Light Front Field Theory And Coherent State Basis

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- Method of Asymptotic Dynamics
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3 **Mass renormalization upto $O(e^2)$ in Fock state basis**

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Summary
What is a light front?

- **Dirac**: “...the three-dimensional surface in space-time formed by a plane wave front advancing with the velocity of light....” . For example, $x^+ = x^0 + x^3 = 0$, is called a front
- **Dirac (1949)**: Three possible forms of relativistic dynamics corresponding to 3 different ways of quantizing corresponding to 3 different surfaces of quantization
  Instant Form, Point Form, Front Form
Light Front Coordinates

\[ x^\mu = (x^+, x^-, x^\perp) \]

where

\[ x^+ = \frac{(x^0 + x^3)}{\sqrt{2}}, \quad x^- = \frac{(x^0 - x^3)}{\sqrt{2}}, \quad x^\perp = (x^1, x^2) \]

The metric tensor

\[ g_{\mu\nu} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \]

Momentum is given by \( p = (p^+, p^-, p^\perp) \)

Mass shell condition \( p^- = \frac{p^2 + m^2}{2p^+} \)
Why quantize on the Light Front?

- Quantization of QCD at fixed light-front time can provide a first principles method for solving non-perturbative QCD
- Dispersion relation $k^- = \frac{k_\perp^2 + m^2}{2k^+}$ is suitable for bound state calculations because
- No square root operator (unlike instant form)
- Dependence of $k^-$ on $k_\perp$ similar to non-relativistic dispersion relation
- Due to the form of the dispersion relation, for an on-shell particle,
  $k^+ \geq 0$ implies that $k^- \geq 0$ and
  $k^+ \leq 0$ implies that $k^- \leq 0$.
- Thus, for physical particles $k^+ \geq 0$ always.
- Simpler vacuum structure
Hamiltonian light front approach

- aims at solving the Hamiltonian eigenvalue problem in the spirit of Tamm Dancoff method
  - $H|\Psi\rangle = \frac{M^2 + P_+^2}{2P_+} |\Psi\rangle$

- Discretized light Cone Quantization (DLCQ):
  - Project the LF Hamiltonian eigenvalue equation on a truncated Fock space
  - Discretize the momentum space
  - Result: a matrix equation which can be solved on a computer
  - DLCQ - used for solving bound state problems in 1 +1 dimension and even for positronium spectrum

- Recently developed method Basis Light Front Quantization- useful in NP bound state calculation
Light Front Field Theory
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LF Hamiltonian formalism

- More suitable for bound state calculations as compared to its equal time counterpart
- However, there are problems that need to be addressed before one can do that
- Renormalization is different \( P^- = \frac{P_+^2 + M^2}{2P_+} \)
- IR divergences pose a big challenge
- Need to separate the "true" IR divergences from the "spurious" ones
- Coherent State Formalism provides a solution


Coherent state formalism is based on the method of asymptotic dynamics
**Method of Asymptotic Dynamics**

The LSZ formalism is based on the assumption

\[ H_{as} = \lim_{|t| \to \infty} H = H_0 \]

- Not always true
- \( H_{as} \neq H_0 \)
  
  , when
  - there are long range interaction
  - incoming and outgoing states are bound states
In the limit $|x^+| \to \infty$, $H \longrightarrow H_{as}$

$$H_{as} = H_0 + V_{as}$$

The total Hamiltonian can be written as

$$H = H_{as} + H'_I$$

where

$$H_{as}(x^+) = H_0 + V_{as}(x^+)$$

The associated $x^+$ evolution operator $U_{as}(x^+)$ in the Schrodinger representation satisfies the equation

$$i \frac{dU_{as}(x^+)}{dx^+} = H_{as}(x^+)U_{as}(x^+)$$
Coherent states

- The asymptotic evolution operator $\Omega^A(x^+)$ is defined by

$$U_{as}(x^+) = \exp[-iH_0 x^+] \Omega^A(x^+)$$

where

$$\Omega^A_{\pm}(x^+) = T^+ \exp\left[-i \int_{\mp}^0 V_{as}(x^+)dx^+\right]$$

- KF : Use $\Omega^A_{\pm}(x^+)$ to define a new set of asymptotic states

$$|n : coh\rangle = \Omega^A_{\pm}|n\rangle$$

$|n\rangle$ is a Fock state, $\Omega^A_{\pm}$ are the asymptotic Möller operators

