Written Exam MFN 1072 Quantum Mechanics

18 February 2014, 2.00–4.00 PM

Please read the following IMPORTANT INSTRUCTIONS

Answer at most TWO questions. Only two questions will be marked. A pass is obtained for one complete answer.

You may not use notes or textbooks, but the lecture notes etc are available for consultation at the front desk.

Answer at most TWO questions

1. The wave function $\psi(x) = A\ x\ \mathrm{e}^{-\alpha x^2}$, with A, α constants, describes an energy eigenstate of a one–dimensional harmonic oscillator of mass m and angular frequency ω.

a) Is this an eigenstate of parity?

b) What is α in terms of m and ω?

c) Which state is this? The ground state, the first excited state, the second excited state etc?

d) Normalise the wave function $\psi(x)$ and calculate the expectation values $< x >$ and $< p >$ of position and momentum.

(*You may need* $\int_{-\infty}^{\infty} dx\, \mathrm{e}^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$, $\quad \int_{-\infty}^{\infty} dx\, x^2\, \mathrm{e}^{-\lambda x^2} = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$)

2. A proton of mass m is confined in an infinite square well with walls at $x = 0$ and $x = L$.

a) Sketch the first three normalised energy eigenstates ψ_n, $n = 1, 2, 3$ and calculate their energies.

b) Determine the probability $P_n(\frac{1}{a})$ that the proton is between $x = 0$ and $x = \frac{L}{a}$ in the state ψ_n.

c) What is $P_n(\frac{1}{a})$ for n large? How does this compare with the classical result?

d) What are the energy and wavelength of the photon that is emitted when the proton jumps down from the first excited state to the ground state?

3. Assume that, in spherical polar coordinates (r, θ, ϕ), the z-component of angular momentum L_z is represented by $-i\hbar\partial/\partial\phi$, where ϕ is the polar angle, $\phi \,\epsilon\, [0, 2\pi]$.

a) Show that L_z has eigenvalues $m\hbar$, with m an integer.

b) The following three states describe the motion of a particle moving in three dimensions

$$\psi_1(r) = \frac{c}{\sqrt{2}}(x + iy)\exp -r^2$$

$$\psi_2(r) = \frac{c}{\sqrt{2}}(x - iy)\exp -r^2$$

$$\psi_3(r) = cz\exp -r^2$$

where c is a normalisation constant and $r^2 = x^2 + y^2 + z^2$.

Verify, using Cartesian coordinates x, y, z, that each of these states is an eigenstate of L_z and find the corresponding eigenvalues.

c) Describe the possible outcomes of measuring L_z for a particle in the state

$$\psi(r) = \frac{c}{\sqrt{5}}(2z - x)\exp -r^2$$

4.

a) Define a hermitian operator in Quantum Mechanics

b) Show that the eigenvalues of a hermitian operator are real, and give two examples.

c) Define parity in Quantum Mechanics. Is the corresponding operator hermitian?

d) Show that wave functions of opposite parity are orthogonal, and give an example.

e) Show that the wavefunctions of the infinite potential well ($V = 0$ if $|x| < a$, $V = \infty$ otherwise), have definite parity.