in $^{52}$Sc that deexcites a level located 2332 keV above the $4^+$ state. The spectrum gated on the 2120-keV $\gamma$ ray in the prompt coincidence matrix, given in Fig. 2(b), and the spectrum double-gated on the 212- and 2120-keV lines in the prompt coincidence cube provided clear identification of several weaker transitions in $^{52}$Sc at energies 391, 742, 1271, 1351, and 1622 keV. The observed coincidence relationships together with the measured intensities subsequently lead to the level scheme proposed in Fig. 3(b) where additional states were established at 3603, 3954, 4345, and 5696 keV.

III. DISCUSSION

The yrast structures identified in the present work for the two $N = 31$ isotones $^{51}$Ca and $^{52}$Sc have to be interpreted in a somewhat speculative manner because no rigorous spin-parity assignments could be made for the states in Figs. 3(a) and 3(b). Indeed, the available statistics did not allow for a meaningful analysis of angular correlations. In such cases shell-model calculations often provide good guidance for spin-parity assignments. Here, however, the situation is somewhat more difficult as both proton and neutron excitations outside the $^{48}$Ca core need to be considered. As pointed out, for example, in Ref. [23], effective interactions for proton excitations across the $Z = 20$ gap are not nearly as well developed as those describing neutrons in the $fp$ shell. Thus, any interpretation that has to take the existence of both types of excitations into account will rely heavily on the results of shell-model calculations for states involving neutron excitations and on arguments based on either systematics or on noted similarities with features in neighboring nuclei for proton states.

![Figure 2](image1.png)

**FIG. 2.** (Color online) (a) $\gamma$-ray spectrum measured in the thin-target experiment with a gate on $^{52}$Sc reaction products identified at the focal plane of the PRISMA spectrometer. Note the change of scale at 1400 keV; (b) coincidence spectrum from the thick-target Gammasphere data gated on the 2120-keV transition. See text for details.

![Figure 3](image2.png)

**FIG. 3.** Comparisons between shell-model calculations with the GXPF1A Hamiltonian and data for $^{51}$Ca and $^{52}$Sc. Only the two calculated states with the lowest energy at each spin are provided. The $^{51}$Ca states shown as dashed lines are given for completeness and were observed only in the $\beta$-decay data of Ref. [6]. The width of the arrows is proportional to the measured relative intensities. The intensities of ground-state transitions were obtained from the thin-target data and were used to normalize appropriately all the other intensities derived from the thick-target data. The energy uncertainty for the strongest transitions in each nucleus is 0.2 keV and increases to 0.6 keV for the weakest lines.
As pointed out in the introduction, a new effective interaction for the full fp shell, labeled GXPF1A, was developed recently by Honma et al. [11]. Shell-model calculations with this interaction were found to be successful in describing the properties of neutron-rich nuclei around the \(N = 32\) subshell closure. Specifically, the GXPF1A Hamiltonian gives a good description of variations in the \(E(2^+_1)\) energies in the Cr, Ti, and Ca isotopic chains near \(N = 32\) [11] and also accounts for the extended level structures in the \(50,52-54\)Ti isotopes [3,14,17]. In addition, the interaction reproduces the magnitude of the \(B(E2; 2^+ \rightarrow 2^+_1)\) reduced transition rates in \(^{48,50,52,54}\)Ti, although it fails to describe some of the measured variations of this quantity with mass [4]. One would then expect the same interaction to account for the yrast states arising from excitations within the fp shell in the two nuclei of interest here. As stated above, the issue is of special relevance because some of the \(^{51}\)Ca and \(^{52}\)Sc low-lying yrast states are expected to involve the promotion of neutrons into the \(\nu p_{1/2}\) and \(\nu f_{5/2}\) orbitals, and the relative location in energy of the latter two orbitals near \(Z \sim 20\) plays a decisive role in the presence or absence of a significant \(N = 34\) shell gap in \(^{54}\)Ca.

