Lecture I: Introduction, general ideas
• The baseline: e+p and p+p collisions
• pQCD evolution equations
• Saturation: non-linear evolution equations

Lecture II: The Color Glass Condensate effective theory
• Classical methods. McLerran Venugopalan
• Quantum evolution: BK and B-JIMWLK equations
• Particle Production in dense environments

Lecture III: Phenomenology, from RHIC to the LHC
• e+p collisions
• d+Au, A+A collisions
• Challenges and open issues
At a theoretical level, the need of non-linear corrections and small-$x$ is clear

Are such corrections present and relevant in current experimental data?

Where to look for them?
Deep inelastic e+p (A) scattering

Kinematic coverage of DIS experiments

Most of the data lie in the large-x large-$Q^2$ domain and is well described within standard DGLAP approaches.
Experimental data on $e+p$ (A) scattering at small-$x$ ($x<10^{-2}$) exhibit the property of geometric scaling [Stasto, K. Golec-Biernat and J. Kwiecinski]

$$\sigma^* h(x, Q^2) \rightarrow \sigma^* h(\tau = Q^2 / Q^2_s(x))$$

$$Q^2_{sp}(x) \approx 1 \left( \frac{x_0}{x} \right)^{\lambda} \text{GeV}^2 \quad \left\{ \begin{array}{l}
\lambda \approx 0.2 \div 0.3 \\
x_0 = 3 \cdot 10^{-4}
\end{array} \right.$$
Deep inelastic e+p (A) scattering

Experimental data on e+p (A) scattering at small-x (x<10^{-2}) exhibit the property of geometric scaling [Stasto, K. Golec-Biernat and J. Kwiecinski]

\[ \sigma^{\gamma^* h}(x, Q^2) \to \sigma^\gamma h(\tau = Q^2/Q_s^2(x)) \]

\[ Q_{sp}^2(x) \approx 1 \left( \frac{x_0}{x} \right)^\lambda \text{ GeV}^2 \]

\[ x_0 = 3 \cdot 10^{-4} \]

Such scaling is immediate in saturation approaches...

\[ \sigma_{T,L}^{\gamma^* h}(x_{Bj}, Q^2) = \int dr \int_0^1 dz |\Psi_{T,L}^{\gamma^*-q\bar{q}}(z, r, Q^2)|^2 \sigma^{dip}(x_{Bj}, r) \]

\[ \sigma^{dip}(x, r) = \sigma^0 \left( 1 - \exp \left[ -r^2 Q_{sp}^2(x) \right] \right) \]

...the “new scale” being the saturation scale

In the absence of saturation the only other scales than can cutoff the interaction are non-perturbative \( \Lambda_{QCD} \), \( m_f \) ...
Deep inelastic $e+p$ (A) scattering

Experimental data on $e+p$ (A) scattering at small-$x$ ($x < 10^{-2}$) exhibit the property of geometric scaling \cite{Stasto, Golec-Biernat and Kwiecinski}.

Geometric scaling also seen in diffractive and exclusive data

\[ \frac{d\sigma_{diff}}{d\beta} \]

\[ \tau = Q^2 / Q_s^2(x) \]

\[ \mu_b \]

proton
nuclei

Gonzalves et al

Marquet et al

DVCS

VM=ρ
A First principle calculation of the saturation scale (more generally, the dipole amplitude) in terms of the running coupling BK equation is possible.

\[ \lambda(Y) = \frac{d \ln Q_s(Y)}{dY} \]

MV Initial conditions:
\[ \mathcal{N}(r, x = x_0) = 1 - \exp \left[ -\frac{r^2 Q_0^2}{4} \ln \left( \frac{1}{r \Lambda} + e \right) \right] \]

✓ NLO corrections are large, rendering evolution compatible with experimental data.

\[ \lambda^{LO} \approx 4.8 \alpha_s \]
Deep inelastic e+p (A) scattering

Using more sophisticated approaches (BK with running coupling evolution) it is possible to get a very good description of total (in e+p and e+A), diffractive and longitudinal structure functions.

