Phase diagram, fluctuations, thermodynamics and hadron chemistry

Observables and concepts

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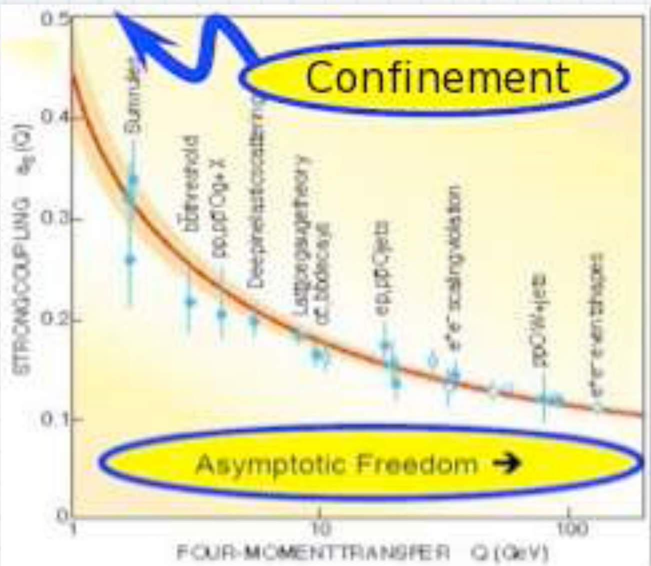
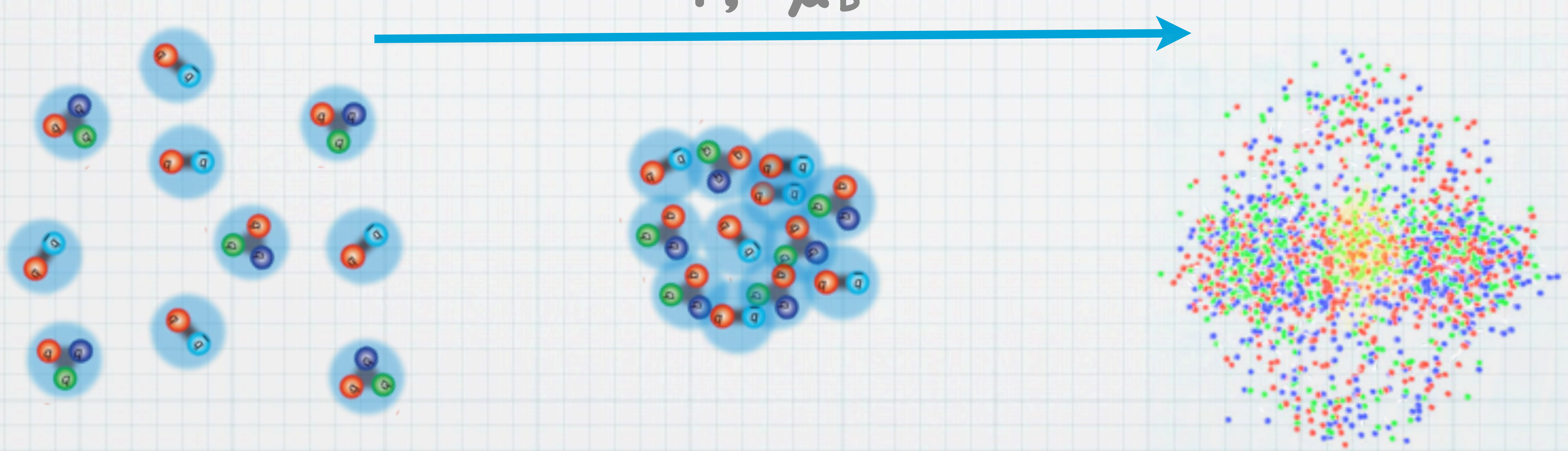
QCD Thermodynamics



- Confinement
  - At large distances the effective coupling is large
  - Free quarks are not observed in nature
- Asymptotic freedom
  - At short distances the effective coupling decreases
  - Quarks and gluons appear to be quasi-free

$T,\ \mu_B$



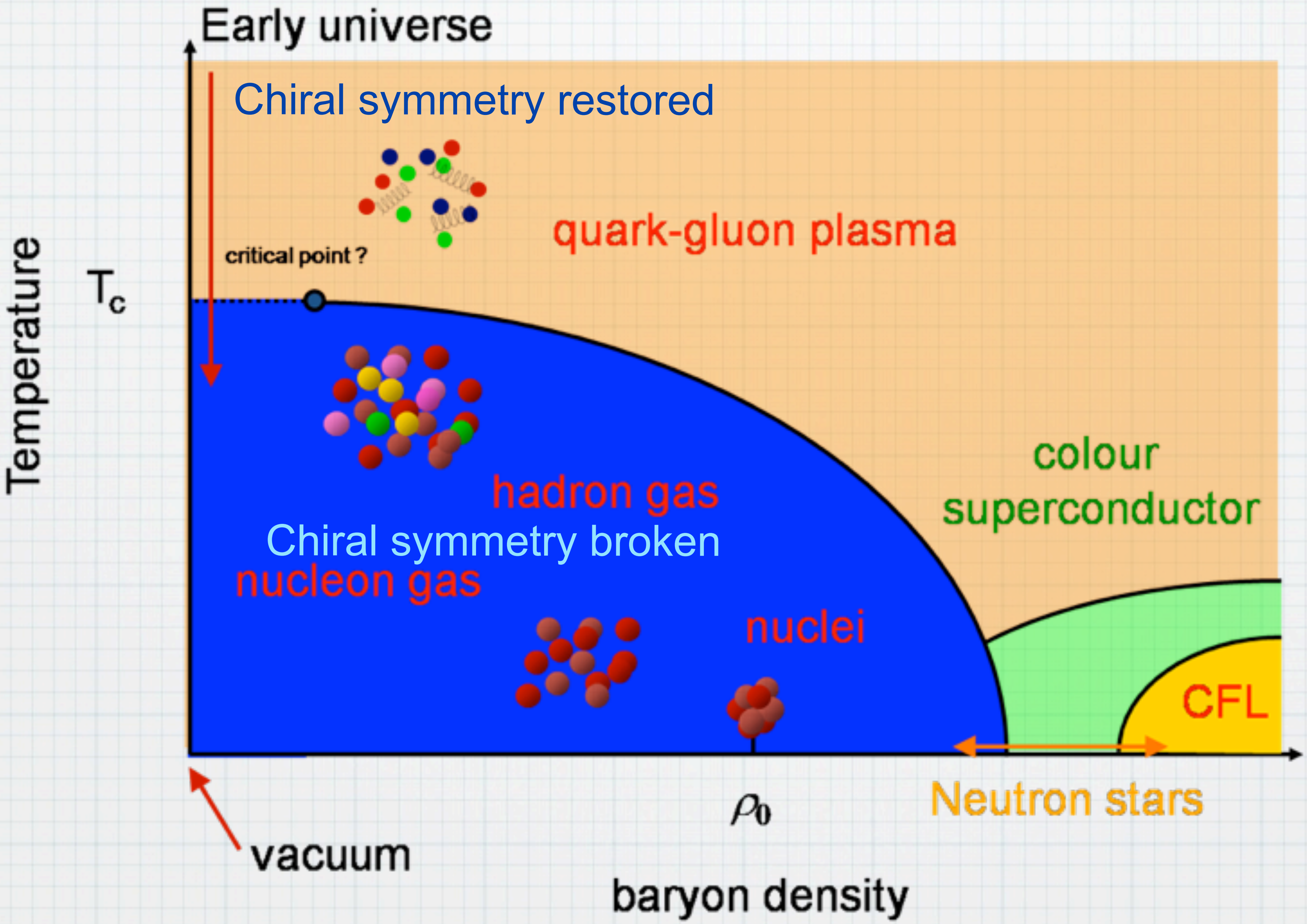
Hadron Gas

Quark-Gluon Plasma

Chiral Symmetry: broken

Chiral Symmetry: restored

QCD Phase Diagram



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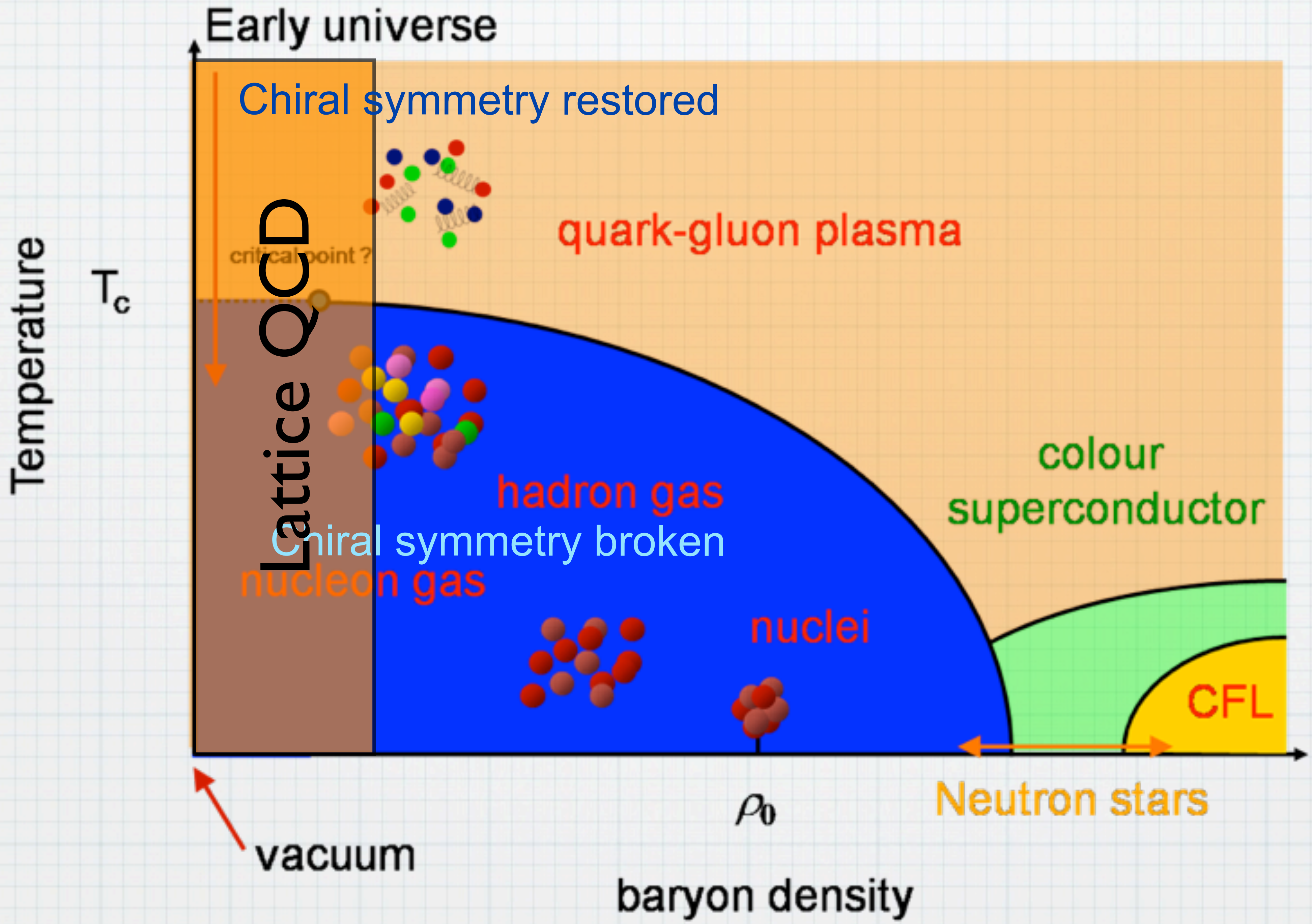

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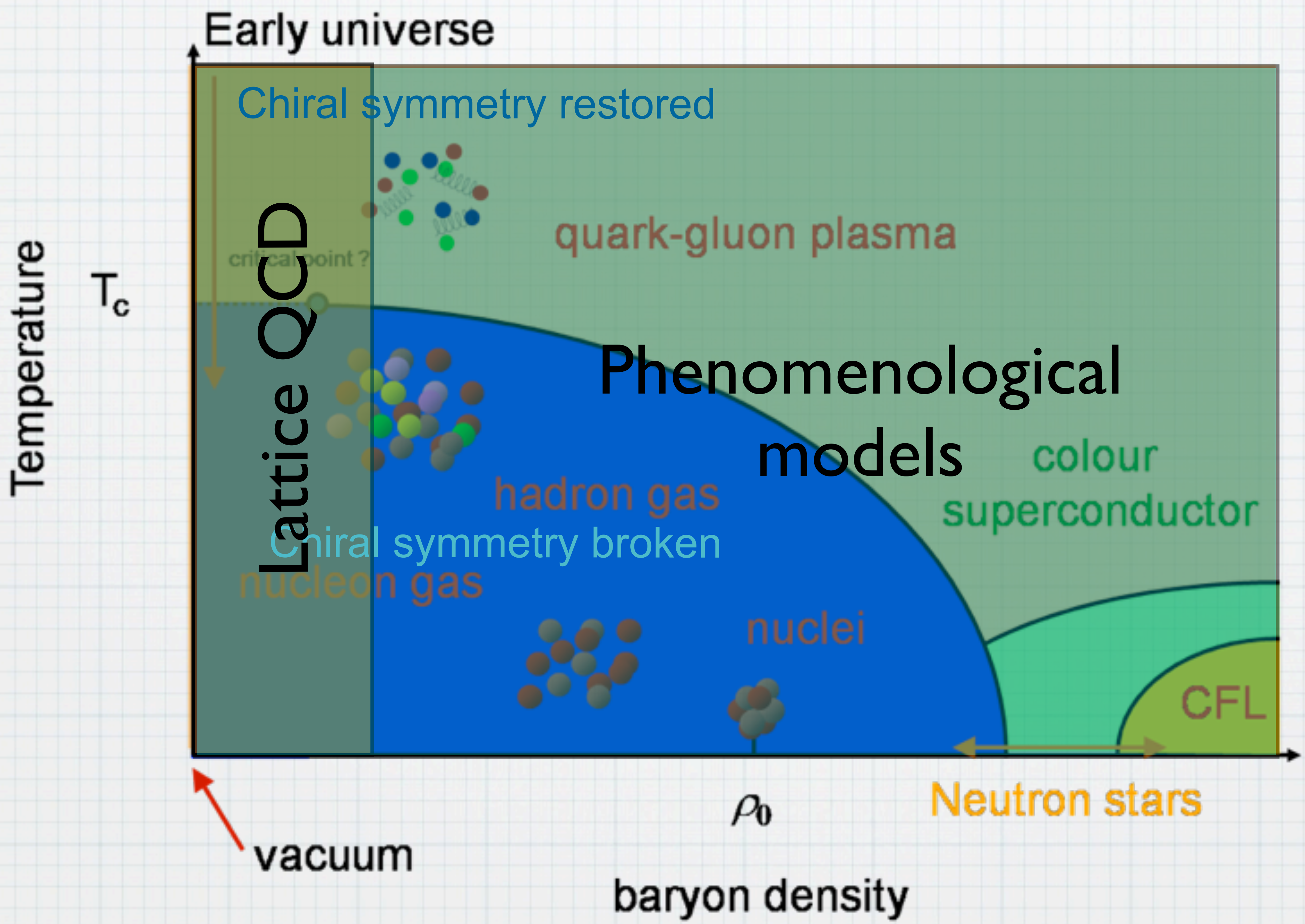


