

# SOME ASPECTS OF TRANSFER REACTIONS IN LIGHT AND HEAVY ION COLLISIONS\*

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Transfer reactions played a key role in unraveling many properties of the nucleus, in particular they have been essential to define the relevance of the independent particle motion and to define the properties of two-particle correlations. In view of the now available radioactive beams, it is important to summarize some aspects of the transfer processes that will be relevant to define the properties of nuclei close to the neutron and proton drip lines.

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## 1. Introduction

From the sixties to the middle of the seventies of last century, light ions transfer reactions have been one of the most important line of research in nuclear physics. By following the idea of Butler [1] one could, in fact, use these reactions to extract important information, namely spectroscopic factors, that have been at the base for the understanding the independent particles description of the low energy spectra of nuclei. After these very prolific years the availability of heavy ions beams and the discovery of Deep Inelastic Reactions (DIC), where a large fraction of the relative motion energy and angular momentum are dissipated in intrinsic excitation and spin, the interest on transfer reactions shifted toward the understanding of the friction and diffusion coefficients introduced to describe the yields of these reactions.

The coming in operation of facilities able to accelerate intense radioactive beams will stir a revival of interest on transfer reactions, being these the reactions that allow to study the evolution of single particle levels away from the  $\beta$ -stability valley and to unravel the role of correlations in defining the actual location of the neutron and proton drip lines.

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Since the spectroscopic factors are extracted by comparing the experimental angular distributions with theoretical calculations performed in the Distorted Wave Born Approximation (DWBA), I think useful to start this paper by summarizing the main steps [2-4] leading to the above approximation and then pass to illustrate, with few examples, the role played by the transfer degrees of freedom in the description of the evolution of a heavy ions collision from the quasielastic to the very complex deep-inelastic regime where the exchange of several quanta is present.

## 2. The DWBA

Direct transfer reactions are ideal for the extraction of spectroscopic information since these reactions are very fast (the collision time  $\tau \sim 3 \times 10^{-21}$  s is comparable with the nucleon orbiting time) and have forward picked angular distributions that are sensitive to the transferred angular momentum.

The fact that these reactions are very fast suggests, for their description, the use of the Born approximation. In this approximation, the transition amplitude for the transfer reaction  $a(b+1) + A \rightarrow b + B(A+1)$  may be written in the form

$$T_{\alpha\beta} = \left\langle \Psi^{(-)} \left| V_{\alpha} - U_{\alpha} \right| \psi_a \psi_A \chi_{\alpha}^{(+)} \left( \vec{k}_{\alpha}, \vec{r}_{\alpha} \right) \right\rangle, \quad (1)$$

where with  $\alpha$  ( $a+A$ ) and  $\beta$  ( $b,B$ ) I have indicated the entrance and exit channel respectively and where the following notations have been used:

- $V_{\alpha}$  sum of all two-body interactions between the nucleons in target and projectile,
- $\psi_a, \psi_A$  intrinsic wave functions of projectile and target,
- $\chi_{\alpha}^{(+)}(\vec{\kappa}_{\alpha}, \vec{r}_{\alpha})$  incoming solution of the Schrödinger equation for the entrance channels  $\alpha$ ,  $\vec{r}_{\alpha}$  is the relative motion coordinate and  $\vec{\kappa}_{\alpha}$  is related to the linear momentum,
- $U_{\alpha}$  the Optical Potential that fits elastic scattering,
- $\Psi^{(-)}$  total wave function of the system with outgoing boundary conditions.

To arrive at the DWBA from (1), one has to apply two approximations: the first one concerns the total wave function of the system  $\Psi^{(-)}$ , while the second one concerns the interaction.