Reduced cross sections in e+p HERA collisions

(JLA-Armesto-Milhano-Quiroga-Salgado)

Fit including heavy quarks

Ratios of nuclear structure functions (SHADOWING!)

Dusling-Gelis-Lappi-Venugopalan

Important: from these studies one can constrain the proton and nuclei ugd and then use them in the study of proton and nuclear reactions (RHIC, LHC)
Using more sophisticated approaches (BK with running coupling evolution) it is possible to get a very good description of total (in e+p and e+A), diffractive and longitudinal structure functions. Reduced cross sections in e+p HERA collisions (JLA-Armesto-Milhano-Quiroga-Salgado)

Ratios of nuclear structure functions (SHADOWING!)
Dusling-Gelis-Lappi-Venugopalan

\[ Q_{sA}^2(x) \sim A^{1/3} Q_{sp}^2(x) \]

Note: \( x \lesssim 10^{-2} \) Above that value description of data is hard (fit tensions...)
An effective way of probing small-x is by going to forward (pseudo)-rapidities.
(approximate) RHIC Kinematics: \( \sqrt{s} = 200 \text{ GeV} \)
\[ x_{1(2)} \sim \frac{m_t}{\sqrt{s}} \exp(\pm y_h) \]

\( \eta = 0 \) Intermediate values of \( x \) in projectile and target

\[ p_t^{\text{had}} \sim 0.5 \div 10 \text{ GeV} \quad \Rightarrow \quad x \sim 0.005 \div 0.1 \]

\( \eta = 3.2 \) Small-\( x \) gluons (projectile) and valence quarks (target) dominate

\[ p_t^{\text{had}} \sim 0.5 \div 3.5 \text{ GeV} \quad \Rightarrow \quad x_2 \sim 2 \cdot 10^{-4} \div 5 \cdot 10^{-3} \]
\[ x_1 \sim 0.1 \div 0.9 \]
Saturation / Color Glass Condensate modeling of multiplicities

- Most of particles produced in the collision originate from small-x gluons in the saturation domain
- Other sources (genuinely soft processes, contribution from valence quarks etc) neglected
- Initial gluon production is calculated via kt-factorization and then mapped to final hadron spectra assuming local parton-hadron duality

\[
\varphi_A(x, p_t, b) = \frac{d\sigma^{A+B\rightarrow g}}{dy d^2p_t d^2R} = \kappa \frac{2}{C_F} \frac{1}{p_t^2} \int_{p_t}^{p_t} \int d^2b \alpha_s(Q) \varphi \left( \frac{|p_t + k_t|}{2}, x_1; b \right) \varphi \left( \frac{|p_t - k_t|}{2}, x_2; R - b \right)
\]

\[
x_{1(2)} = \left( \frac{p_t}{\sqrt{s_{NN}}} \right) \exp(\pm y)
\]

“Leading order”: N_part scaling

\[
\frac{dN_{AA}}{d\eta} \Bigg|_{\eta=0} \propto Q^2_{sA}(\sqrt{s}, b) \sim \sqrt{s}^\lambda N_{part} \quad \text{with} \quad \lambda \sim 0.2 \div 0.3
\]

Phenomenological models:

\[
\frac{1}{N_{part}} \frac{dN^g_{AB}}{d^2b d\eta} \Bigg|_{\eta=0} = \begin{cases} 
\sqrt{s}^\lambda \ln \left( \sqrt{s}^\lambda N_{part} \right) \\
\sqrt{s}^\lambda N_{part}^{1-\delta} \\
\sqrt{s}^\lambda N_{part}^{-\frac{1}{3}} 
\end{cases}
\]

- Scaling violations from running coupling, KLN model (Kharzeev-Levin-Nardi, NPA 747 609)
- Data driven ASW model ASW (Armesto-Salgado-Wiedemann PRL94 022002)
The energy, rapidity and centrality dependence of multiplicities measured at RHIC are in very good agreement with saturation models.