Lattice QCD

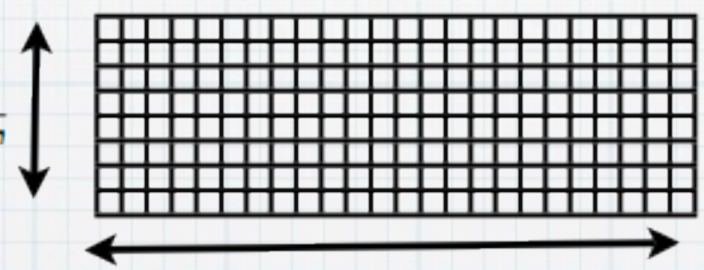
* Analytic or perturbative solutions **in low-energy QCD** are hard or impossible due to the **highly nonlinear nature** of the strong force

* Lattice QCD: well-established **non-perturbative approach** to solving QCD

* Solving QCD on a grid of points in space and time

* The lattice action is the parameterization used to discretize the Lagrangian of QCD on a space-time grid

$$N_t = \frac{1}{aT}$$



$$N_s = L/a$$

* From the partition function Z, knowledge of all the thermodynamics

$$F = -T \ln Z \,,$$

$$p = \frac{\partial (T \ln Z)}{\partial V} \,,$$

$$S = \frac{\partial (T \ln Z)}{\partial T} \,,$$

$$\bar{N}_i = \frac{\partial (T \ln Z)}{\partial \mu_i} \,,$$

$$E = -pV + TS + \mu_i \bar{N}_i$$

Sign problem

* The QCD path integral is computed by **Monte Carlo algorithms** which samples field configurations with a weight proportional to the **exponential of the action**

$$Z(\mu_B,T) = \mathrm{Tr}\left(e^{-\frac{H_{\rm QCD}-\mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U,\mu_B]$$

$\det M[\mu_B]$ **complex** $\Longrightarrow$ **Monte Carlo simulations are not feasibile.**

* If the action is **complex**, its exponential is **oscillating**: it cannot be used as a probability

* This is the reason why lattice QCD simulations cannot presently be performed at finite chemical potential

* Possible solutions:

  - ➡ Taylor expansion around μ_B=0
  - ➡ Imaginary chemical potential
  - ➡ Reweigthing techinque

All valid at small chemical potentials

Phase transitions and order parameters

- We want to study the transition from hadrons to the QGP: deconfinement and chiral symmetry restoration

- A phase transition is the transformation of a thermodynamic system from one phase or state of matter to another

- During a phase transition of a given medium certain properties of the medium change, often discontinuously, as a result of some external conditions

- The measurement of the external conditions at which the transformation occurs is called the phase transition point

- Order parameter: some observable physical quantity that is able to distinguish between two distinct phases

- We need to find observables which allow us to distinguish between confined/deconfined system and between chirally broken/restored phase

Polyakov loop: deconfinement

* We consider a system of gluons in which we put a heavy quark-antiquark pair as a probe

$$< \Phi > \sim e^{-F/T}$$

➡ How much energy F is needed to extract the heavy quark from the system?

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Confined system
Infinite energy is needed

$$\langle \Phi \rangle = 0$$

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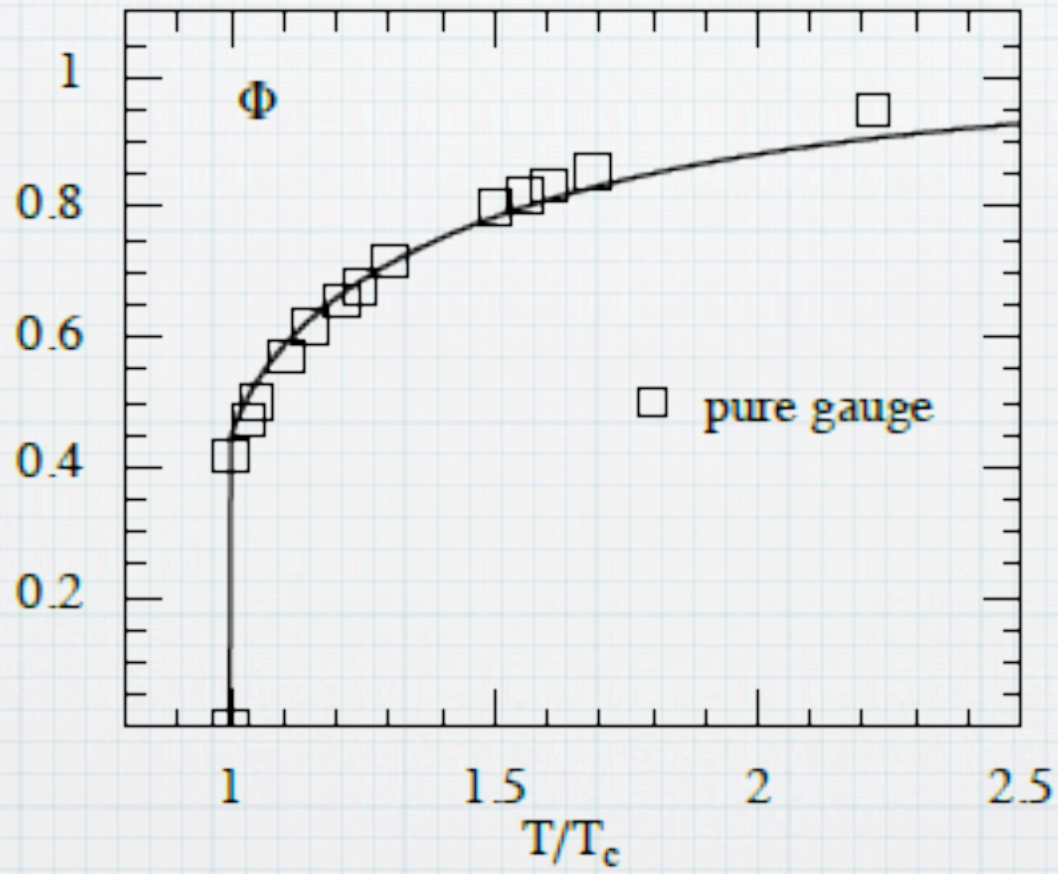

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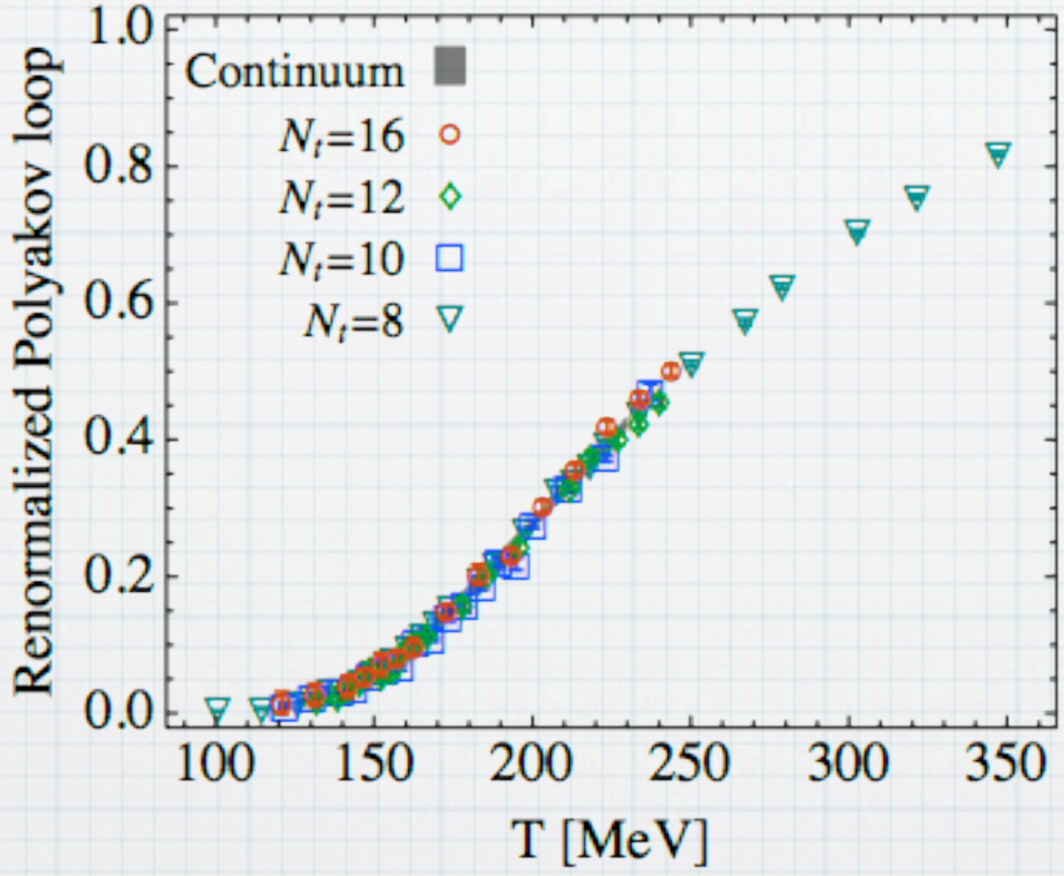

For QCD with physical quark masses the transition is a smooth crossover

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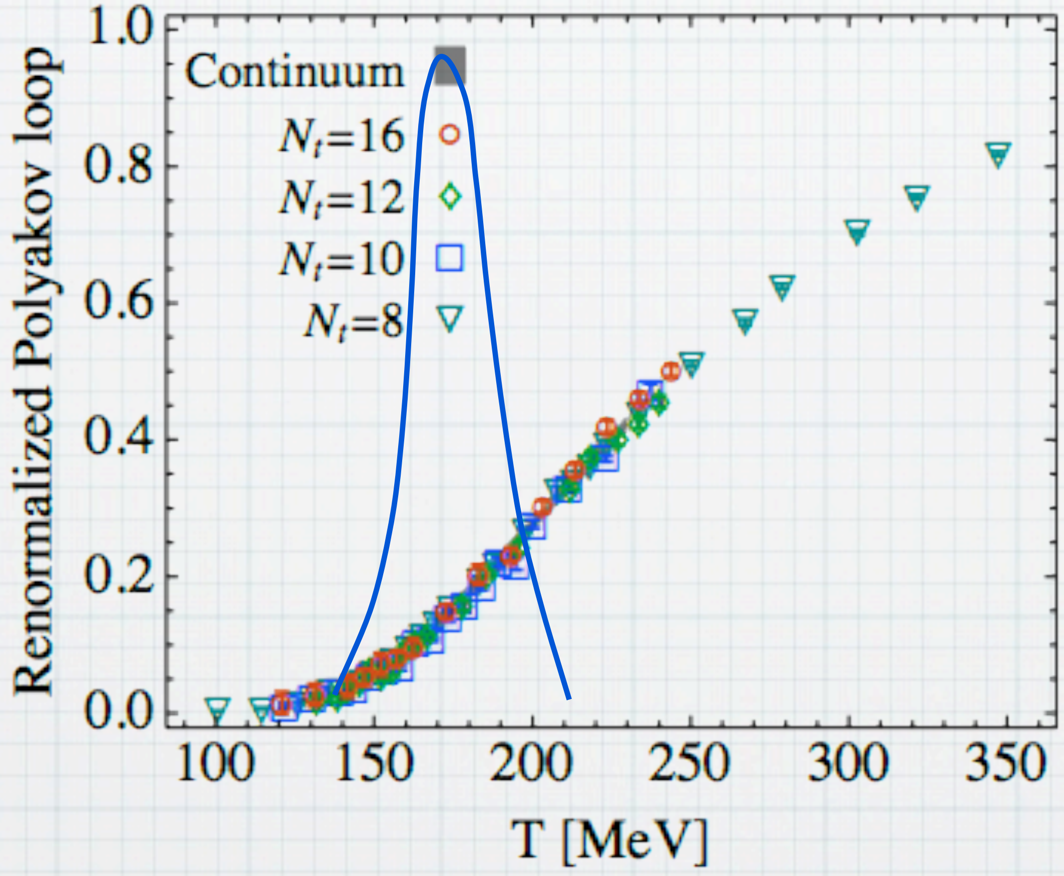

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Polyakov loop: order parameter for deconfinement

Chiral condensate: chiral transition

- The chiral condensate $\langle\bar{\psi}\psi\rangle$ is the vacuum expectation value of the operator $\bar{\psi}\psi$.

- The magnitude of the constituent quark mass is proportional to it

  ➡ Even if the "bare" quark mass in the QCD Lagrangian is small, they develop a constituent one, through interaction with the chiral condensate

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Chirally broken system
Large effective quark mass

$$\langle\bar{\psi}\psi\rangle \neq 0$$

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Small effective quark mass

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Chirally broken system	Chirally restored system
Large effective quark mass	Small effective quark mass
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Chiral condensate: order parameter for chiral phase transition

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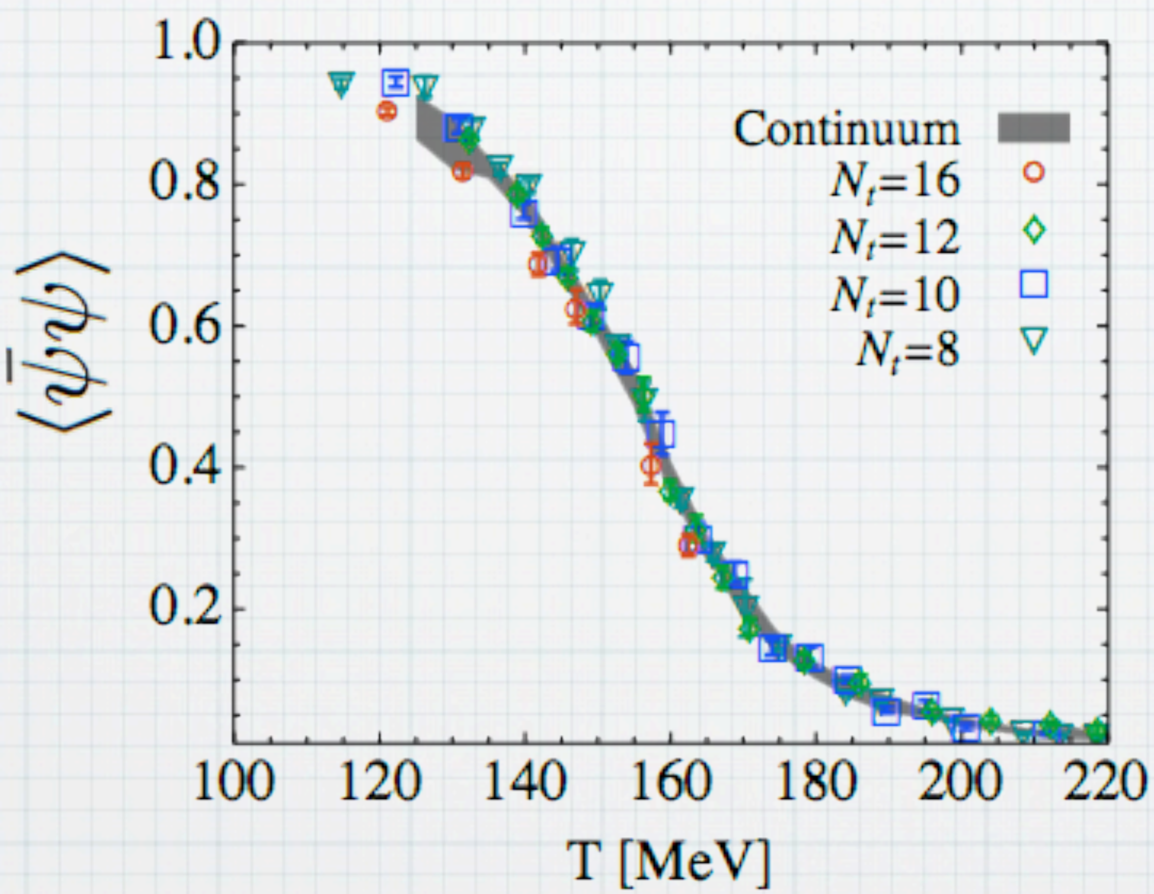