- *First.* The exact wave function  $\Psi^{(-)}$  is replaced by the product

$$\Psi^{(-)} \sim \psi_b \psi_B \chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta), \quad (2)$$

where with  $\psi_b$  and  $\psi_B$  I have indicated the intrinsic wave functions of the asymptotic mass partition for the exit channel and with  $\chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta)$  the wave function describing the relative motion with the outgoing boundary conditions. With this approximation the amplitude (1) is written in the form

$$T_{\alpha\beta} = \int d\vec{r}_\beta \int d\vec{r}_\alpha \chi_\beta^{(-)}(\vec{k}_\beta, \vec{r}_\beta) \langle bB | (V_\alpha - U_\alpha) | aA \rangle \chi_\alpha^{(+)}(\vec{k}_\alpha, \vec{r}_\alpha), \quad (3)$$

where the six-dimensional integral have to be performed over the relative motion coordinates of entrance ( $\vec{r}_\alpha$ ) and exit ( $\vec{r}_\beta$ ) channels. Notice that this approximation implies the introduction of an Optical Potential (OP) also for the exit channel in order to construct the relative motion wave function. The information about the spectroscopic factor is contained in the expression on the nuclear matrix element

$$\langle bB | (V_\alpha - U_\alpha) | aA \rangle = \int d\xi_a d\xi_A \psi_b \psi_B (V_\alpha - U_\alpha) \psi_a \psi_A. \quad (4)$$

- *Second.* To show how the matrix element in (4) is evaluated, let us consider the stripping ( $d, p$ ) reaction. By introducing the fractional parentage expansion for the wave function of the target-like nucleus, we write

$$\psi_B = [\psi_A \otimes \varphi_{n(A)}], \quad (5)$$

where  $\varphi_{n(A)}$  is the single particle wave function of the neutron in the nucleus  $B$ . By inserting this expansion in (4) one obtains

$$\langle bB | (V_\alpha - U_\alpha) | aA \rangle = \int d\xi_n d\xi_A \chi_p [\psi_A \otimes \varphi_{n(A)}] (V_\alpha - U_\alpha) \varphi_d \psi_A, \quad (6)$$

where  $\chi_p$  is the spin part of the proton wave function (it is convenient to leave the spin part of the proton wave function here in order to define the formfactor is a form suitable also for heavier projectiles, in the following, we will see that this wave function will be replaced by the single particle wave function in the projectile) and  $\varphi_d$  is the deuteron wave function. By integrating over all intrinsic coordinates  $\xi_A$ , one can write

$$= \sqrt{S_{n(A)}} \int d\xi_n \chi_p \varphi_{n(A)} (V_{pn}) \varphi_d = F_{\beta\alpha}(\vec{r}_\alpha, \vec{r}_\beta), \quad (7)$$

where  $\sqrt{S_{n(A)}}$  is the spectroscopic factor corresponding to the  $\varphi_{n(A)}$ . Notice that with this integration the interaction  $(V_\alpha - U_\alpha)$  is replaced by the potential binding the neutron to the proton to form a deuteron (for more details on this point refer to the above references). If the projectile is a more complex nucleus, this interaction will be replaced by the shell model potential of the projectile  $(V_{nA})$ . This expression constitutes the so called formfactor that has been here constructed in the PRIOR representation.

It is now easy to write the cross section of the transition. It is given by

$$\frac{d\sigma_{\alpha\beta}}{d\Omega_\beta} = (\dots)|T_{\alpha\beta}|^2 = S_{n(A)} \frac{d\sigma_{\alpha\beta}^{\text{DW}}}{d\Omega_\beta}. \quad (8)$$

Since the cross sections are directly proportional to the spectroscopic factors, the above formula provides a direct recipe to extract them from the experimental angular distributions. In the extraction of the spectroscopic factors, one has to take into account that the entrance and exit channels wave functions of relative motion are obtained by using Optical Potentials coming from fits of elastic scattering angular distributions. Unfortunately, there is a full family of potentials that derives from this fitting procedure and their predictions for the transfer cross section are not unique. One is guided in the choice of the OP parameters by the fact that direct reactions are strongly localized in that only a small number of partial waves, close to the grazing angular momentum, contributes to the cross section. One has to be very wary when potentials provide sizable contributions also for small partial waves. The localization in  $\ell$ -space may be easily translated in a localization in  $r$ -space. From this it follows that transfer probes the nucleus only in the surface region, *i.e.* only the tail of the bound state wave function is relevant for the nuclear matrix element in Eq. (4). For this reason, most DWBA codes construct the bound state wave function by using a Wood–Saxon form of the Shell Model Potential whose parameters have to be properly adjusted.