Models of independent particle production (hard or soft) predict $\frac{dN}{d\eta} \sim N_{coll}$.
From RHIC to the LHC

The energy dependence of the multiplicities follows a power-law behaviour similar to the one predicted in saturation models:

\[ \frac{dN_{AA}}{d\eta} \bigg|_{\eta=0} \propto Q_{sA}^2(\sqrt{s}, b) \sim \sqrt{s}^\lambda N_{\text{part}} \]

ALICE Pb-Pb (2.76 TeV, 5% central)

Why is this power higher in A+A than in p+p collisions?

NOTE: If a A+A collision was like a incoherent superposition of p+p collisions, one would have expected to get ~ 6000 charged particles in central Pb+Pb collisions at the LHC. COHERENCE effects are crucial.!!!
Some care must be exercised in choosing the transverse area of the local density corresponds to the presence of a single nucleon. The density of large fluctuations can result in collisions of a large number of nucleons at the same transverse position. We first generate a configuration of nucleons for each of the colliding nuclei. To complete our discussion of the initial conditions we explain how we construct a list of random coordinates where the evolution enforces a minimal distance between nucleons. Factorizing the fluctuations of the nucleons in a nucleus from possible fluctuations appears to be justified by the scale hierarchy within a nucleon (not accounted for at present), and finally from sequential production of nucleons in the transverse plane. Factorizing the fluctuations of the nucleons in a nucleus from possible fluctuations appears to be justified by the scale hierarchy within a nucleon (not accounted for at present), and finally from sequential production of nucleons in the transverse plane. Factorizing the fluctuations of the nucleons in a nucleus from possible fluctuations appears to be justified by the scale hierarchy within a nucleon (not accounted for at present), and finally from sequential production of nucleons in the transverse plane.

1. Generate configurations for the positions of nucleons in the transverse plane \( (r_i, i=1...A) \). Wood-Saxons thickness function \( T_A(R) \)
2. Count the number of nucleons at every point in the transverse grid, \( R \):

\[
N(R) = \sum_{i=1}^{A} \Theta \left( \sqrt{\frac{\sigma_0}{\pi}} - |R - r_i| \right)
\]

\( \sigma_0 \approx 42 \text{ mb} \)

3. Assign a local initial \( (x=x_0=0.01) \) saturation scale at every point in the transverse grid, \( R \):

\[
Q_{s0}^2(R) = N(R) Q_{s0,\text{nucl}}^2
\]

\( Q_{s0,\text{nucl}}^2 = 0.2 \text{ GeV}^2 \)

\[
\varphi(x_0 = 0.01, k_t, R) \quad \text{rcBK equation} \quad \varphi(x, k_t, R)
\]
1. Generate configurations for the positions of nucleons in the transverse plane \((r_i, i=1...A)\). Wood-Saxons thickness function \(T_A(R)\).

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\]

4. Gluon production is calculated at each transverse point according to \(k_t\)-factorization

\[
\frac{d\sigma^{A+B \rightarrow g}}{dy \, d^2p_t \, d^2R} = \kappa \frac{2}{C_F} \frac{1}{p_t^2} \int_{p_t}^{P} \frac{d^2k_t}{4} \int d^2b \alpha_s(Q) \varphi \left( \frac{|p_t + k_t|}{2}, x_1; b \right) \varphi \left( \frac{|p_t - k_t|}{2}, x_2; R - b \right)
\]

\[
\frac{dN_{\text{ch}}}{d\eta} = \frac{\cosh \eta}{\sqrt{\cosh^2 \eta + m^2/P^2}} \frac{dN_{\text{ch}}}{dy} \text{ with } m = 350 \text{ MeV and } P = 400 \text{ MeV}
\]
With this method a very good description of RHIC and LHC multiplicities is achieved. Monte Carlo methods based that successfully describe data incorporate strong coherent mechanisms (shadowing and energy-dependent cut-off of particle production) parallel to those naturally incorporated in the CGC description.
Particle spectra in p+p and d+Au collisions