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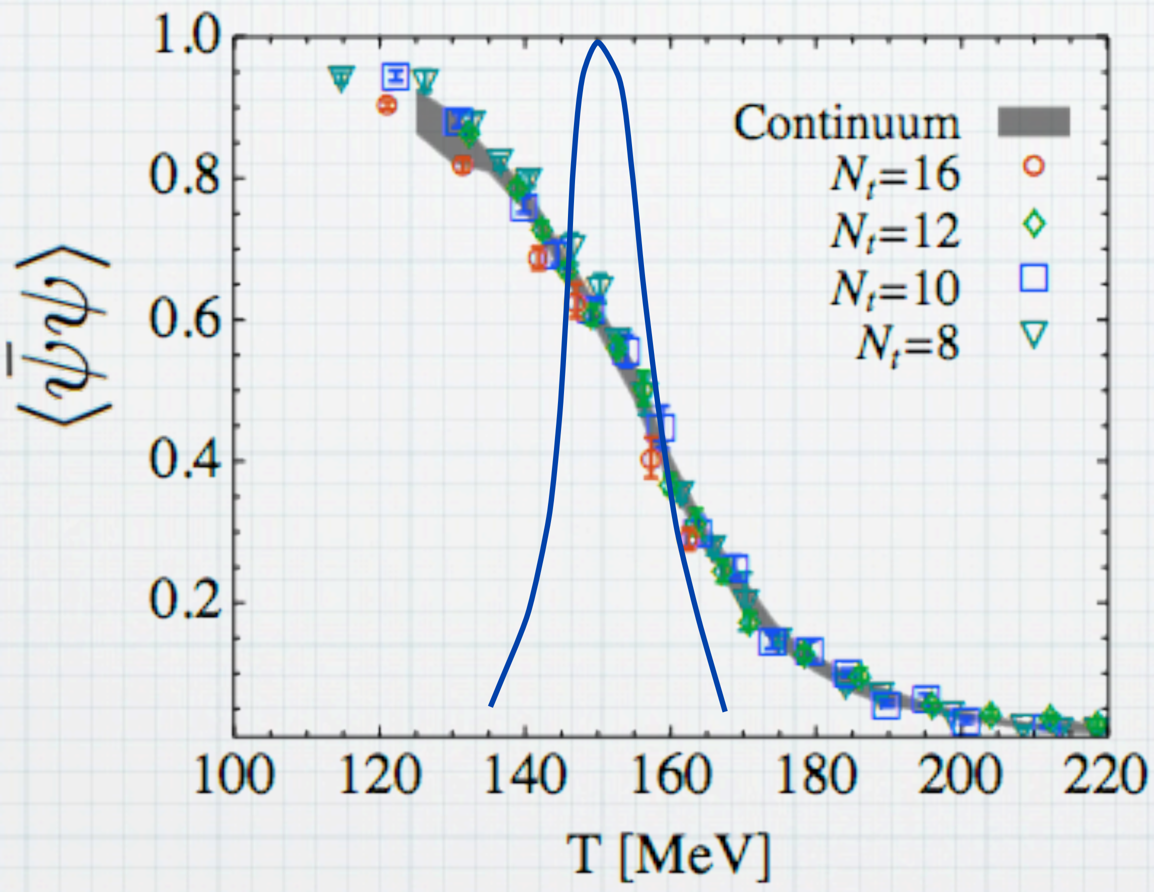

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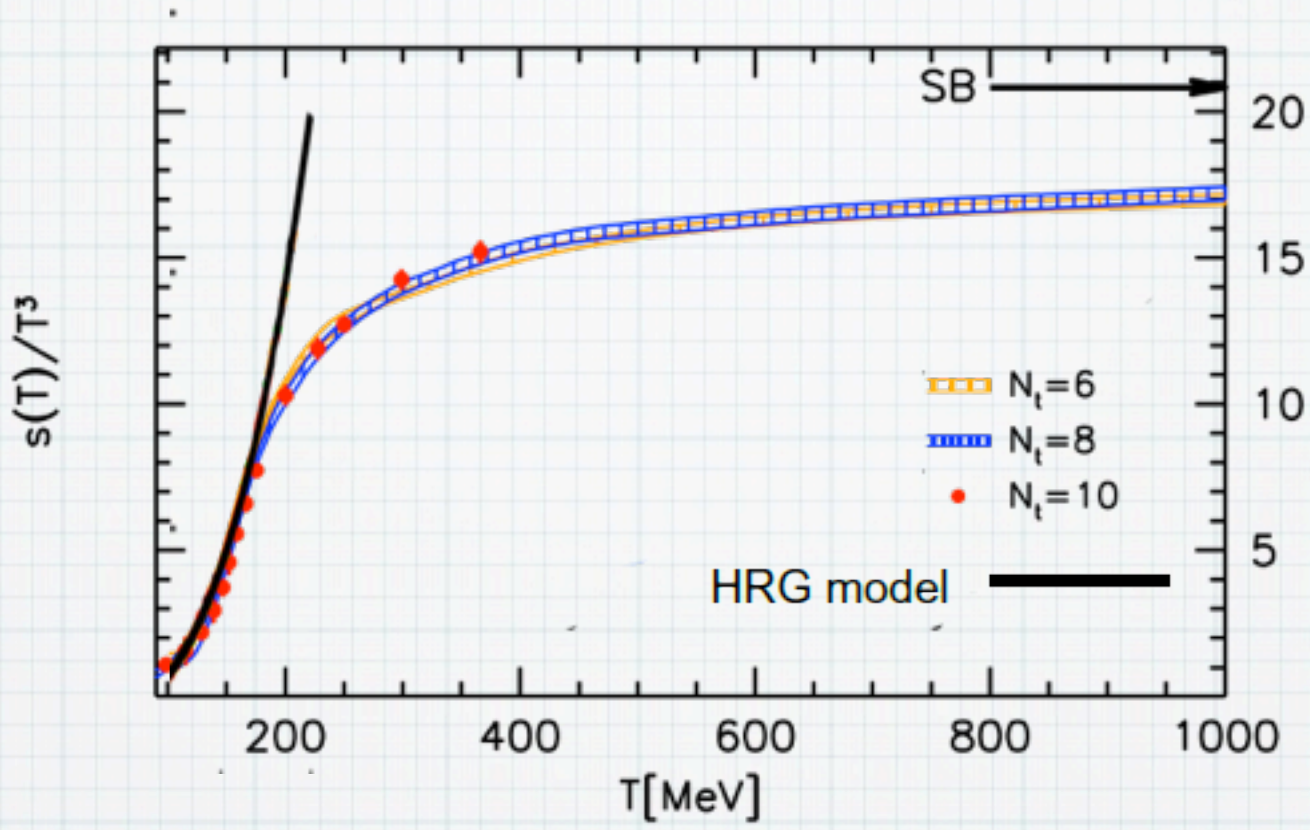
Chiral condensate: order parameter for chiral phase transition

Transition from QCD Thermodynamics



* s/T^3 indicates the number of particle species
* Rapid rise = liberation of degrees of freedom
* Compare to an ideal gas of quarks and gluons

$$s = \frac{4g}{\pi^2} T^3$$

* This gives us an idea of how strong is the interaction

What happens below T_c?

* At low T and μ_B=0, QCD thermodynamics is dominated by **pions**

* as T increases, heavier hadrons start to contribute

* Their mutual interaction is suppressed:

$$n_i n_k \sim \exp[-(M_i + M_k)/T]$$

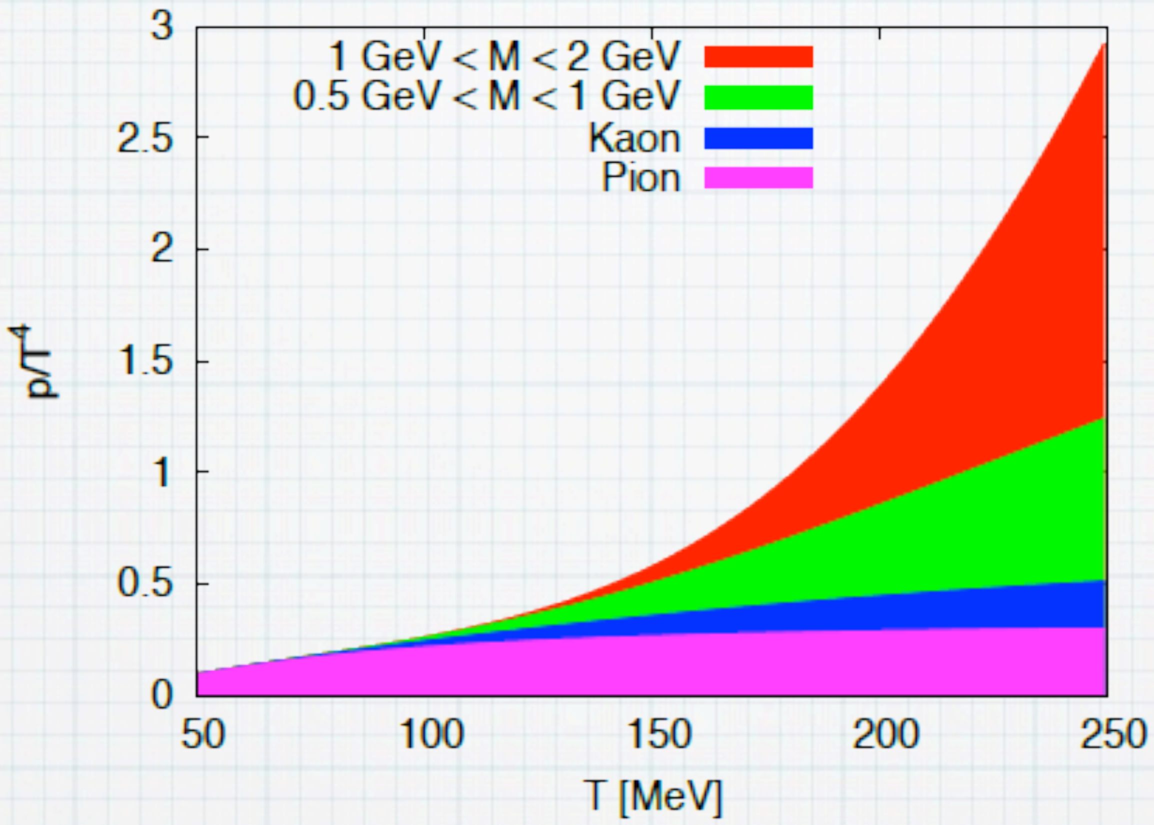
* **Interacting** hadronic matter in the ground state can be well approximated by a **non-interacting** gas of **hadronic resonances**

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \, mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \, baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a}) \, ,$$

with $\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T})$, $\varepsilon_i = \sqrt{k^2 + m_i^2}$,

$z_i = \exp\left((\sum_a X^a_i \mu_{X^a})/T \right)$ and X^a are all conserved charges.

How many resonances do we include?

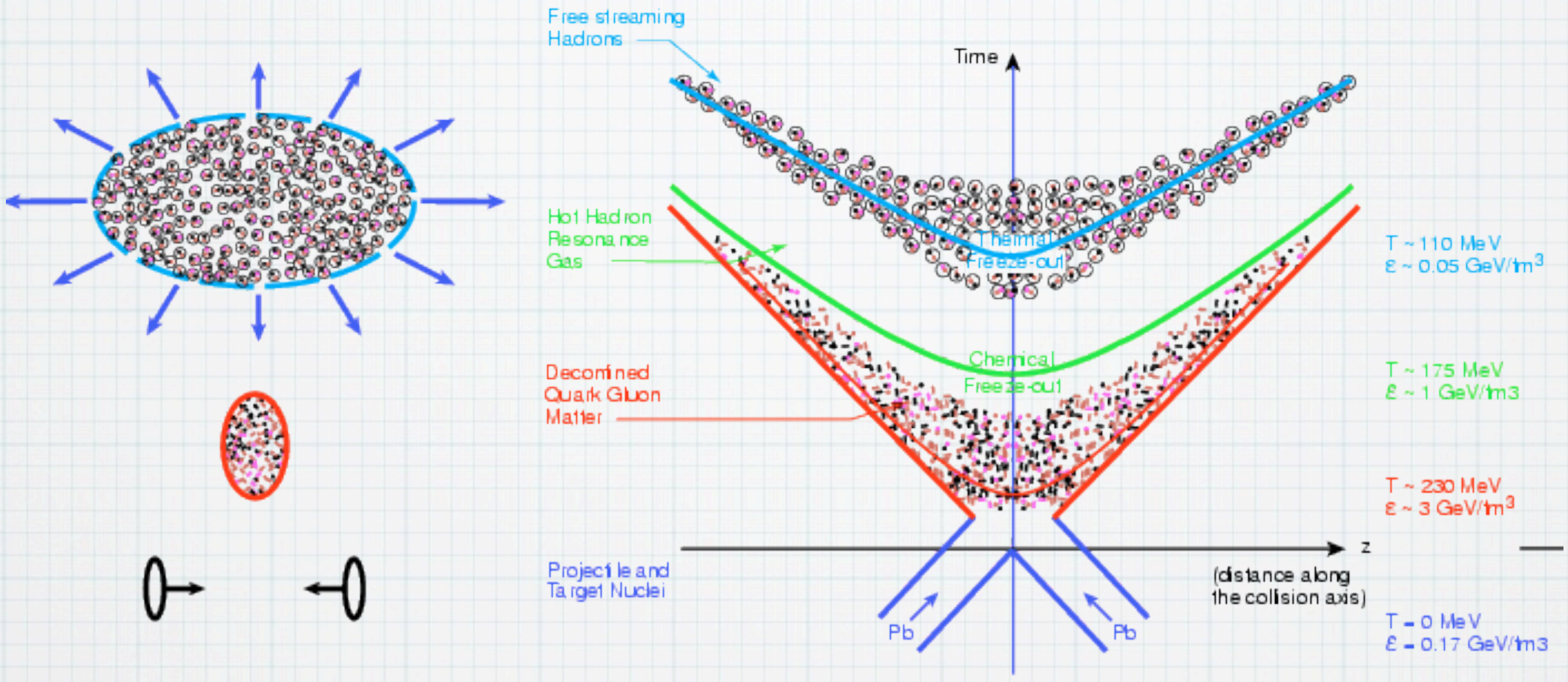


* With different mass cut-offs we can separate the contribution of different particles