### A. \(^{51}\)Ca

As mentioned earlier, deep-inelastic processes, used here to produce the nuclei of interest, populate preferentially the yrast and near-yrast states and this fact provides a strong indication of the nature of the observed excitations. The level scheme established in the present work for \(^{51}\)Ca is compared in Fig. 3 with the results of shell-model calculations with the GXPF1A interaction. For clarity, only the lowest two calculated states at each spin are given in the figure.

In the \(^{51}\)Ca ground state, the three neutrons outside the \(N = 28\) core occupy the \(p_{3/2}\) orbital, resulting in straightforward \(3/2^−\) spin and parity assignments. There is little doubt that the strongly populated level at 2378 keV corresponds to the first excited yrast state arising from the \(\nu p_{3/2} p_{1/2}\) coupling and a spin and parity assignment of \(5/2^−\) follows. Its excitation energy is reproduced rather well by the calculations, and this agreement between theory and experiment is another indication in favor of the proposed configuration. Of all the \(^{51}\)Ca levels seen in \(\beta\) decay, only the 3462-keV state was assigned a spin and a parity (of \(7/2^−\)) in the work of Perrot et al. [6]. As can be seen in Fig. 3, the calculations predict the presence of a state in this energy domain dominated by the \(\nu p_{3/2}f_{5/2}\) configuration. In addition, the computed transition probabilities predict the decay of this \(7/2^−\) level into the \(3/2^−\) ground state to be strongly favored over a \(7/2^− \rightarrow 5/2^−\) branch and this explains the absence of such a branch in the experimental data.

The yrast neutron excitation predicted by the shell model to lie immediately above the first \(7/2^−\) state is associated with a \(9/2^+\) level of dominant \(\nu p_{3/2}f_{5/2}\) character, calculated to be located at \(\sim 4800\) keV. According to the calculations, this state should decay mostly to the \(5/2^−\) level, with a negligible branch toward the \(7/2^−\) state. Moreover, the computed \(B(E2)\) value for the \(9/2^− \rightarrow 5/2^−\) transition corresponds to a few-picoseconds lifetime for this state (assuming a transition energy of the order of 2 MeV), and the de-exciting transitions should translate into sharp peaks as a result. All these criteria are satisfied for the 4320-keV state and, hence, spin-parity quantum numbers of \(9/2^−\) are proposed in Fig. 3.

The decay patterns of the two states at 3844 and 4155 keV strongly suggest that their spin must be higher than \(5/2\). However, in the energy range of interest the \(fp\)-shell calculations do not predict any states with these energy and spin requirements other than those already discussed. The observed excitations most likely arise from the promotion of a proton across the \(Z = 20\) gap. The level at 4155 keV probably corresponds to a \(9/2^+\) excitation mostly of the \(\pi f_{7/2}^+ s_{1/2}^− \nu p_{3/2}^−\) type, similar in character to the \(9/2^+\) state at 4018 keV in \(^{49}\)Ca [27]. In turn, the feeding and decay pattern of the 3844-keV level is then in line with a \(7/2^+\) spin-parity assignment. This assignment, however, raises an issue because a corresponding excitation does not appear to be present below the \(9/2^+\) state in \(^{49}\)Ca. It is known that, in the case of the \(\pi f_{7/2}^+ s_{1/2}^− \nu p_{3/2}^−\) multiplet in \(^{50}\)Sc, the \(5^+\) state arising from the maximum spin coupling is lowest in excitation energy [28]. However, once a \(\pi s_{1/2}^+\) configuration is involved, the situation changes: the level associated with the \(I_{\text{maximum}} − 1\) coupling becomes lower in energy than the one corresponding to \(I_{\text{maximum}}\) [29]. Thus, in the case of \(^{52}\)Sc where a \(p_{3/2}\) neutron hole (with respect to the \(N = 32\) closed subshell) has to be considered, one should expect the ordering of the \(4^+\) and \(5^+\) states to be reversed with respect to the situation in \(^{50}\)Sc; i.e., the \(\pi f_{7/2}^+ \times \nu p_{3/2}^−\) configuration should have its \(4^+\) level at lower energy than its \(5^+\) state. This ordering has indeed been suggested on the basis of the experimental data [23] and is also in line with the experimental results on \(^{52}\)Sc presented below. By analogy it then follows that the \(\pi f_{7/2}^+ s_{1/2}^− \nu p_{3/2}^−\) configuration producing the \(7/2^+\) and \(9/2^+\) states in \(^{49}\)Ca will be characterized by an higher excitation energy for the \(7/2^+\) level than for the \(9/2^+\) state. In \(^{51}\)Ca, however, the relevant configuration is of \(\pi f_{7/2}^+ s_{1/2}^− \nu p_{3/2}^−\) character, and the \(7/2^+\) state should naturally be below the \(9/2^+\) level.