Mid-rapidity (moderate-x):
Cronin enhancement in p-A compared to pp
Interpreted as due to multiple scatterings

Forward rapidity (small-x):
Explained in the CGC as due to non-linear evolution

$\left( p_t, y_h \gg 0 \right)$
Particle spectra in p+p and d+Au collisions

Mid-rapidity (moderate-x):
Cronin enhancement in p-A compared to pp
Interpreted as due to multiple scatterings

Forward rapidity (small-x):
Explained in the CGC as due to non-linear evolution

\((p_t, y_h >> 0)\)
Particle spectra in p(d)+Au collisions

⇒ Collinear factorization
\[
\frac{d\sigma^{AB\rightarrow hX}}{dyd^2k_t} \propto f_{a/A}(x_1, k_t^2) f_{b/B}(x_2, k_t^2) \otimes \frac{d\sigma^{ab\rightarrow cd}}{dyd^2k_t} \otimes D_{h/c}(z)
\]

⇒ nPDF's: All nuclear effects included in modification of collinear pdf's (EPS08, EPS09, HKN)
\[
f_{a/Au}(x, Q^2) = f_{a/p}(x, Q^2) R_{a/Au}(x, Q^2)
\]

⇒ Glauber-Eikonal multiple independent scatterings + unintegrated pdf's:
\[
f_{a/A}(x, Q^2) \rightarrow F_{a/A}(x, Q^2, <k_T^2>) \rightarrow F_{a/A}(x, Q^2, <k_T^2> + \Delta k_T^2(\sqrt{s}, b, p_t))
\]

⇒ Intrinsic kt momentum broadening
\[
\frac{d\sigma^{iA}}{dyd^2k_t}(b) \sim \sum_{n=1}^{\infty} \frac{1}{n!} \int [d^2k] \frac{d\sigma^{iN}}{d^2k_1} T_A(b) \cdots \frac{d\sigma^{iN}}{d^2k_n} T_A(b) \exp \left[ -\sigma_{in}(p_0) T_A(b) \right] \delta^2(\sum \vec{k}_i - \vec{k}_t)
\]

⇒ CGC: Full Coherence limit. Nucleus described as a saturated small-x glue ensemble

kt factorization for small-x gluon production + valence quark production
\[
\frac{dN^{AB\rightarrow gX}}{dy \, d^2p_t} = \frac{S_A C_F \alpha_s}{\pi} \frac{1}{p_t^2} \int d^2q \varphi_A(x_A, q) \varphi_B(x_B, p_t - q)
\]

Semiclassical MV multiple scatterings + quantum corrections
• Comparable description of data from different approaches

• *nPDF’s* studies do not access the $p_T<2$ GeV region (suppression)

• Both *CGC* and *GE* calculation invoke some intrinsic “non-perturbative” transverse scale $\sim 1$ GeV
Forward spectra. Qualitative expectations


Non-Linear Evolution of Cronin Enhancement

\[ \frac{\frac{\partial \varphi(x)}{\partial \ln(x_0/x)}}{Q_{SAu}^2 = 2 \text{ GeV}^2, Q_{Sd}^2 = 0.1 \text{ GeV}^2} \approx \tilde{K} \otimes \varphi - \varphi^2 \]

Semiclassical MV model always yield a Cronin-peak (i.e., MV has unitarity but no shadowing)

Forward suppression originates in the dynamical shadowing generated by the quantum non-linear BK-JIMWLK evolution equations
Single Inclusive forward hadron production in the CGC

\[ x_{1(2)} \sim \frac{m_t}{\sqrt{s}} \exp(\pm y_h) \]

(dumitru, jalilian-marian)

\[
\frac{dN_h}{dy_h d^2p_t} = \frac{K}{(2\pi)^2} \sum_q \int_{x_F}^1 \frac{dz}{z^2} \left[ x_1 f_{q/p}(x_1, p_t^2) \tilde{N}_F \left( x_2, \frac{p_t}{z} \right) D_{h/q}(z, p_t^2) \right. \\
\left. + x_1 f_{g/p}(x_1, p_t^2) \tilde{N}_A \left( x_2, \frac{p_t}{z} \right) D_{h/g}(z, p_t^2) \right]
\]