* Known resonances up to M=2.5 GeV

* ~170 different masses ⟷ 1500 resonances

Evolution of a heavy-ion collision

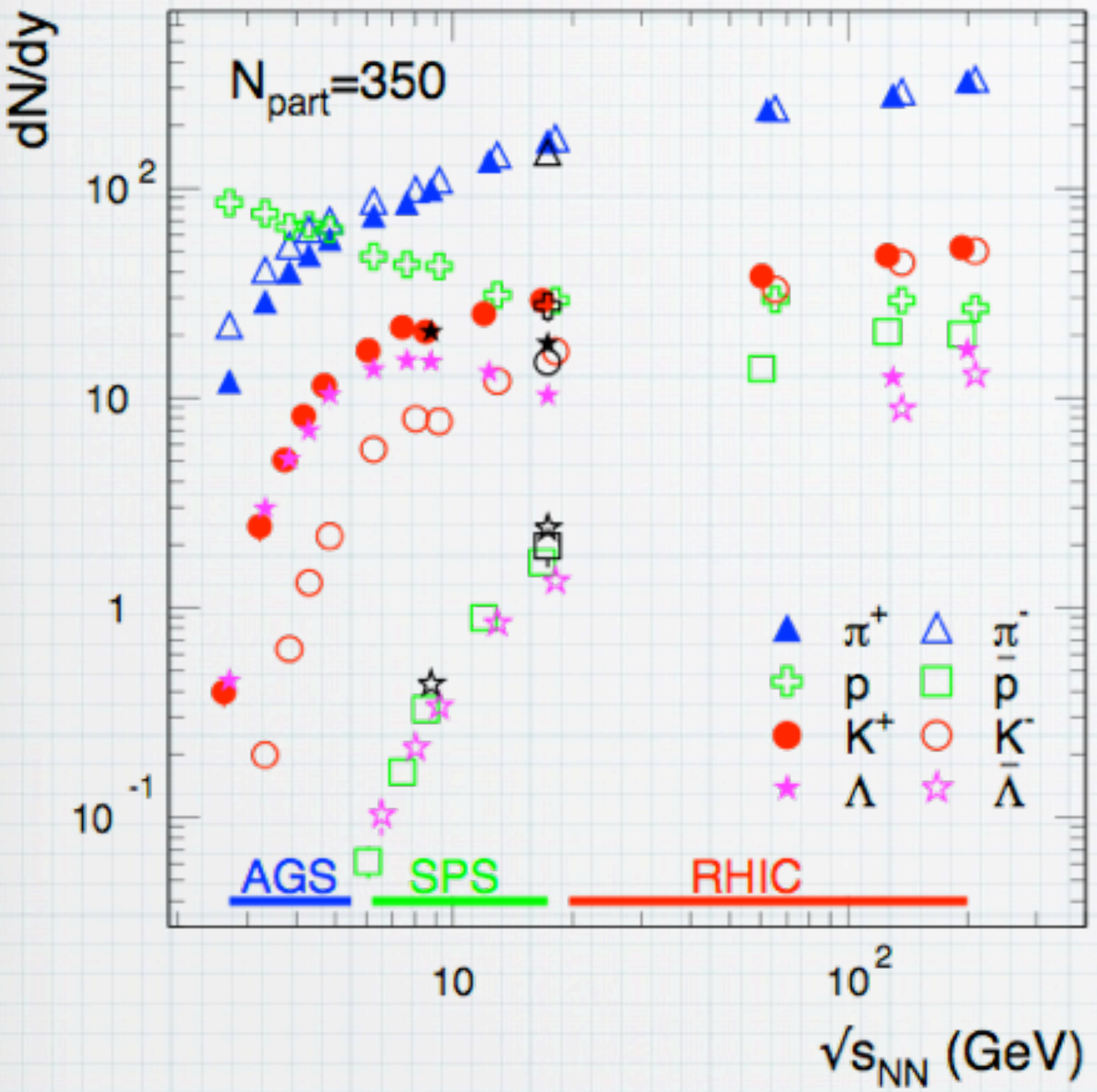


* **Chemical freeze-out**: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)

* **Kinetic freeze-out**: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)

* Hadrons reach the detector

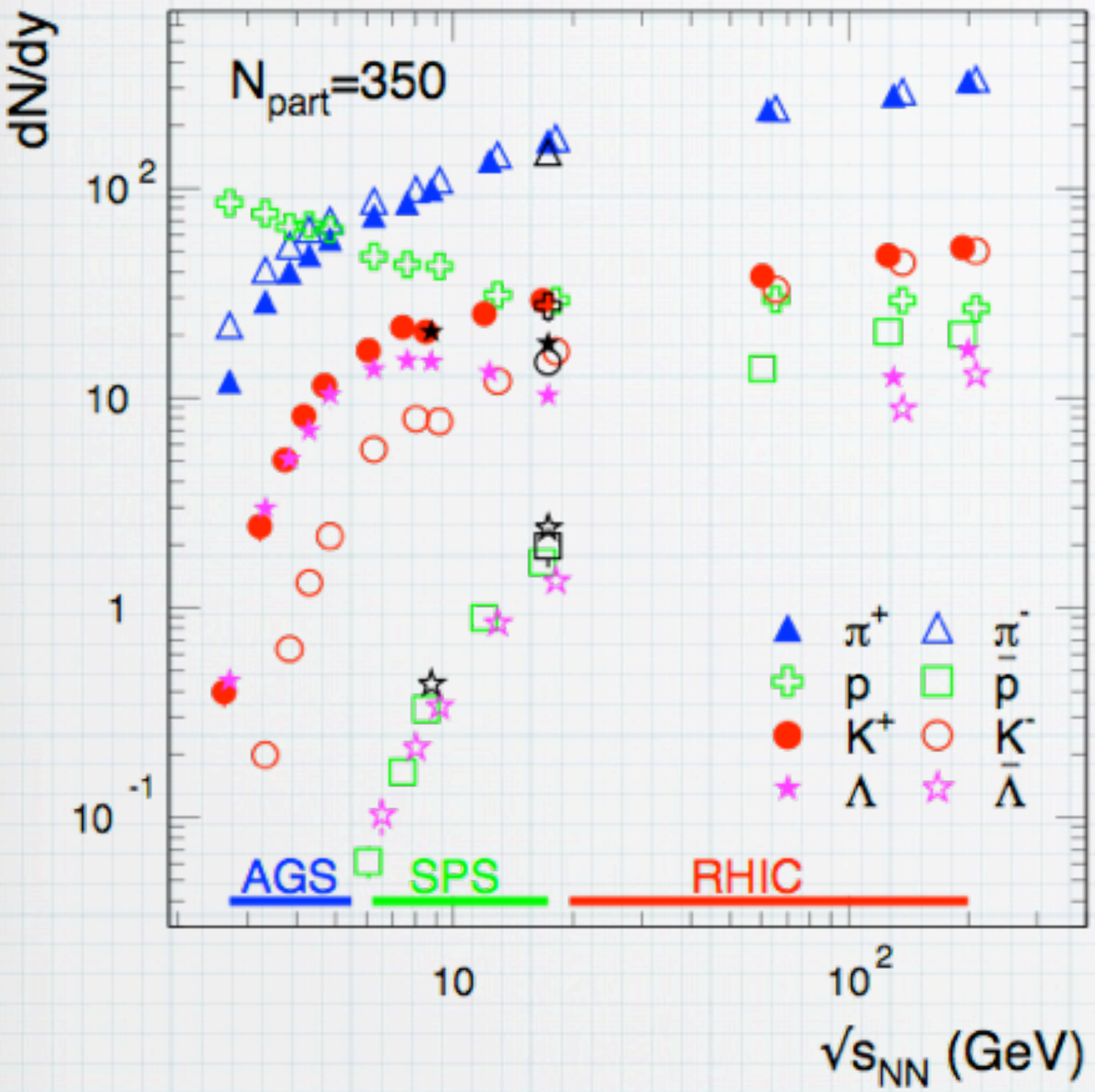
Hadron yields



- **$E=mc^2$**: lots of particles are created
- **Particle counting** (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays
- **HRG model**: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T] \pm 1}$$

Hadron yields



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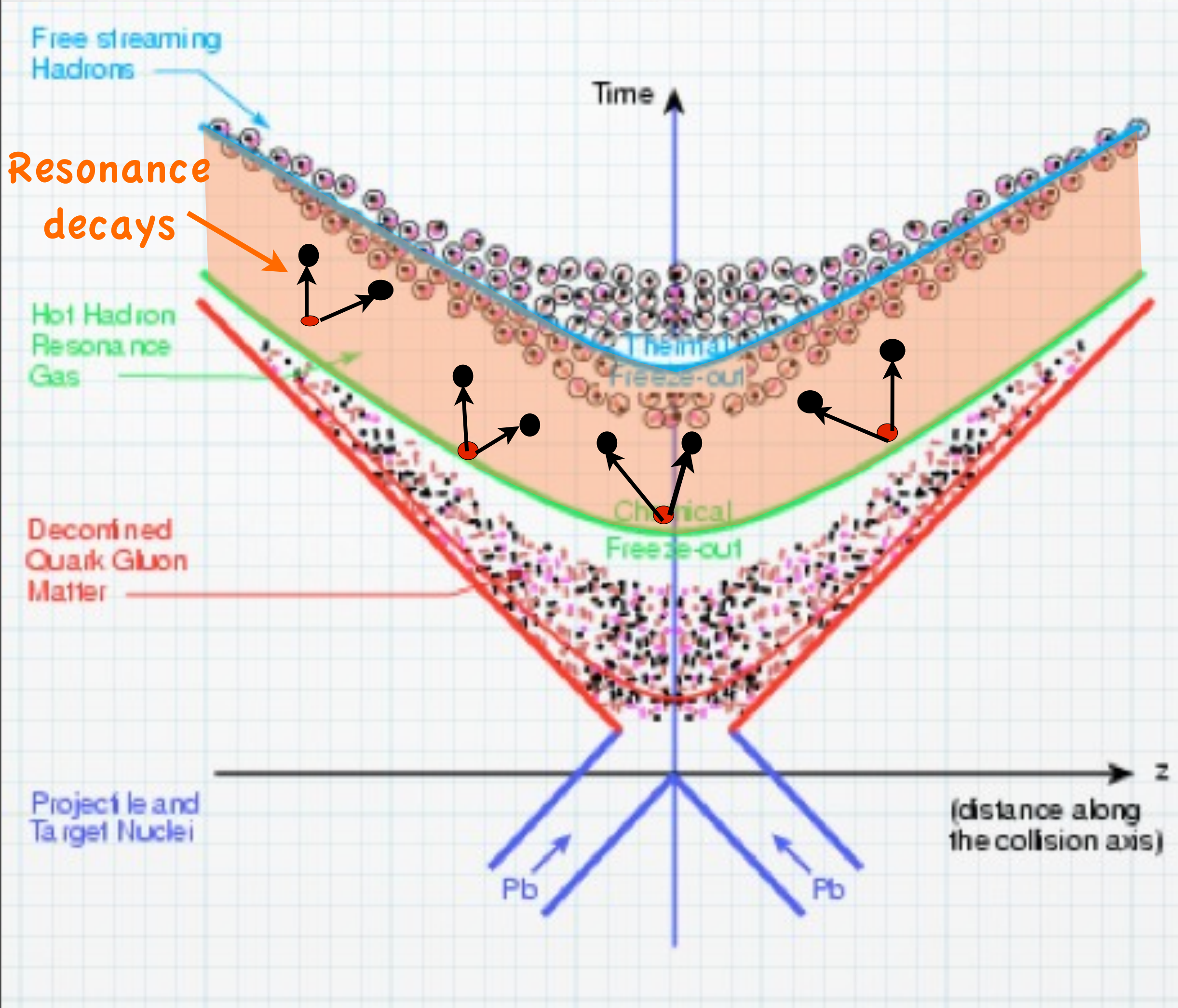
- **HRG model**: test hypothesis of hadron abundancies in equilibrium
- We need:
  - a complete hadron spectrum
  - control the hadron fraction from decays

Decays



* All hadrons are subject to **strong and electromagnetic decays**

$$\Delta \to p(n) + \pi,\ \rho \to \pi + \pi$$

* e.g. pions: **1/4** primordial, **3/4** from strong decays

* Weak decays can be treated too:

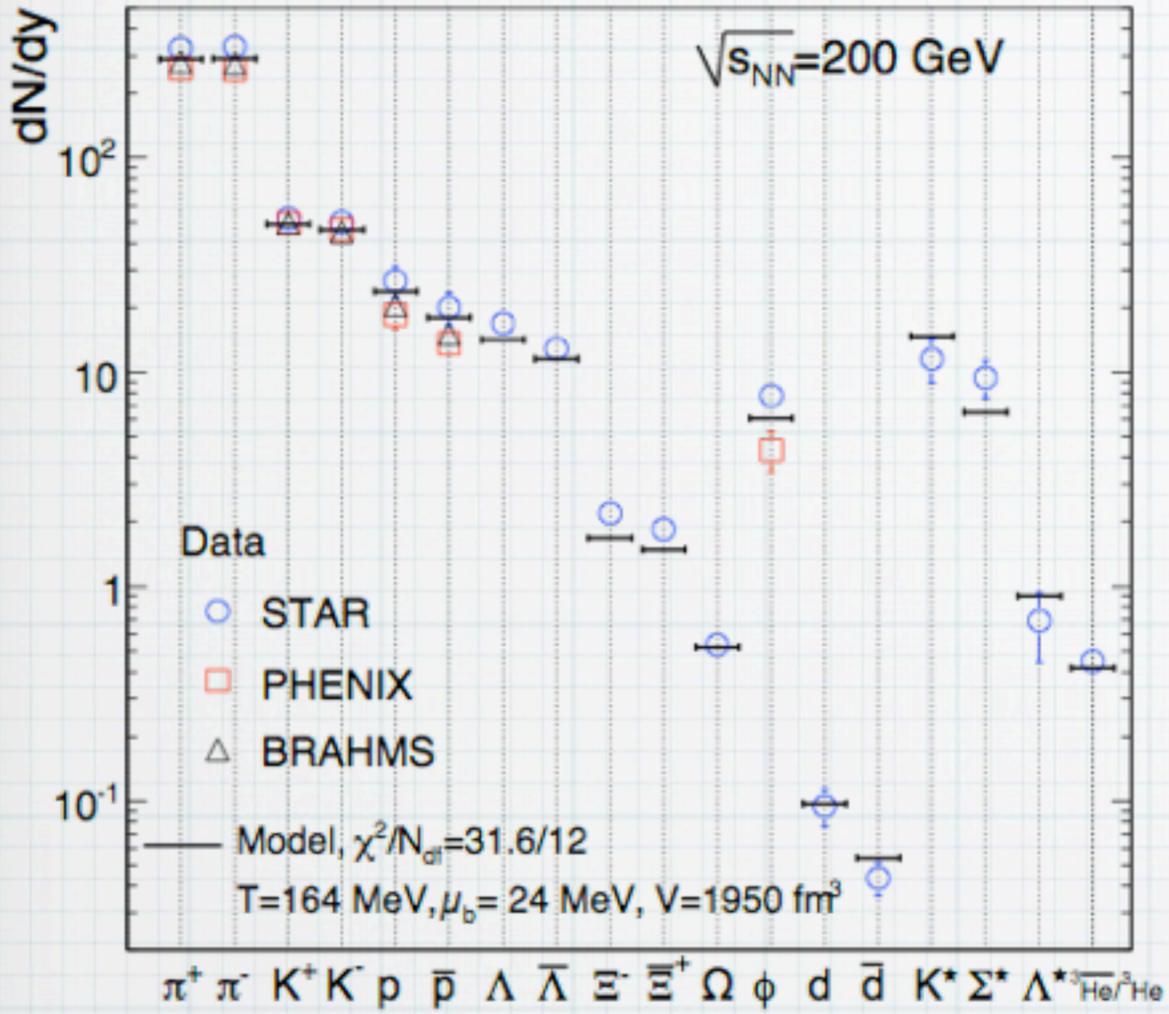
$$\Sigma \to \Lambda + \gamma$$

* **after chemical freeze-out:** only elastic and quasi-elastic scatterings take place:

$$\pi\pi \to \rho \to \pi\pi \qquad p\pi \to \Delta \to p\pi \qquad K\pi \to K^* \to K\pi$$

$$\bar{N}_i = N_i + \sum_r d_{r\to i} N_r$$

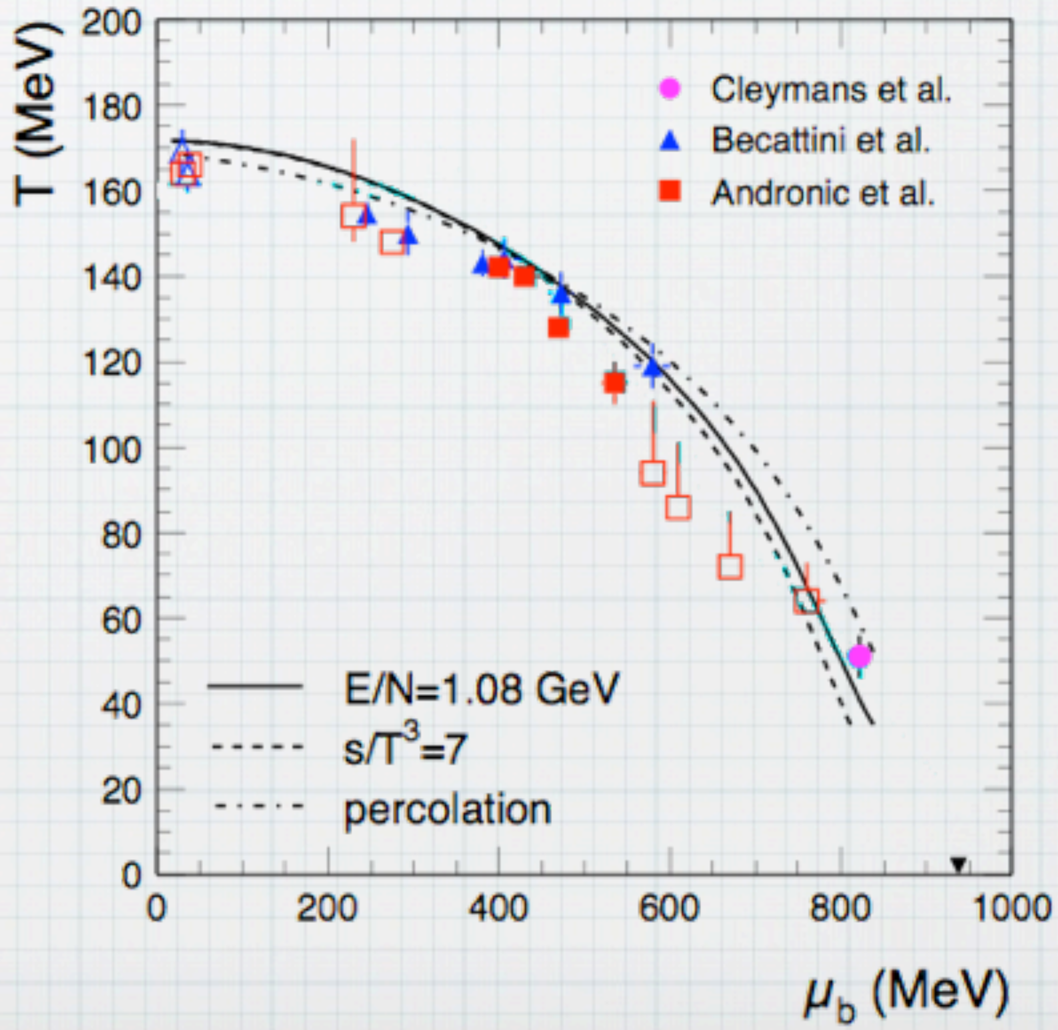
The thermal fits



* Fit is performed minimizing the χ^2

* **Fit to yields**: parameters T, μ_B, V

* **Fit to ratios**: the volume V cancels out



* Changing the collision energy, it is possible to draw the freeze-out line in the T, μ_B plane

Caveats

* These results are model-dependent
    * they depend on the particle spectrum which is included in the model
    * possibility of having heavier states with exponential mass spectrum
    * not known experimentally but can be postulated
    * their decay modes are not known

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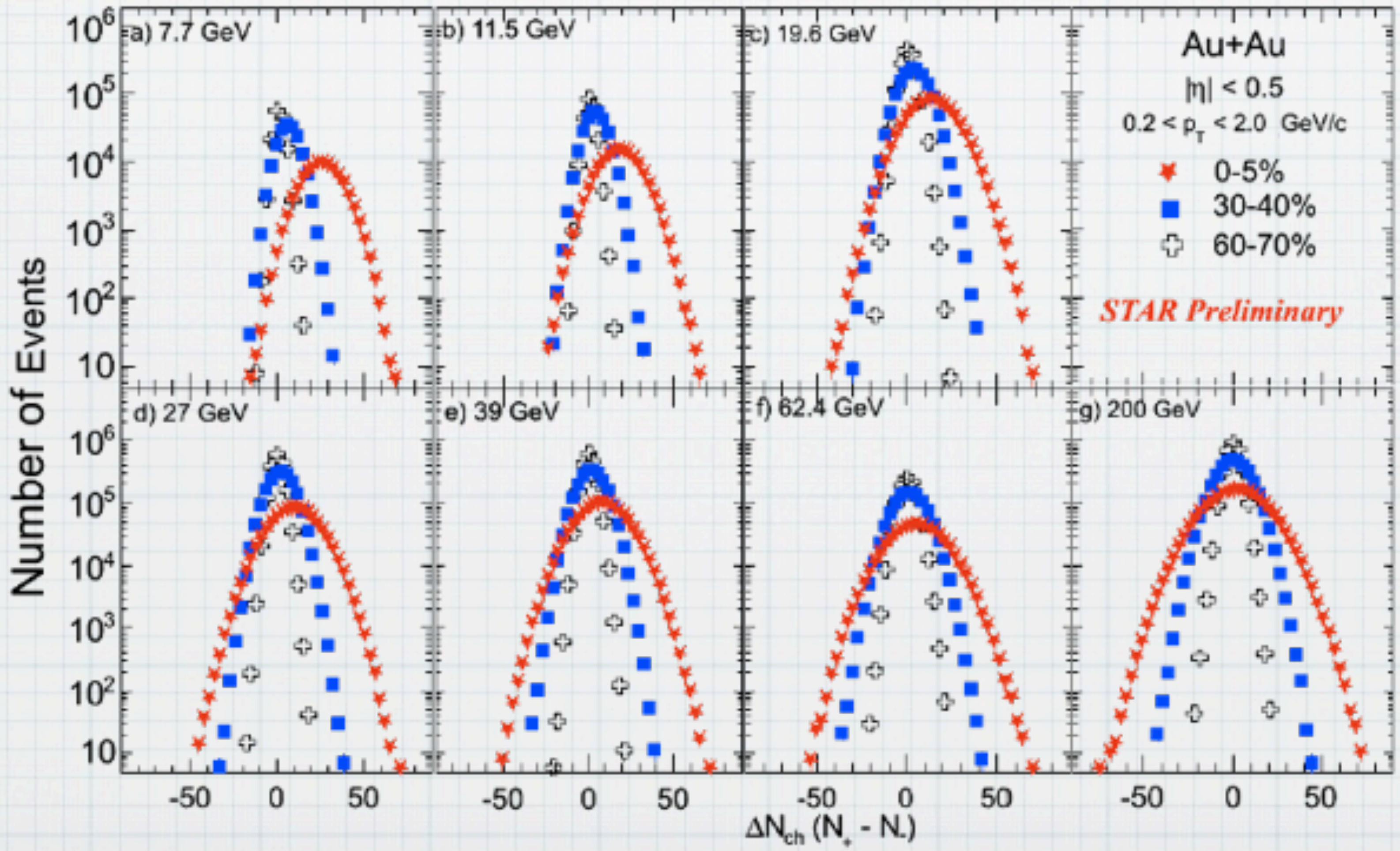
* Purpose: extract freeze-out parameters from first principles
  * direct comparison between experimental measurement and lattice QCD results
  * observable: fluctuations of conserved charges (electric charge, baryon number and strangeness)
  * directly related to moments of multiplicity distribution (measured)
  * lattice QCD looks at conserved charges rather than identified particles

Fluctuations of conserved charges

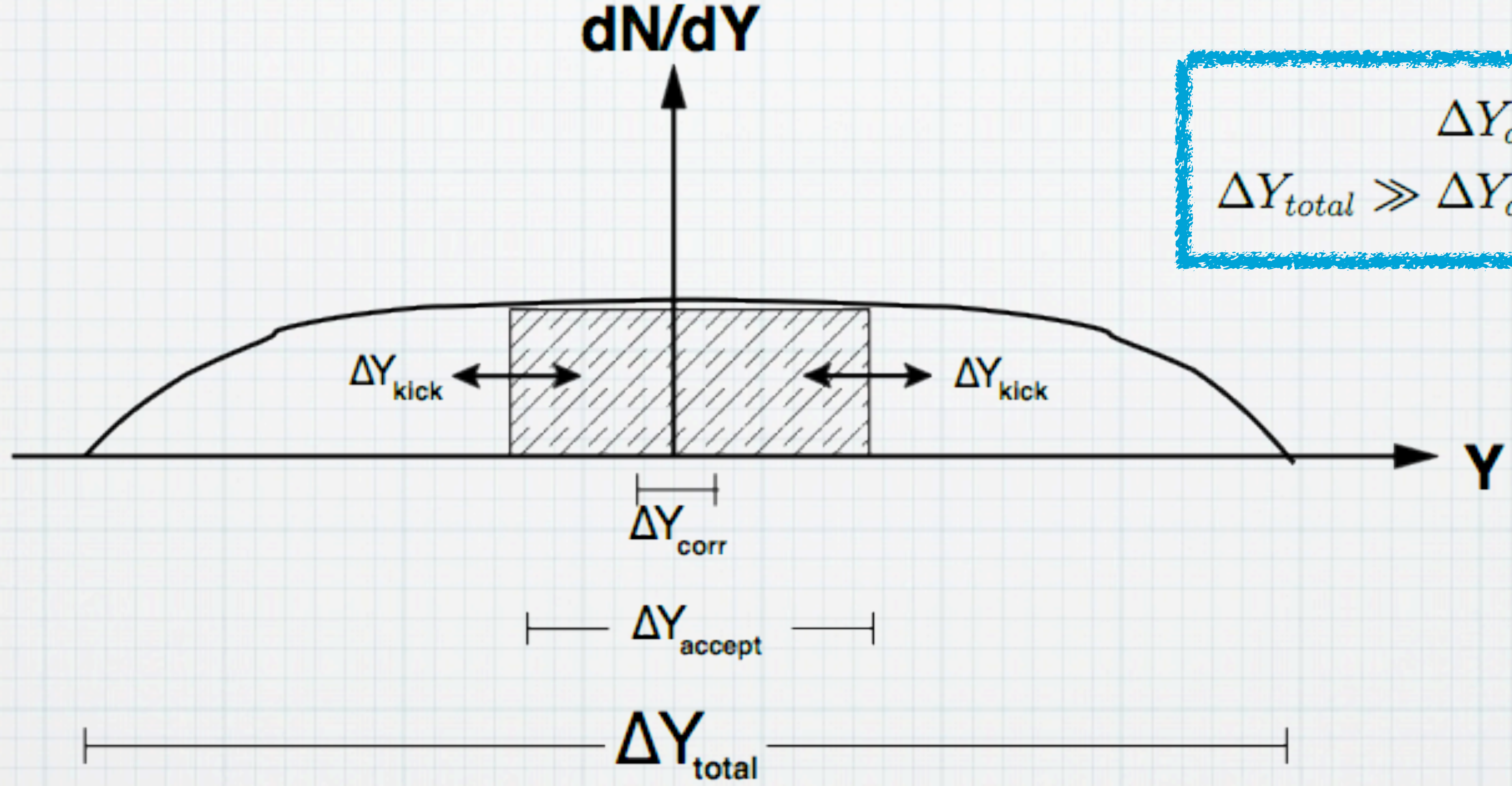
- Consider the number of electrically charged particles N_Q
- Its average value over the whole ensemble of events is $\langle N_Q \rangle$
- In experiments it is possible to **measure its event-by-event distribution**



Fluctuations of conserved charges???