Recently, we became aware that independently from our investigations, the yrast structure of \(^{51}\)Ca has been studied in a thin-target experiment employing the same reaction system and the reaction product-\(\gamma\)-ray coincidence technique [30]. There is an overall agreement between our results and the results of Ref. [30].

### B. \(^{52}\)Sc

The ground state of \(^{52}\)Sc was assigned \(3^+\) tentative spin and parity quantum numbers based on the population pattern of \(^{52}\)Ti excited levels in the \(\beta\) decay of the \(^{52}\)Sc ground state [7]. According to the shell model, the ground state and the low-lying excitations can be understood as members of the \(\pi f_{7/2}^+ \nu p_{3/2}^−\) multiplet. In the calculations, the two lowest states, i.e., the \(3^+\) and \(4^+\) levels, are almost degenerate, whereas the \(5^+\) state is computed to be approximately 270 keV above them. As mentioned, investigations employing \(\gamma\)-ray spectroscopy following secondary fragmentation [23] reported a \(\gamma\) ray at 212(3) keV that was tentatively assigned to the \(5^+ \rightarrow 4^+\) transition.
transition. The level scheme of $^{52}$Sc established in present study is given in Fig. 3(b) together with the results of the full fp shell-model calculations using the GXPF1A Hamiltonian (full fp shell-model calculations for $^{52}$Sc employing other available effective interactions were performed in Refs. [23,31]).

Our results strengthen the 4$^+$ and the 5$^+$ assignments to the levels connected by the 212-keV $\gamma$ ray. Unfortunately, the separation between the 4$^+$ level and the 3$^+$ ground state remains unknown. The lack of an identifiable candidate line for the $4^+ \rightarrow 3^+_g$ transition in the Gammasphere data results in an upper limit on its energy of 60 keV. Because of this lack of evidence for the ground-state transition, all the level-energy values are henceforth expressed relative to the 4$^+$ state. Within the shell model, the nature of yrast excitations above the 4$^+$ and 5$^+$ states is quite straightforward. The calculations predict two distinct yrast levels: the 6$^+$ state of dominant $\pi f_{3/2}^2\nu_{p_{1/2}}$ character at 2237 keV and the 8$^+$ excitation arising predominantly from the $\pi f_{1/2}^2\nu_{p_{3/2}} f_{5/2}$ configuration at 3740 keV. Thus, the former state can be viewed as the analog of the 5/2$^-$ level in $^{51}$Ca, whereas the latter is the analog of the corresponding 9/2$^-$ level. There is little doubt that the observed levels at 2332 and 3603 keV correspond to these two computed states. The reduced $B(E2)$ transition probability calculated for the 8$^+ \rightarrow 6^+$ transition results in a lifetime of $\sim 100$ ps, consistent with the experimental observation that the associated $\gamma$-ray transition is a sharp line. In the vicinity of the 8$^+$ state, the calculations predict two 7$^+$ excitations at 3730 and 4139 keV, respectively. According to the computed transition probabilities, they both should have strong decay branches toward the 5$^+$ state. Experimentally, however, none of the states located around 4 MeV exhibits such a pattern. Instead, the decay of the two states located at 3954 and 4345 keV is consistent with respective spin and parity assignments of 7$^+$ and 8$^+$. Those states very likely arise from proton excitations across the Z = 20 shell gap and are probably related to the 7/2$^+$ and 9/2$^+$ levels of $^{51}$Ca.