In order to ensure \( x_1 \geq x_0, x_2 \leq x_0 \) with \( x_0 \approx 0.01 \rightarrow y_h \geq 2 \)

We allow for a rapidity dependent K-factors to account for the normalization

We use CTEQ6 pdf’s and de Florian-Sassot ff’s
Comparison to RHIC data
(JLA, C. Marquet 10)

• Very good description of data for p+p and d+Au collisions at $s^{1/2} = 200$ GeV; $y_h \geq 2$

• No need for K-factors! (...almost)
  K=1 (charged hadrons)
  K=0.4 (neutral pions) ??

Fit parameters:

Proton

0.005 \leq x_0 \leq 0.01

$Q^2_{s0} = 0.2 \text{ GeV}^2$

parameters compatible with analysis of HERA data for structure functions in e+p scattering
(JLA, Armersto,Milhano,Salgado 09)

Nucleus:

0.01 \leq x_0 \leq 0.025

$Q^2_{s0} = 0.4 \text{ GeV}^2$

parameters compatible analysis of multiplicities
Proton-proton yields at higher energies McLerran Praszalowicz

- The spectra measured in p+p collisions from 200 GeV to 7 TeV energies also seem to obey a scaling law

\[
\frac{1}{\sigma} \frac{dN_{\text{ch}}}{d\eta d^2p_T} = F \left( \frac{p_T}{Q_{\text{sat}}(p_T/\sqrt{s})} \right)
\]

\[
\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda
\]

![Graph showing the scaling of transverse momentum spectra](image)
nPDF’s description of forward suppression involves a huge nuclear shadowing at small-x

Forward RHIC data not included in most recent NLO parametrizations (EPS09)
Small-x effects would also lead to strong suppression of initial gluon production in Pb+Pb collisions @ LHC:

\[
R_{PbPb}^g = \frac{1}{N_{\text{coll}}} \frac{dN_{h}^{PbPb}}{dy_h d^2p_t} \bigg/ \frac{dN_{h}^{pp}}{dy_h d^2p_t}
\]

“Naked” n.m.f.

\[\sqrt{s_{NN}}=5.5 \text{ TeV}\]
\[y_g=0, 4\]

Pb+Pb @ LHC

\[Q_{g0}^2=1 \text{ GeV}^2\]
\[Q_{g0}^2=0.8 \text{ GeV}^2\]

It is crucial to control these possible initial state effects in order to disentangle them from final state effects (due to a QGP) in observed spectra!!
Alternative explanations of forward suppression as due to large-\(x\) effects and energy loss have been proposed. Such effects are not present in the CGC description.

\[
x_1 \to 1
\]

Probability for a large-\(x\) of not losing energy:

\[
P(\Delta y) \approx e^{-n_G(\Delta y)} \approx (1 - x_F)^\# \]
Double Inclusive forward hadron production

\[ x_p = \frac{|k_1|e^{y_1} + |k_2|e^{y_2}}{\sqrt{s}} \]

\[ x_A = \frac{|k_1|e^{-y_1} + |k_2|e^{-y_2}}{\sqrt{s}} \]

Cyrille Marquet 07:

\[
\frac{d\sigma^{dAu\rightarrow qgX}}{d^2k_d dy_d d^2q_d dy_q} = \alpha_s C_F N_c x_d q(x_d, \mu^2) \left( \int \frac{d^2x d^2x'}{(2\pi)^2} \frac{d^2b d^2b'}{(2\pi)^2} e^{ik_\perp (x'-x)} e^{iq_\perp (b'-b)} \right) \\
\left| \Phi^{q\rightarrow qg}(z, x-b, x'-b') \right|^2 \left\{ S_{qgg}^{(4)}[b, x, b', x'; x_A] - S_{qgg}^{(3)}[b, x, b'+z(x'-b'); x_A] \right\} \\
- S_{qgg}^{(3)}[b+z(x-b), x', b'; x_A] + S_{qg}^{(2)}[b+z(x-b), b'+z(x'-b'); x_A] \\
\]

q\rightarrow qg splitting (pQCD)