- If we look at the entire system, none of the conserved charges will fluctuate

- By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{accept} \gg \Delta Y_{corr}$$
$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

➡ ΔY_{total}: range for total charge multiplicity distribution

➡ ΔY_{accept}: interval for the accepted charged particles

➡ ΔY_{corr}: charge correlation length characteristic to the physics of interest

➡ ΔY_{kick}: rapidity shift that charges receive during and after hadronization

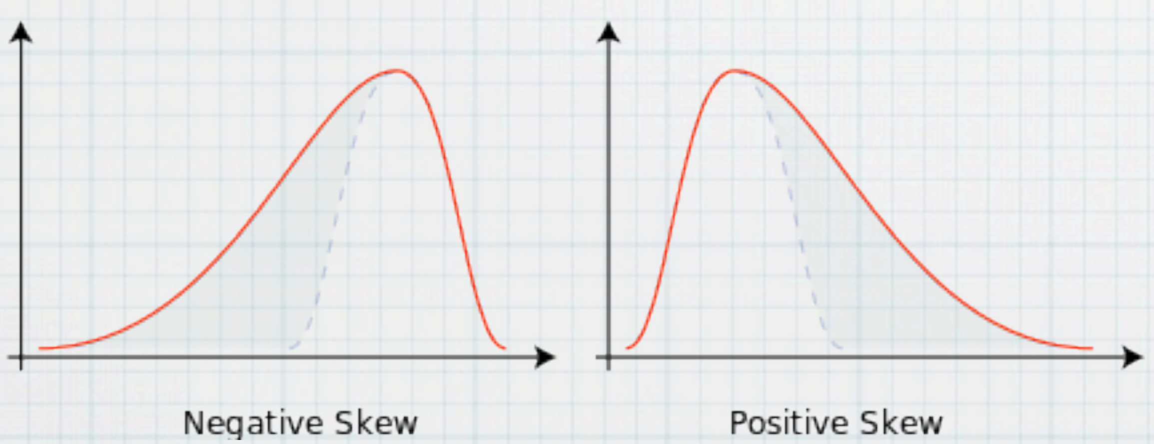
Cumulants of multiplicity distribution

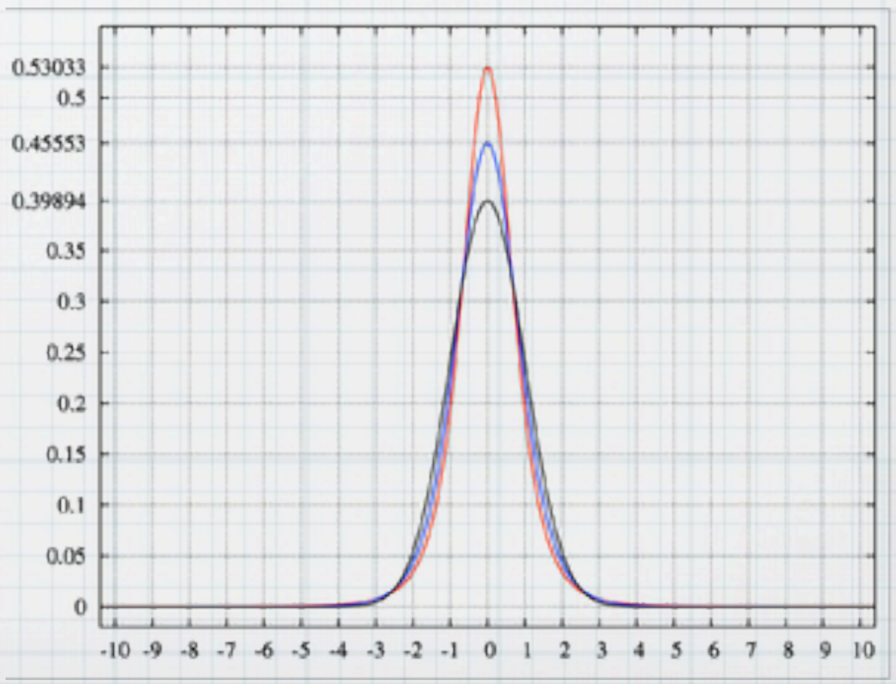
* Deviation of N_Q from its mean in a single event: $\delta N_Q = N_Q - \langle N_Q \rangle$
* The cumulants of the event-by-event distribution of N_Q are:

$$K_2 = \langle (\delta N_Q)^2 \rangle \qquad K_3 = \langle (\delta N_Q)^3 \rangle \qquad K_4 = \langle (\delta N_Q)^4 \rangle - 3\langle (\delta N_Q)^2 \rangle^2$$

* The cumulants are related to the central moments of the distribution by:

variance: $\sigma^2 = K_2$ Skewness: $S = K_3/(K_2)^{3/2}$ Kurtosis: $\kappa = K_4/(K_2)^2$

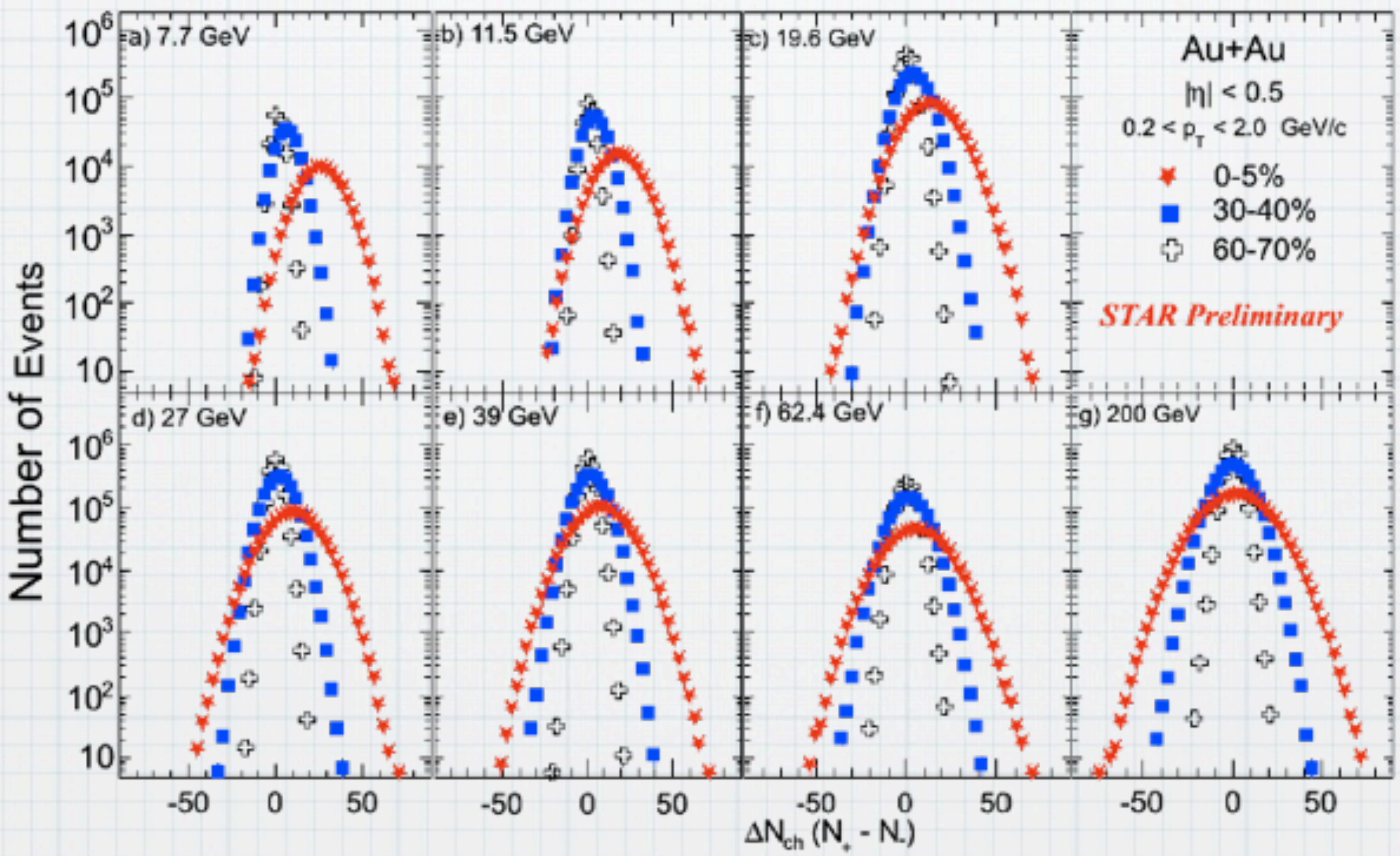




Experimental measurement

* Volume-independent ratios:

$M/\sigma^2 = K_1/K_2$ $\quad S\sigma = K_3/K_2$ $\quad \kappa\sigma^2 = K_4/K_2$ $\quad S\sigma^3/M = K_3/K_1$



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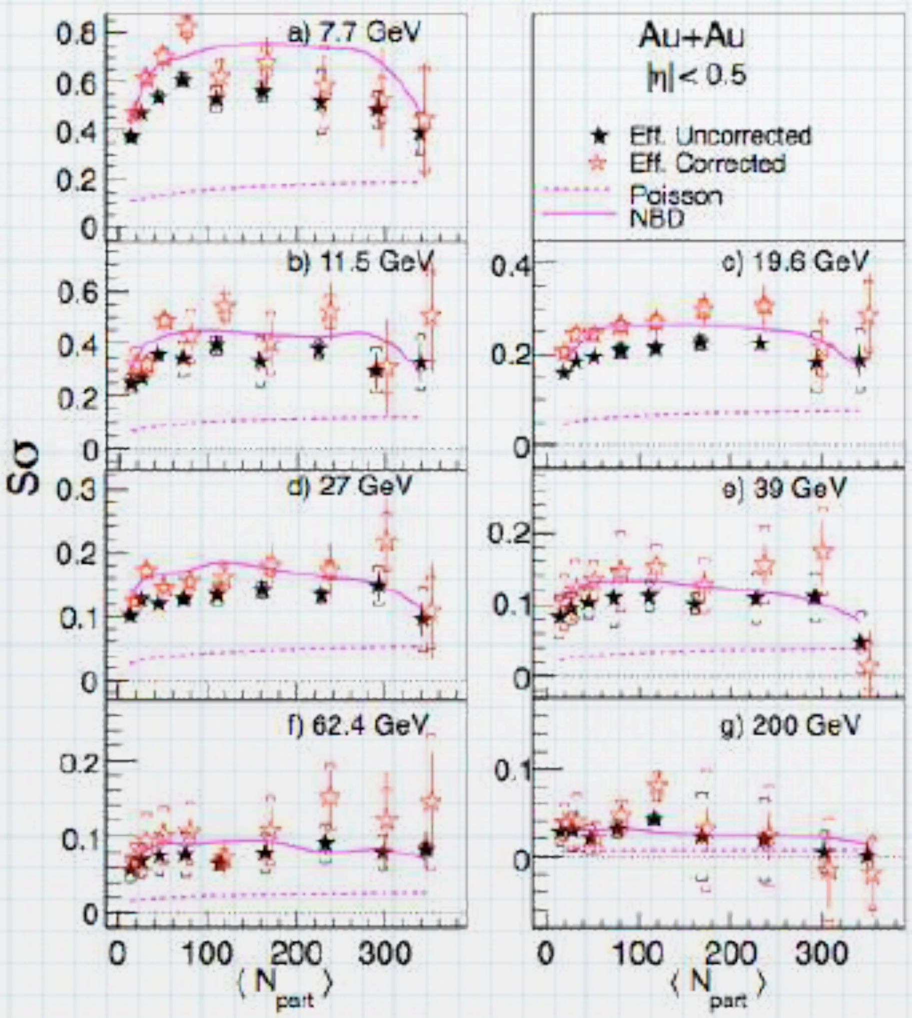
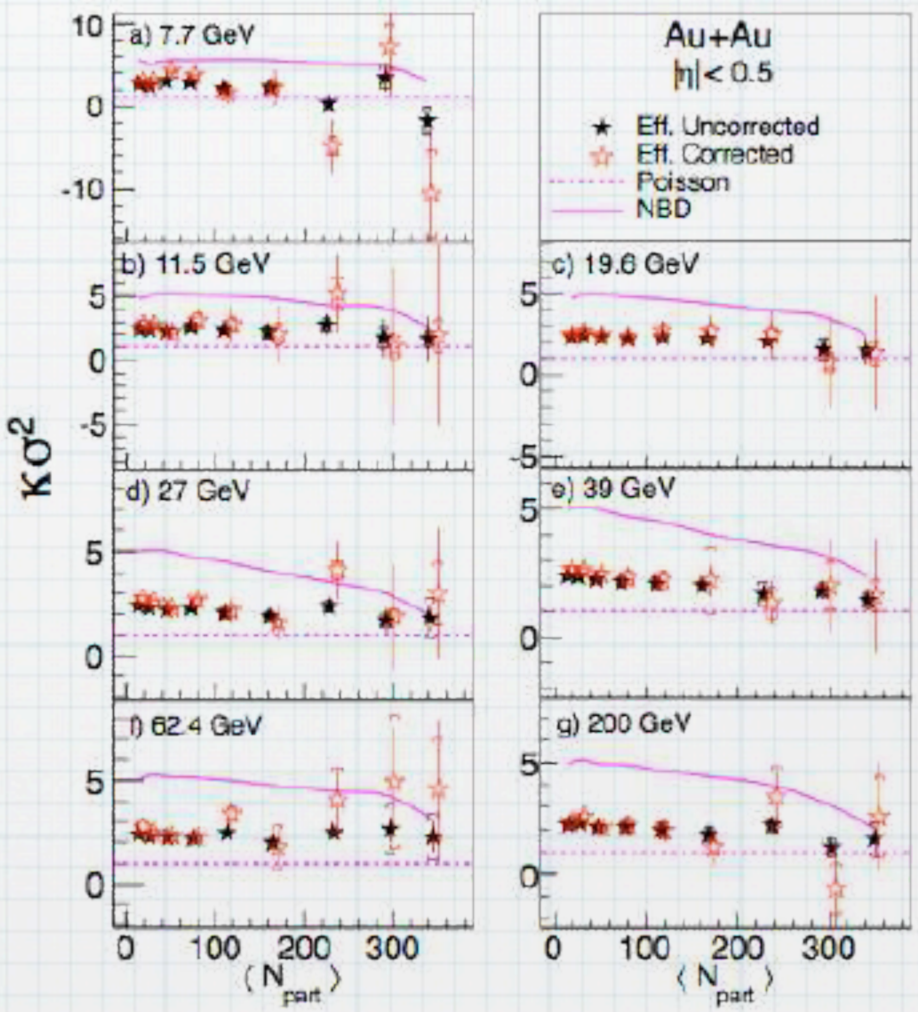




Susceptibilities of conserved charges

* Susceptibilities of conserved charges

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

* Diagonal second-order susceptibilities measure the response of the **charge density** to an infinitesimal change in the **chemical potential**

$$\chi_2^X = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)^2} = \frac{\partial}{\partial(\mu_X/T)} \left(n_X/T^3\right)$$

➡ A **rapid increase** of these observables in a certain temperature range signals a **phase transition**

* **Non-diagonal** susceptibilities measure the **correlation** between different charges

$$\chi_{11}^{XY} = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)\partial(\mu_Y/T)} = \frac{\partial}{\partial(\mu_Y/T)} \left(n_X/T^3\right)$$

➡ Information about the **strength of the interaction**

Linking lattice QCD and experiment

* **Susceptibilities** of conserved charges are the **cumulants** of their event-by event distribution

$$\text{mean}: \; M = \chi_1 \qquad \text{variance}: \; \sigma^2 = \chi_2$$

$$\text{skewness}: \; S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \; \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

* Lattice QCD results are functions of **temperature** and **chemical potential**

➡ By comparing lattice results and experimental measurement we can **extract the freeze-out parameters from first principles**