### 3. Transfer degrees of freedom in heavy-ion collisions

In the case of heavy-ion collisions, to calculate the matrix element for one-nucleon transfer (*cf.* Eq. (4)) one has to introduce the parentage expansion also for the projectile, thus for the DWBA one obtains the expression

$$\frac{d\sigma_{\alpha\beta}}{d\Omega_\beta} = S_{m(b)} S_{n(A)} \frac{d\sigma_{\alpha\beta}^{\text{DW}}}{d\Omega_\beta}, \quad (9)$$

*i.e.* the cross section depends from the product of the projectile and target spectroscopic factors. This fact, together with the poor energy resolution of the experiment that makes difficult to distinguish individual final states, prevented the use of these reactions for spectroscopic studies.

With the discovery of DIC, where a large fraction of energy and angular momentum from the relative motion is dissipated in intrinsic excitation of the two fragments, the relevance of transfer reactions shifted toward the understanding of the friction and diffusion coefficients introduced to describe the yields of these reactions. A large fraction of the fragments emerging from these reactions have an energy that is smaller than the Coulomb barrier of two touching spheres, indicating that the fragments must have large deformations at the detaching point. To illustrate the interplay between collective and transfer degrees of freedom in defining the evolution of a reaction, I will show some results obtained with the semi-classical model GRAZING [6–9]. This model incorporates explicitly the surface degree of freedom (important for the treatment of the large deformations that may occur in the collision) and transfer degrees of freedom. The excitation of these last degrees of freedom is obtained by using the actual formfactors for the transfer of one nucleon (neutron and proton, stripping and pick-up). Multinucleon transfer is, in the model, treated in the successive approximation.

### 3.1. Fusion excitation function

The importance of transfer reactions in the description of a heavy-ion reaction has been underlined in several papers [10, 11]. These transfer degrees of freedom are weak, very numerous, span a wide range of  $Q$ -values and are governed by long range formfactors. Unfortunately, fusion reactions have been very elusive in pinning down the role of particle transfer. Many good fits of the data could, in fact, be obtained by including only surface modes. In trying to elucidate the role of transfer in fusion, I will discuss the fusion excitation functions of  $^{132}\text{Sn}$  on  $^{58}\text{Ni}$  and  $^{64}\text{Ni}$ . This tin isotope has a quite pronounced nuclear skin, so that the effect of transfer channels on barrier penetration could in principle be relevant.

Before discussing the results, let me point out that fusion is sensitive to the potential that determines the relative motion and to the correct treatment of the surface degrees of freedom. The model uses the empirical potential of Ref. [13] and includes, beside the transfer channels, the low lying 2+ and 3- states of projectile and target. In Fig. 1, in comparison with the experimental data [12, 14], there are shown the excitation functions for the two indicated reactions (full line). For the light nickel isotopes the model is somewhat over predicting the fusion cross section but a small correction ( $\Delta R = 0.1$  fm) of the radius of the nuclear potential fixes the problem

