Written Exam MFN 1072 Quantum Mechanics

22 January 2013, 2.30–4.30 PM

Please read the following IMPORTANT INSTRUCTIONS

Answer at most TWO questions. Only two questions will be marked. A pass is obtained for one complete answer .

You may not use notes or textbooks, but the lecture notes etc are available for consultation at the front desk.

1 A particle is in the n-th energy state $\psi_n(x)$ of an infinite square well potential with width L.
a) Calculate the functions $\psi_n(x)$ and their energies E_n.
b) Determine the probability $P_n(\frac{1}{a})$ that the particle is confined to the first $\frac{1}{a}$ of the width of the well.
c) What is $P_n(\frac{1}{a})$ for n large? How does this compare with the classical result?

2 The wave function $\psi(x) = A\ x\ \mathrm{e}^{-\alpha x^2}$ describes a state of a one–dimensional harmonic oscillator, provided the constant α is chosen appropriately.
a) Determine an expression for α in terms of the oscillator mass m and the classical frequency of vibration ω.
b) Determine the energy of this state and normalise the wave function $\psi(x)$.

(*You may need* $\int_{-\infty}^{\infty} dx\, \mathrm{e}^{-\lambda\, x^2} = \sqrt{\frac{\pi}{\lambda}}, \quad \int_{-\infty}^{\infty} dx\, x^2\, \mathrm{e}^{-\lambda\, x^2} = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$)

3 A quantum system has only two normalised energy eigenstates $|1\rangle$ and $|2\rangle$, corresponding to energy eigenvalues E_1, E_2. An operator A acts on these energy eigenstates as $A\ |1\rangle = |2\rangle$, $A\ |2\rangle = |1\rangle$ (A can be regarded as a sort of parity operator).
a) What are the eigenvalues of A and what are its eigenstates?
b) At time $t = 0$ the system is in a positive/even parity eigenstate. Determine the probability of finding the system with positive/even parity at times $t > 0$.
(*You may find it useful to express* $|1\rangle$ *and* $|2\rangle$ *in terms of the eigenstates of* A)

4 The normalised wave function for the ground state of the hydrogen atom is $\psi = A\mathrm{e}^{-\alpha r}$ where A, α, are constants and r is the electron–nucleus distance.
a) Show that $A^2 = \frac{\alpha^3}{\pi}$
b) Calculate the expectation value $< r >$
c) What is the probability of finding the electron between r and $r + dr$? For what value of r is it a maximum?

(*You may need* $\int_0^{\infty} x^n \mathrm{e}^{-x} dx = n!$)