Baryometer and thermometer

* Let us look at the Taylor expansion of R^B_{31}

$$R^B_{31}(T,\mu_B) = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B(T,0) + \chi_{31}^{BQ}(T,0)q_1(T) + \chi_{31}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)} + \mathcal{O}(\mu_B^2)$$

* To order μ^2_B it is independent of μ_B: it can be used as a **thermometer**

* Let us look at the Taylor expansion of R^B_{12}

$$R^B_{12}(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

* Once we extract T from R^B_{31}, we can use R^B_{12} to extract μ_B

Caveats

* Effects due to volume variation because of finite centrality bin width

* Finite reconstruction efficiency

* Spallation protons

* Canonical vs Gran Canonical ensemble

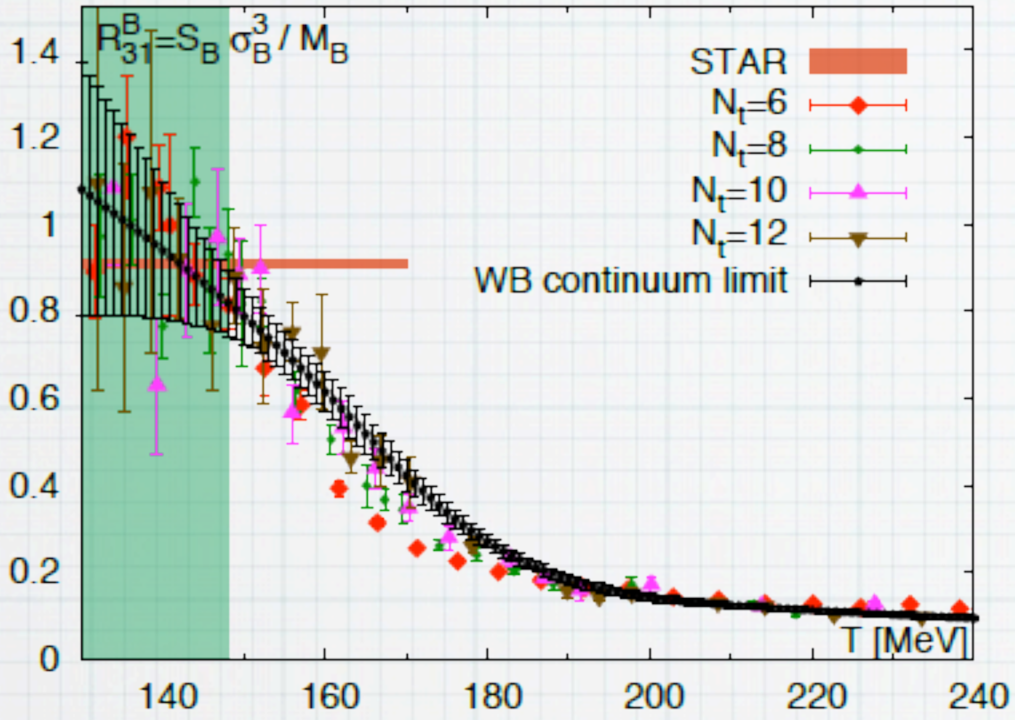
* Proton multiplicity distributions vs baryon number fluctuations

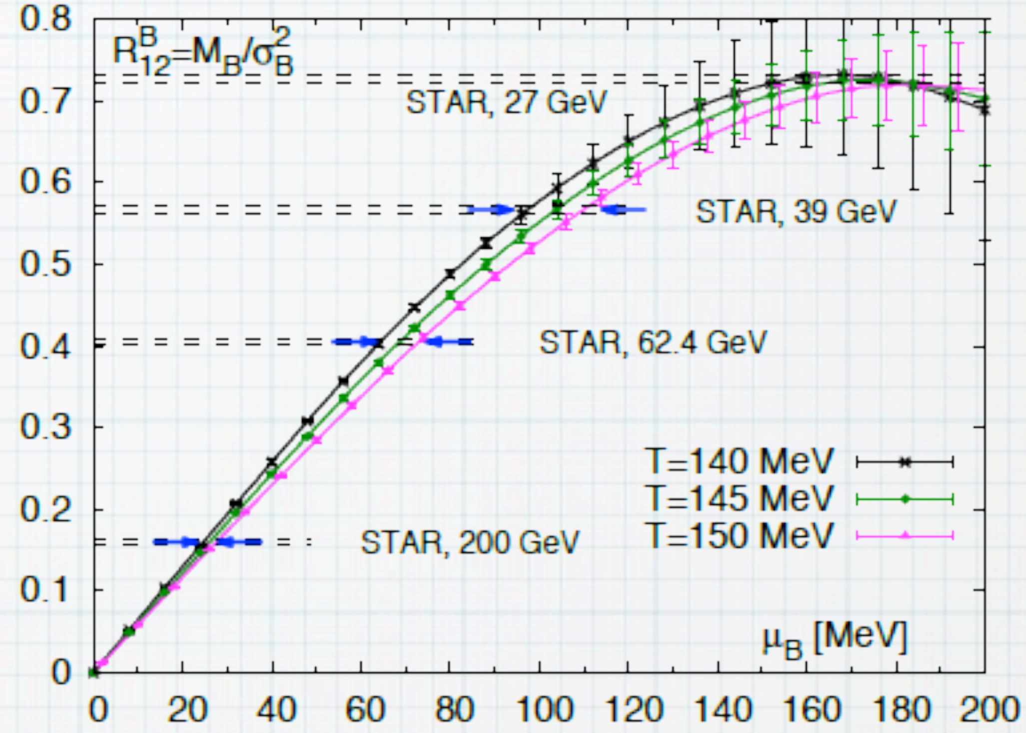
* Final-state interactions in the hadronic phase

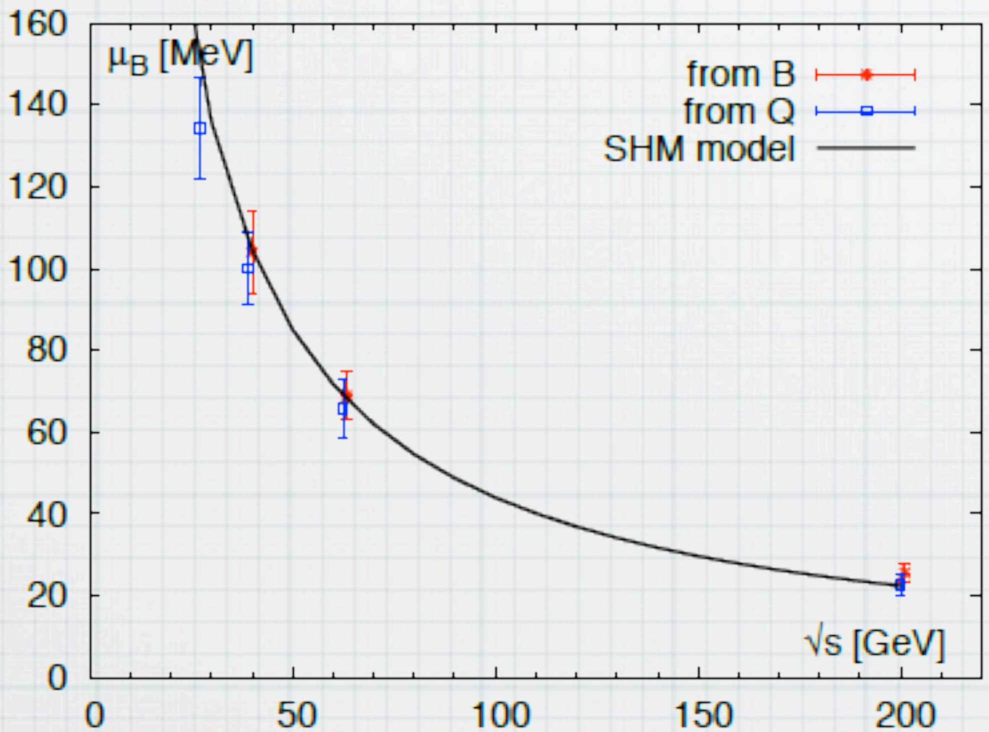
Caveats

- Effects due to volume variation because of finite centrality bin width
  - ➡ Experimentally corrected by centrality-bin-width correction method
- Finite reconstruction efficiency
  - ➡ Experimentally corrected based on binomial distribution
- Spallation protons
  - ➡ Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - ➡ Experimental cuts in the kinematics and acceptance
- Proton multiplicity distributions vs baryon number fluctuations
  - ➡ Numerically very similar once protons are properly treated
- Final-state interactions in the hadronic phase
  - ➡ Consistency between different charges = fundamental test

Results



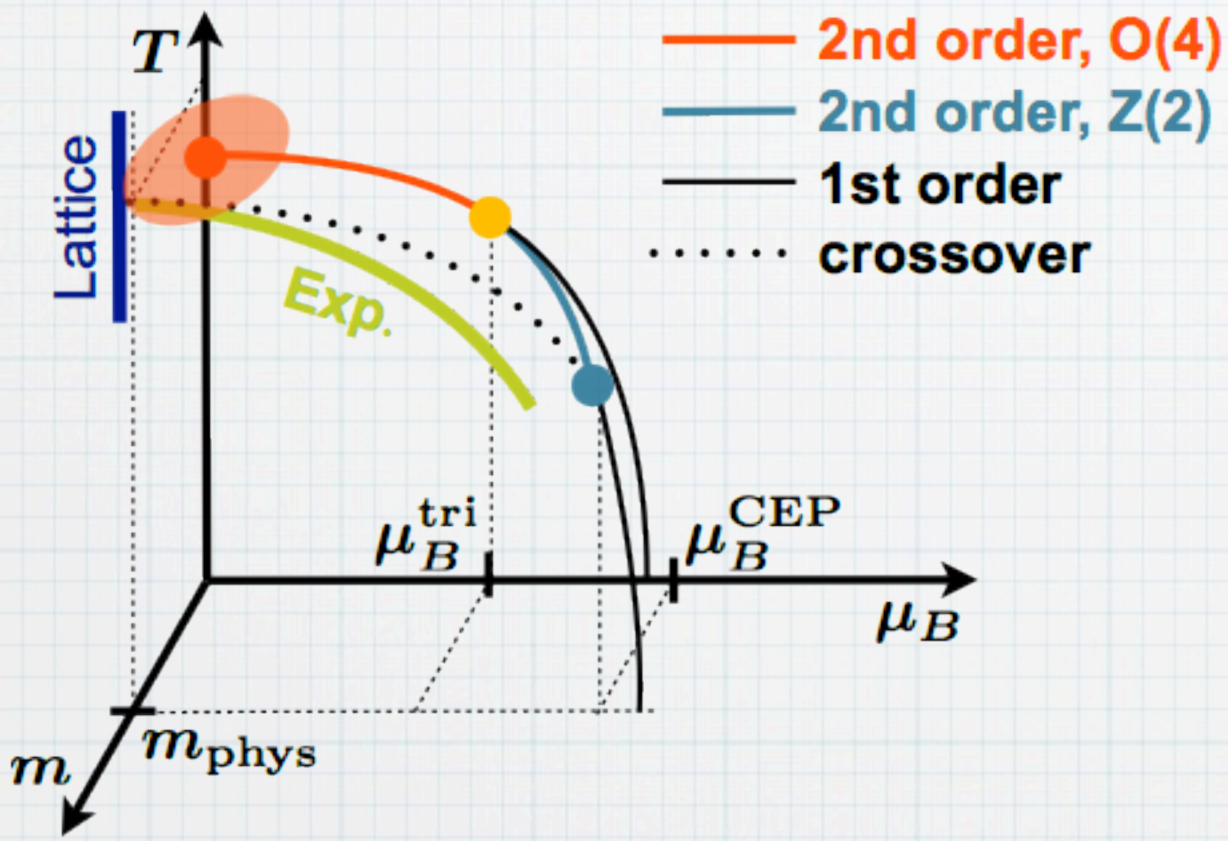




Consistency between different charges and with SHM fits

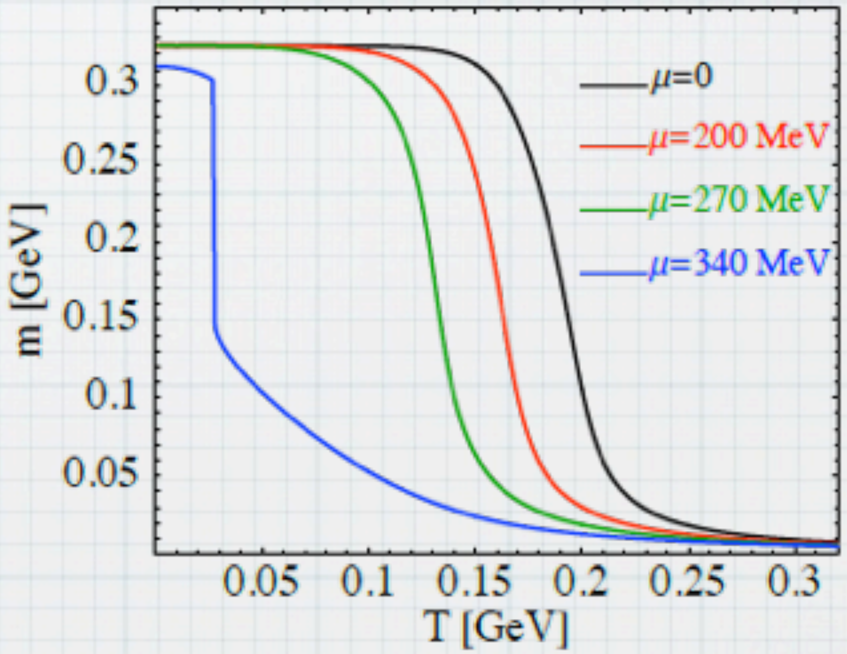
Our world is not ideal:

neither chiral symmetry (m_q=0) nor confinement (m_q=∞) is well defined.