KF: The transition matrix calculated using such coherent states is IR divergence free in equal time QED.
• Cancellation of IR divergences in QCD (lowest order)

• Coherent States in LFFT
  Relevance of coherent state formalism in Light Front Field Theory (LFFT)

• Coherent state formalism in LFFT : Cancellation of IR divergences in 3-point vertex correction in QED and QCD at one loop level.
IR Divergences in LFFT

- Dispersion relation: \( k^- = \frac{k_\perp^2 + m^2}{2k^+} \)
  
  Two kinds of IR divergences in LFFT

- Spurious IR divergences
  
  \[ k^+ \to 0 \]

- True IR divergences
  
  \[ k_\perp \to 0, \; k^+ \to 0 \]


The coherent state method provides an alternative way of treating the true IR divergences.
Coherent State Formalism in LFFT

- $H_{as}$ is evaluated by taking the limit $|x^+| \to \infty$ in $\exp[-i(p_1^- + p_2^- + \cdots + p_n^-)x^+]$ of the interaction Hamiltonian $H_{int}$.

- If $(p_1^- + p_2^- + \cdots + p_n^-) \to 0$ for some vertex, then the corresponding term in $H_{int}$ does not vanish in large $x^+$ limit.

- Use KF method to construct asymptotic Hamiltonian and coherent state basis.
Light-Front QED in LF gauge

- The LFQED Hamiltonian \( (P) \) in LF gauge is

\[
H_I(x^+) = V_1(x^+) + V_2(x^+) + V_3(x^+)
\]

where

- \( V_1(x^+) \) is the standard three point vertex of QED
- \( V_2(x^+) \) is an instantaneous 4-point interaction which appears when we write the fermionic part of \( P^- \) in terms of independent component shown - hence we eliminate it and write \( P^F_- \) in terms of \( \psi_+ \) only
- \( V_3(x^+) \) is an instantaneous 4-point interaction which appears when we write \( A^\mu \) only in terms of physical degrees of freedom
Light-front QED Hamiltonian in the light-front gauge \((A^+=0)\)

\[
P^- = H \equiv H_0 + V_1 + V_2 + V_3 ,
\]

Here

\[
H_0 = \int d^2x_\perp dx^- \left\{ \frac{i}{2} \bar{\xi} \gamma^- \lessdot \partial^- \xi + \frac{1}{2} (F_{12})^2 - \frac{1}{2} a_+ \partial^- \partial_k a_k \right\}
\]

\[
V_1 = e \int d^2x_\perp dx^- \bar{\xi} \gamma^\mu \xi a_\mu
\]

\[
V_2 = -\frac{i}{4} e^2 \int d^2x_\perp dx^- dy^- \epsilon(x^- - y^-)(\bar{\xi} a_k \gamma^k)(x) \gamma^+(a_j \gamma^j \xi)(y)
\]

\[
V_3 = -\frac{e^2}{4} \int d^2x_\perp dx^- dy^- (\bar{\xi} \gamma^+ \xi)(x)|x^- - y^-|(\bar{\xi} \gamma^+ \xi)(y)
\]
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QED
\( \xi(x) \) and \( a_\mu(x) \) can be expanded in terms of creation and annihilation operators as

\[
\xi(x) = \int \frac{d^2 p_\perp}{(2\pi)^{3/2}} \int \frac{dp^+}{\sqrt{2p^+}} \sum_{s=\pm \frac{1}{2}} \left[ u(p, s) e^{-i(p^+ x^- - p_\perp \cdot x_\perp)} b(p, s, x^+) 
\right. \\
+ \left. v(p, s) e^{i(p^+ x^- - p_\perp \cdot x_\perp)} d^\dagger(p, s, x^+) \right],
\]

\[
a_\mu(x) = \int \frac{d^2 q_\perp}{(2\pi)^{3/2}} \int \frac{dq^+}{\sqrt{2q^+}} \sum_{\lambda=1,2} \epsilon_\mu^\lambda(q) \left[ e^{-i(q^+ x^- - q_\perp \cdot x_\perp)} a(q, \lambda, x^+) \right. \\
+ \left. e^{i(q^+ x^- - q_\perp \cdot x_\perp)} a^\dagger(q, \lambda, x^+) \right],
\]