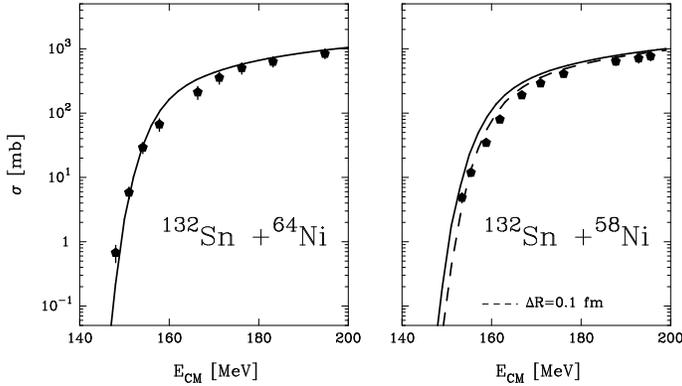


Fig. 1. Fusion excitation functions for the collision of  $^{132}\text{Sn}$  on  $^{64}\text{Ni}$  and  $^{58}\text{Ni}$  in comparison with the data of Refs. [12, 14]. For the  $^{58}\text{Ni}$  case to obtain a better fit (dashed line) the radius of the nuclear potential has been increased by 0.1 fm.

(dashed line). Of course, we would like to know whose degrees of freedom dominate the process. To this purpose, we consider only the reaction with  $^{64}\text{Ni}$ . In Fig. 2 (left side), we show, in comparison with the full calculations and the experimental data, the results when the transfer degrees of freedom are neglected (dashed line). By keeping only the couplings to the transfer degrees of freedom, *i.e.* neglecting the surface mode of both target and projectile, one obtains the results shown with a dashed line on the right-hand side frame of the same figure. Here, with a dotted line, there are also shown the calculations when the surface modes of the  $^{132}\text{Sn}$  are neglected.

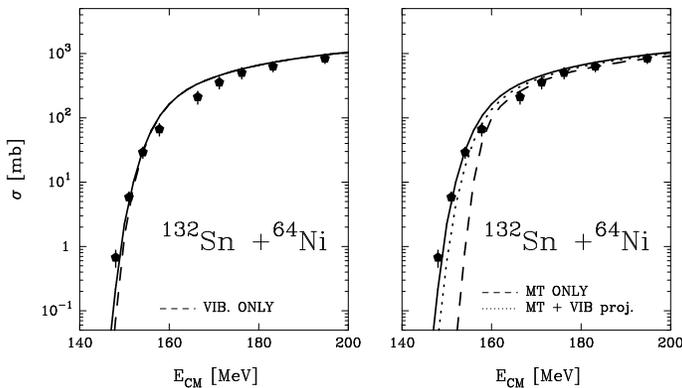


Fig. 2. For the collision of  $^{132}\text{Sn}$  on  $^{64}\text{Ni}$  the fusion excitation function is compared with several GRAZING calculations corresponding to different degrees of freedom included in the calculation (see the text). The data are from Refs. [12, 14].

The GRAZING model seems to indicate that the capture process is dominated by the surface degrees of freedom also in the case of a very neutron rich nucleus (notice that  $^{132}\text{Sn}$  has a one neutron separation energy  $S_n = 7.3$  MeV).

### 3.2. Production of neutron-rich nuclei

The study of nuclei at the border of the  $\beta$ -stability lines, in particular of those close to the neutron drip-line, constitutes one of the main challenges of the new research with radioactive beams. The production of nuclei close to the proton drip-line does not pose special problems since they can be reached, with stable nuclei, via fusion reactions or via fragmentation. Unfortunately, these kind of techniques do not work for the production of heavy neutron rich nuclei. In the following, I will try to show that multinucleon transfer reactions can offer a mechanism for the production of these heavy neutron rich nuclei.

The magnitude of a given transfer process can be estimated easily by writing down its first order Born approximation. For a given impact parameter (incoming partial wave  $\ell$ ), the probability for the transition from the entrance channel  $\alpha$  to the exit channel  $\beta$  may be written in the form

$$P_{\beta\alpha}(\ell) = \left| \frac{i}{\hbar} \int_{-\infty}^{+\infty} dt e^{i\sigma_{\beta\alpha}t} f_{\beta\alpha}(0, \vec{r}) e^{i[(E_{\beta}-E_{\alpha})+(\delta_{\beta}-\delta_{\alpha})]t/\hbar} \right|^2, \quad (10)$$

where the time integral has to be performed along the classical trajectory for the given partial wave  $\ell$ . The form factor  $f_{\beta\alpha}(0, \vec{r})$ , related to the nuclear matrix element in Eq. (4), is a function of the distance  $\vec{r}$  connecting the center of mass of the two nuclei.