Scattering of the 2-parton system with the CGC target

Involves more than 3 and 4 point functions. Calculated in the large Nc limit
Double Inclusive forward hadron production

\[ x_p = \frac{|k_1|e^{y_1} + |k_2|e^{y_2}}{\sqrt{s}} \]

\[ x_A = \frac{|k_1|e^{-y_1} + |k_2|e^{-y_2}}{\sqrt{s}} \]

Cyrille Marquet 07:

\[
\frac{d\sigma^{dAu\rightarrow qgX}}{d^2k_\perp dy_k d^2q_\perp dy_q} = \alpha_S C_F N_c x_d q(x_d, \mu^2) \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} \frac{d^2b'}{(2\pi)^2} e^{ik_\perp.(x'-x)} e^{iq_\perp.(b'-b)}
\]

\[
|\Phi^{q\rightarrow qg}(z, x-b, x'-b')|^2 \left\{ S_{qgg}^{(4)}[b, x, b', x'; x_A] - S_{qgq}^{(3)}[b, x, b'+z(x'-b'); x_A] \right\}
\]

\[
q\rightarrow qg \text{ splitting (pQCD)}
\]

\[
z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}
\]

Scattering of the 2-parton system with the CGC target

Involves more than 3 and 4 point functions. Calculated in the large Nc limit
⇒ “Monojets” in d+Au collisions at RHIC at forward rapidity
⇒ “Coincidence probability” at measured by STAR Coll. at forward rapidities:

\[ CP(\Delta \phi) = \frac{1}{N_{\text{trig}}} \frac{dN_{\text{pair}}}{d\Delta \phi} \]

⇒ Away peak is present in p+p coll.
⇒ Absence of away particle in d+Au coll.

\[ p+p \rightarrow \pi^0\pi^0 + X, \sqrt{s} = 200 \text{ GeV} \]
\[ p_{\text{T}} > 2 \text{ GeV/c}, 1 \text{ GeV/c} < p_{\text{T}} < p_{\text{T}} \]
\[ \langle \eta_L \rangle = 3.2, \langle \eta_S \rangle = 3.1 \]

\[ d+Au \rightarrow \pi^0\pi^0 + X, \sqrt{s} = 200 \text{ GeV}, 2000 < \Sigma Q_{\text{em}} < 4000 \]
\[ p_{\text{T}} > 2 \text{ GeV/c}, 1 \text{ GeV/c} < p_{\text{T}} < p_{\text{T}} \]
\[ \langle \eta_L \rangle = 3.1, \langle \eta_S \rangle = 3.2 \]

Peaks
\[ \Delta \phi \quad \sigma \]
\[ 0 \quad 0.41 \pm 0.01 \]
\[ \pi \quad 0.68 \pm 0.01 \]
“Monojets” in d+Au collisions at RHIC at forward rapidity

“Coincidence probability” measured by STAR Coll. at forward rapidities:

\[ CP(\Delta \phi) = \frac{1}{N_{\text{trig}}} \frac{dN_{\text{pair}}}{d\Delta \phi} \]

- Dependence on the saturation scale of the target (centrality)

[STAR PRELIMINARY]

Parameter free!!: All info about nucleus w.f. from single inclusive analysis
Summary

• RHIC data for inclusive hadron production in d+Au and p+p collisions probe moderate to small values of x in the target wave function. Onset of CGC description is expected

• Mid-rapidity data does not allow to discriminate between different approaches (CGC, Glauber-Eikonal, npdf’s...)

• Very good description of forward data in p+p and d+Au collisions based on CGC @ NLO.

• Energy-momentum conservation corrections (missing in the CGC description) have been argued to lead to a comparable suppression of forward yields as the one stemming from CGC analyses

• The study of more exclusive observables is needed

• The p+Pb program at the LHC may shed light on the problem