Existence of QCD critical point predicted by models

Z(2) universality class



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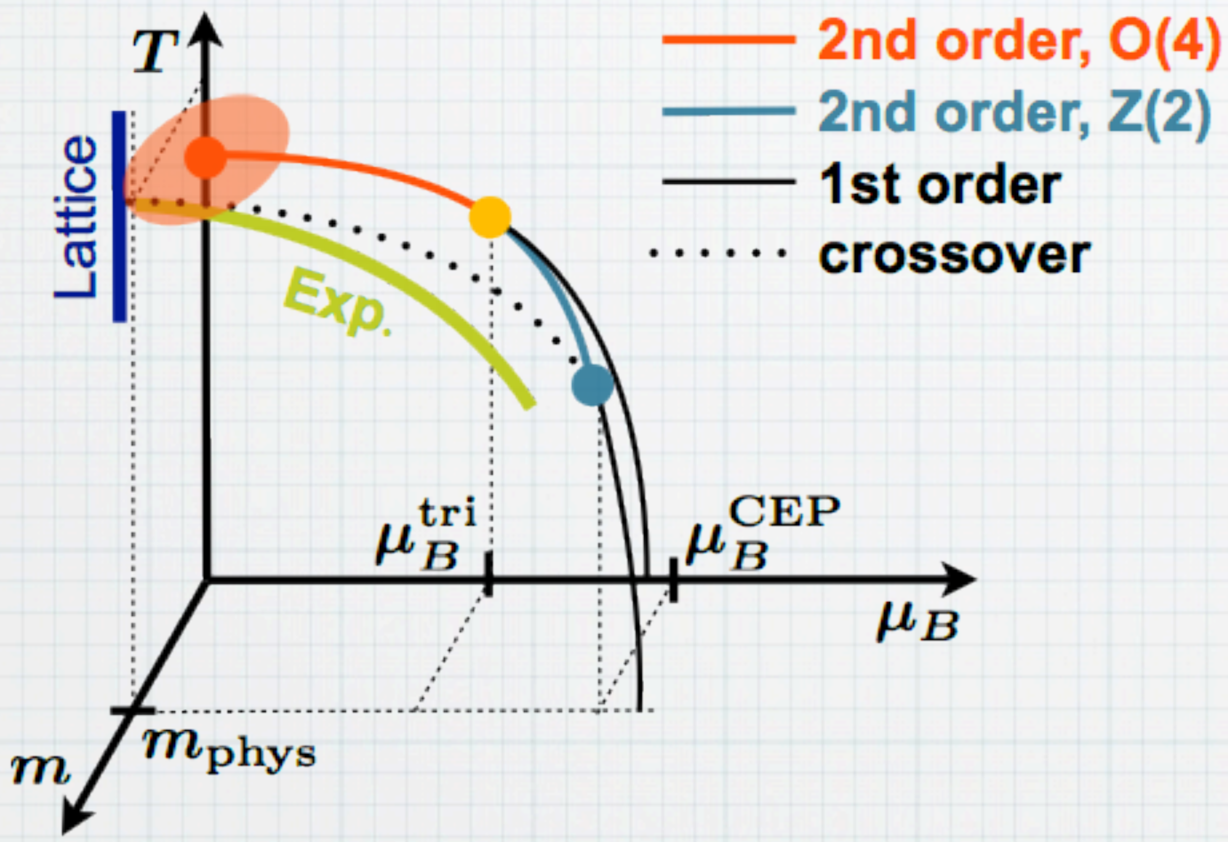

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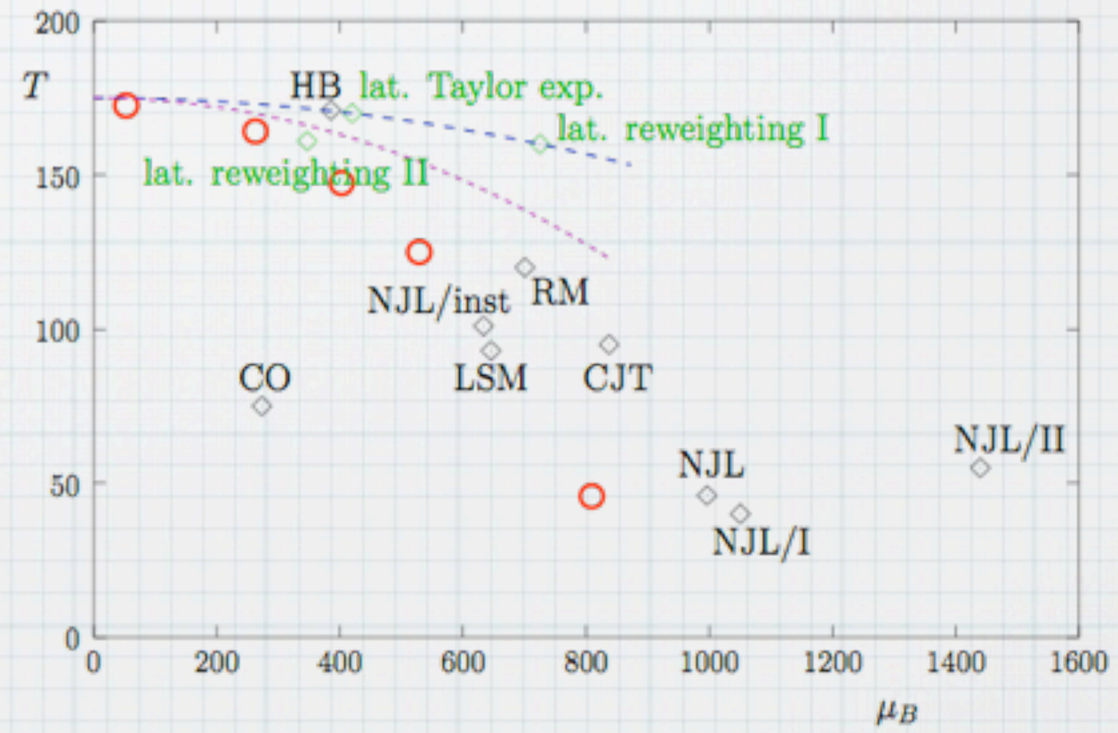
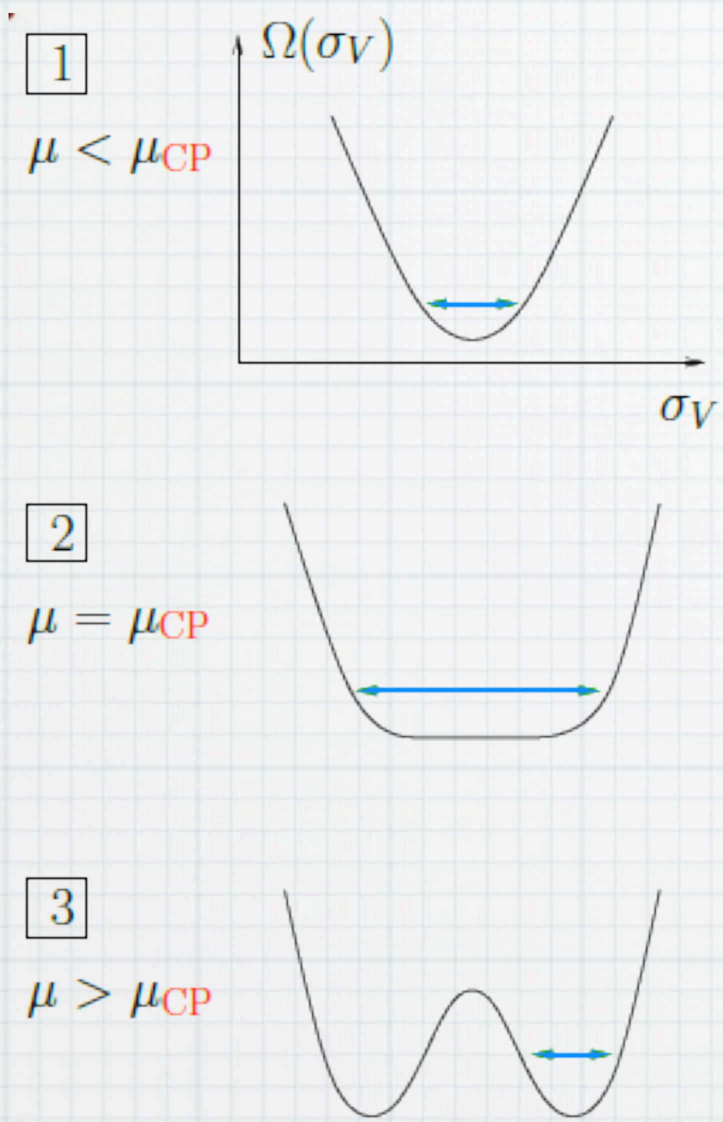


Fluctuations at the critical point



Consider the order parameter for the chiral phase transition σ ~ $\bar{\psi}\psi$

It has a probability distribution of the form:

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right].$$

where: $m_\sigma \equiv \xi^{-1}$

and, near the critical point:

$$\lambda_3 = \tilde{\lambda}_3\, T\, (T\,\xi)^{-3/2}, \quad \text{and} \ \ \lambda_4 = \tilde{\lambda}_4\, (T\,\xi)^{-1}$$

$$\langle \sigma_V^3 \rangle = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}$$

$$\langle \sigma_V^4 \rangle = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7$$

correlation length ξ is limited due to critical slowing down, together with the finite time the system has to develop the correlations: ξ<2-3 fm

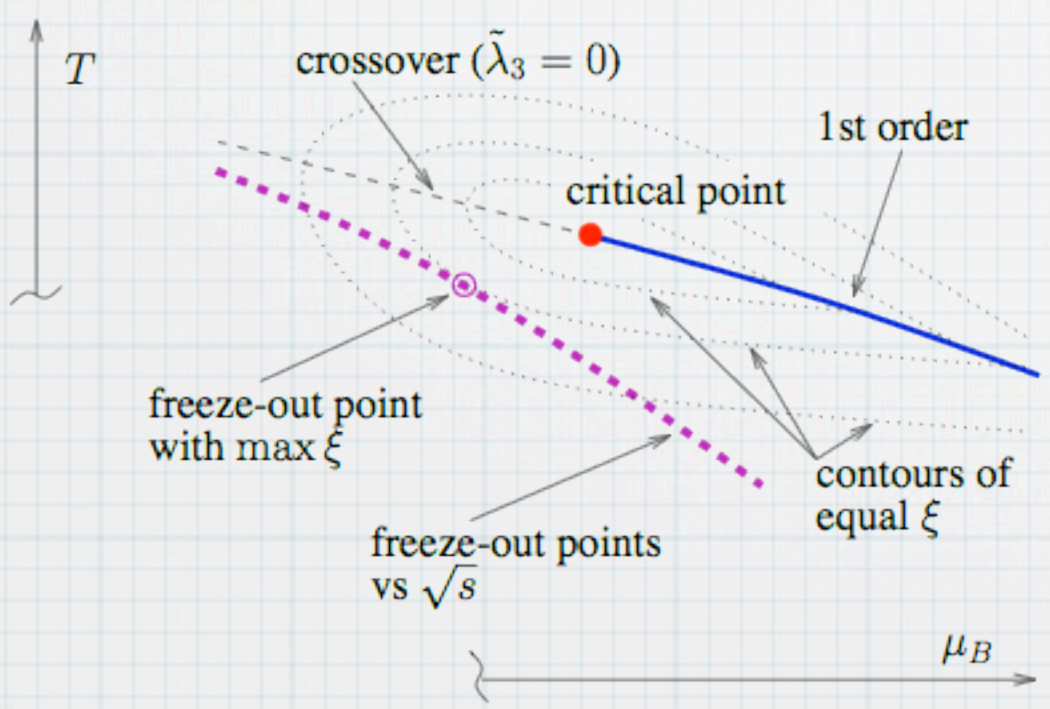
Experimental fluctuations

We consider the fluctuation of an observable (e.g. protons)

$$\delta N = \sum_{\boldsymbol{p}} \delta n_{\boldsymbol{p}}$$

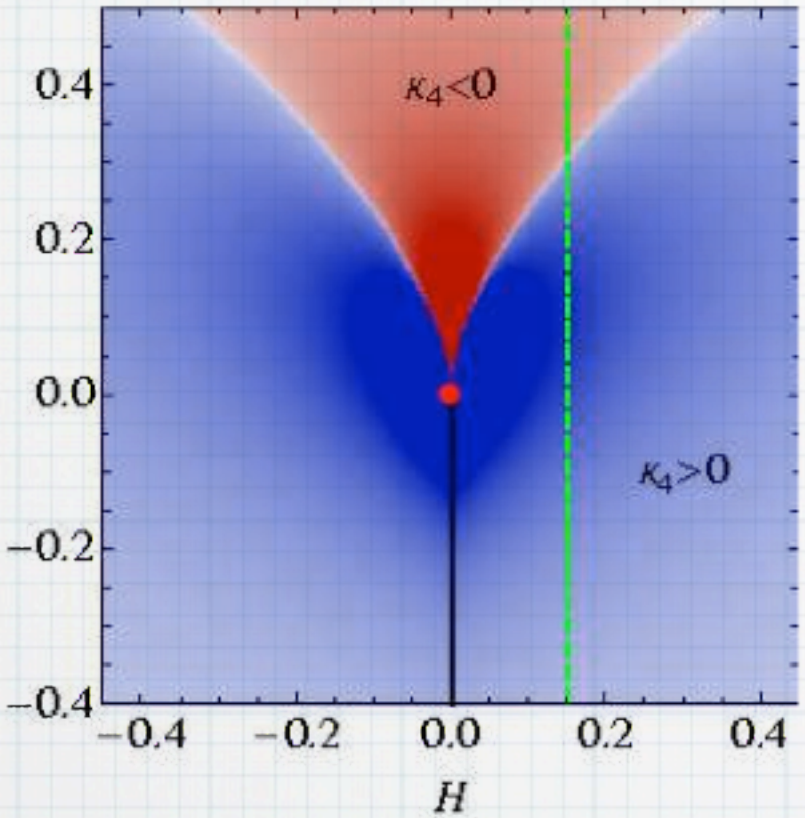
At the critical point, it receives both a regular and a singular contribution. The latter comes from the coupling to the σ field:

$$\delta n_{\boldsymbol{p}} = \underbrace{\delta n^0_{\boldsymbol{p}}}_{\substack{\text{statistical} \\ \text{(Poisson)}}} + \underbrace{\frac{\partial \bar{n}_{\boldsymbol{p}}}{\partial m} g\, \delta\sigma}_{\text{critical}}$$

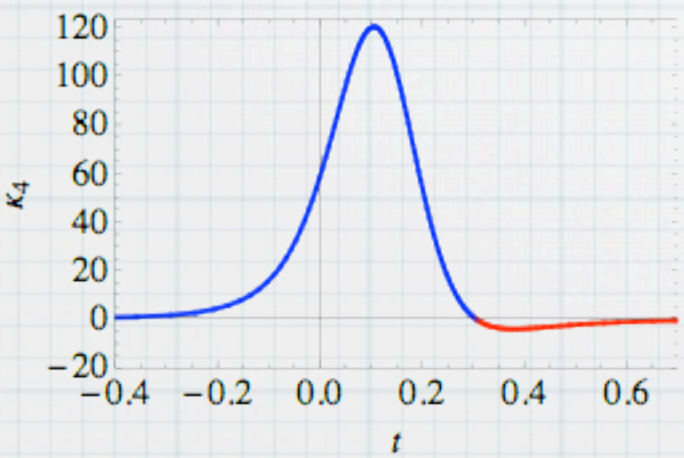


Higher order moments have **stronger dependence on ξ**: they are more sensitive signatures for the critical point

Sign of kurtosis



(a)



The 4th order cumulant becomes negative when the critical point is approached from the crossover side: from Ising model:

$M=R^{\beta}\theta$, $t=R(1-\theta^2)$, $H=R^{\beta\delta}h(\theta)$

$K_4=\langle M^4\rangle$ $(t,H)\to(\mu-\mu_{CP}, T-T_{CP})$

Consequently, the experimental 4th order fluctuation will be smaller than its Poisson value (precise value depends on ξ, on how close the freeze-out occurs to the critical point...)