In direct processes the two nuclei barely overlap, so that only the tail of the formfactor is relevant. By approximating the true trajectory with a parabolic parametrization around the turning point, the above transition probability may be written in the form

$$P_{\beta\alpha} = \sqrt{\frac{1}{16\pi\hbar^2|\ddot{r}_0|\kappa_{a'_1}}} |f_{\beta\alpha}(0, r_0)|^2 g(Q_{\beta\alpha}), \quad (11)$$

where  $\ddot{r}_0$  is the radial acceleration at the distance of closest approach  $r_0$ . The adiabatic cut-off function  $g(Q)$  is defined as

$$g(Q) = \exp\left(-\frac{(Q - Q_{\text{opt}})^2}{\hbar^2\ddot{r}_0\kappa_{a'_1}}\right), \quad (12)$$

where the optimum  $Q$ -value is

$$Q_{\text{opt}} = \left( \frac{Z_d}{Z_A} - \frac{Z_d}{Z_b} \right) E_B + \left( \frac{m_d}{m_b} - \frac{m_d}{m_A} \right) (E - E_B) + \frac{m_d \ddot{r}_0}{m_a + m_A} (R_A m_b - R_a M_B) \quad (13)$$

and  $m_d$  and  $Z_d$  are the mass and charge of the transferred particle,  $E_B$  is the Coulomb barrier.

The adiabatic cut-off function  $g(Q)$  defines the actual value of the transition probability, the maximum being at the optimum  $Q$ -value. This derives from the requirement that the trajectory of entrance and exit channels matches smoothly close to the turning point where the contribution from the form factor peaks.

In Fig. 3, for the  $^{58}\text{Ni}+^{208}\text{Pb}$  reaction we show the adiabatic cut-off function  $g(Q)$  for all one and two particles transfer channels. In the same figure with horizontal lines we represent, for all channels, the location of all possible transitions. Since only the channels whose  $Q$ -values lie below

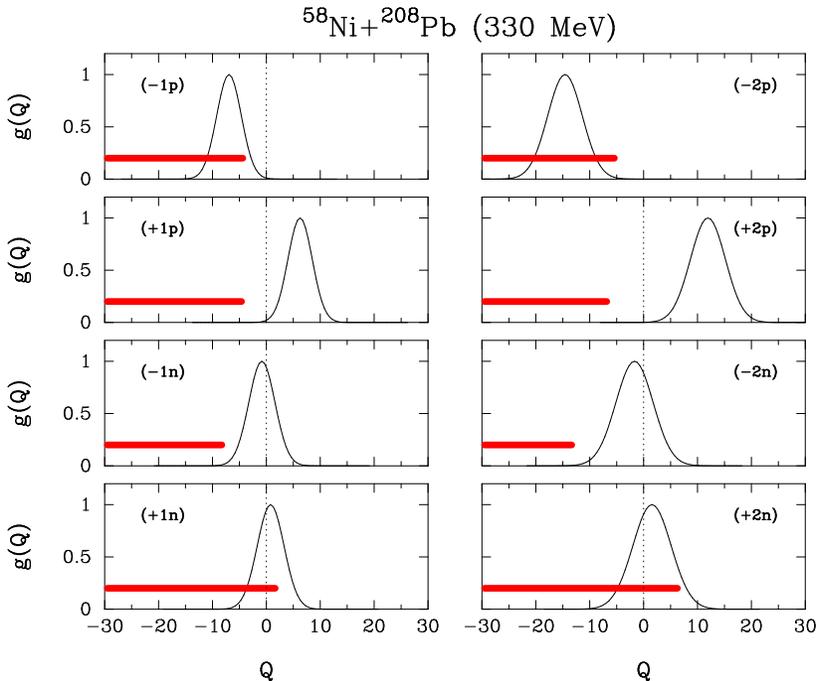


Fig. 3. Adiabatic cut-off functions for one and two neutron and proton transfer channels for the reaction  $^{58}\text{Ni}+^{208}\text{Pb}$  at the indicated energy. The horizontal lines represent the location of all possible transitions.