C. Information on the $\nu_{p_{1/2}}$ and $\nu_{f_{5/2}}$ states

The overall agreement between experiment and theory for the fp-shell states in $^{51}$Ca and $^{52}$Sc is quite satisfactory. With the character of the excitations in these two nuclei established, the opportunity arises to inspect in detail the behavior of the experimental and calculated states with configurations involving the $\nu_{p_{1/2}}$ and $\nu_{f_{5/2}}$ orbitals along an extended isotonic $N = 31$ chain, as detailed spectroscopic information is also available from Ref. [32] for $^{53}$Ti. Figure 4 displays the partial level schemes of the three nuclei and compares the states of interest with the results of shell-model calculations with the GXPF1A interaction. The yrast states containing the $\nu_{f_{5/2}}$ orbital in their main configuration are the 4320-keV, 9/2$^-$ level in $^{51}$Ca, the 3603-keV, 8$^+$ state in $^{52}$Sc, and the 21/2$^-$ level at 6056 keV in $^{53}$Ti. In the heaviest isotope, the agreement between the experimental and the calculated values is very good, $\Delta E = 51$ keV. However, the deviation between the data and the results of the calculation increases slightly to $\Delta E = 138$ keV in $^{52}$Sc and becomes even larger for $^{51}$Ca, $\Delta E = 476$ keV. At the same time, in all three isotones, the calculations reproduce relatively well the yrast states where a neutron occupies the $\nu_{p_{1/2}}$ single-particle state: these are the 5/2$^-$, 6$^+$, and 17/2$^-$ levels in $^{51}$Ca, $^{52}$Sc, and $^{53}$Ti, respectively. Thus, the observed behavior of the states with predominant $p_{1/2}$ and $f_{5/2}$ configurations suggests that the $p_{1/2} f_{5/2}$ energy difference in Ca nuclei might be somewhat smaller than that predicted by the GXPF1A interaction. The same conclusion was reached in Ref. [17] from a comparison between experiment and theory in the case of $^{55}$Ti, where the 1/2$^-$ ground state is of $\nu_{p_{1/2}}$ character, whereas the first excited 5/2$^-$ state is associated with a $\nu f_{5/2}$ excitation. The present GXPF1A calculations correspond to a gap of 3.6 MeV between these two single-particle states at $N = 34$ and $Z = 20$. Lowering this gap by 0.5 MeV in Ca nuclei would result in a very good description of the 9/2$^-$ state in $^{51}$Ca. Calculations with this slightly reduced gap still predict the 2$^+$ energy to lie at rather high energy in $N = 34$ $^{54}$Ca: $\sim 2.5$ MeV, leaving the experimental discovery of this excitation as an interesting challenge.

IV. CONCLUSION

The yrast structures of $^{51}$Ca and $^{52}$Sc have been located using deep-inelastic reactions in two complementary experiments: a thick-target $\gamma$-$\gamma$ and a reaction product-$\gamma$ coincidence measurement. The identified level structures can be understood as resulting from neutron excitations into the $p_{1/2}$ and $f_{5/2}$ orbitals and proton excitations across the Z = 20 shell gap. Shell-model calculations with the GXPF1A effective interaction reproduce the fp-shell neutron
excitations reasonably well, although an increasing deviation between theory and experiment for states with predominant $\nu f_{5/2}$ character was noted along the $N = 31$ isotonic chain when going from Ti to Ca. This observation suggests that the $p_{1/2} - f_{5/2}$ energy difference in Ca nuclei may well be somewhat smaller than that predicted by the GXPF1A Hamiltonian. The results, however, do not rule out the presence of a sizable gap at $N = 34$. Thus, whether a subshell closure occurs in $^{54}$Ca remains a challenging and intriguing issue.

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