the bell-shaped curve can actually occur, it is clear that the only allowed transfers are neutron pick-up and proton stripping. All the other channels are hindered by optimum  $Q$ -value consideration. Similar conclusions can be drawn for most projectile and target combinations with stable nuclei.

By looking at the chart of nuclei it is clear that one should be able to reach the region of neutron rich heavy nuclei just by extracting protons (*i.e.* making protons pick-up reactions) from a heavy stable targets or transferring some neutrons (*i.e.* making neutron stripping reactions). As seen above, proton pick-up and neutron stripping are very weak reactions when stable nuclei are involved. Thus one has to resort to the use of neutron rich projectiles as it was discussed in Refs. [15, 16].

Focusing in the region of the nuclei chart close to the magic number  $N = 126$ , we show in Fig. 4 how multinucleon transfer reactions on  $^{208}\text{Pb}$  evolve by using, as projectile, several isotopes of xenon. We show this evolution by displaying, for the indicated projectile, the production of mercury ( $+2p$  channels) and polonium ( $-2p$  channels) isotopes. The bombarding energy is set at 700 MeV in the center-of-mass system for all the systems. It is clear from the figure that with neutron-rich projectile the population evolves toward proton pick-up thus populating nuclei below the lead that are very neutron rich.

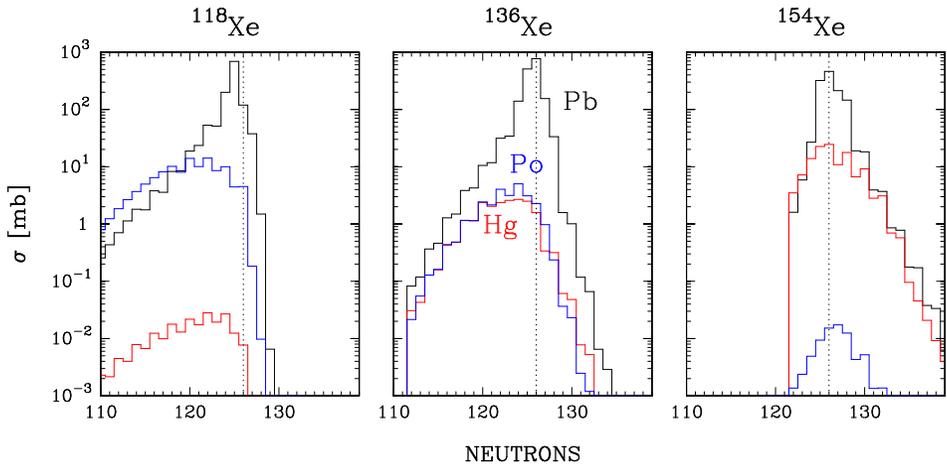


Fig. 4. Isotopic distributions for the  $+2p$  (Hg) and  $-2p$  (Po) channels in the reaction of  $^{118,136,154}\text{Xe}$  on  $^{208}\text{Pb}$  reactions.

#### 4. Conclusions

In this contribution, I outlined the role of transfer reactions in nuclear structure and reaction dynamics studies. For light ions, I emphasized the dependence of the spectroscopic factors from the parameters governing the dynamics of the reactions. For heavy ions, by using a semiclassical model I showed that it is possible to discuss the interplay of transfer and surface degrees of freedom in the evolution of the reaction, in particular, it has been shown that capture is dominated by the dynamics of the nuclear surfaces.

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