operators satisfy

\[
\{ b(p, s), b^\dagger(p', s') \} = \delta(p^+ - p'^+) \delta^2(p_\perp - p'_\perp) \delta_{ss'} = \{ d(p, s), d^\dagger(p', s') \},
\]

\[
[ a(q, \lambda), a^\dagger(q', \lambda') ] = \delta(q^+ q'^+) \delta^2(q_\perp - q'_\perp) \delta_{\lambda \lambda'}. 
\]
Light cone time dependence of $V_1$ is of the form

$$V_1(x^+) = e \sum_{i=1}^{4} \int d\nu_i [e^{-i\nu_i(1)x^+} \tilde{h}_i(\nu_i) + e^{i\nu_i(1)x^+} \tilde{h}^\dagger_i(\nu_i)]$$

where $\tilde{h}_i(\nu_i)$ are the QED interaction vertices:

$$\tilde{h}_1 = \sum_{s, s', \lambda} b^\dagger(\bar{p}, s') b(p, s) a(k, \lambda) \bar{u}(\bar{p}, s') \gamma^\mu u(p, s) \epsilon^\lambda_\mu,$$

$$\tilde{h}_2 = \sum_{s, s', \lambda} b^\dagger(\bar{p}, s') d^\dagger(p, s) a(k, \lambda) \bar{u}(\bar{p}, s') \gamma^\mu v(p, s) \epsilon^\lambda_\mu,$$

$$\tilde{h}_3 = \sum_{s, s', \lambda} d(\bar{p}, s') b(p, s) a(k, \lambda) \bar{v}(\bar{p}, s') \gamma^\mu u(p, s) \epsilon^\lambda_\mu,$$

$$\tilde{h}_4 = \sum_{s, s', \lambda} d^\dagger(\bar{p}, s') d(p, s) a(k, \lambda) \bar{v}(\bar{p}, s') \gamma^\mu v(p, s) \epsilon^\lambda_\mu.$$
\( \nu_i \) is the light cone energy transferred at the vertex \( \tilde{h}_i \).

The integration measure is given by
\[
\int d\nu = \frac{1}{(2\pi)^{3/2}} \int \frac{[dp][dk]}{\sqrt{2\bar{p}^+}} ,
\]
\( \bar{p}^+ \) and \( \bar{p}_\perp \) being fixed at each vertex by momentum conservation.

For example
\[
\nu_1^{(1)} = p^- + k^- - \bar{p}^- = \frac{p \cdot k}{p^+ + k^+}
\]
is the energy transfer at \( ee'\gamma \) vertex.
- At asymptotic limits, non-zero contributions to $H_i(x^+)$ come from regions where $\nu_i$ goes to zero.
- $\nu_2$ and $\nu_3$ are always non-zero, and hence, $\tilde{h}_2$ and $\tilde{h}_3$ do not appear in the asymptotic Hamiltonian.
- The 3-point asymptotic Hamiltonian is defined by

$$V_{1as}(x^+) = e \sum_{i=1,4} \int d\nu_i^{(1)} \Theta_\Delta(k)[e^{-i\nu_i^{(1)} x^+} \tilde{h}_i^{(1)}(\nu_i^{(1)}) + e^{i\nu_i^{(1)} x^+} \tilde{h}_i^{\dagger}(\nu_i^{(1)})]$$

where $\Theta_\Delta(k)$ defines the asymptotic region i.e the region in which $\nu_i^{(1)}$ is zero.
- $\Theta_\Delta(k)$ is 1 in the asymptotic region and 0 elsewhere.
• Define the asymptotic region to consist of all points in the phase space for which

\[ \frac{p \cdot k}{p^+} < \Delta, \]

where \( \Delta \) is an energy cutoff which may be chosen to be related to the experimental resolution.

• For simplicity, choose a frame \( p_\perp = 0 \). In this frame the above condition reduces to

\[ \frac{p^+ k_\perp^2}{2k^+} + \frac{m^2 k^+}{2p^+} < \Delta, \]

where \( \Delta = p^+ \Delta E \).
Sufficient to choose a subregion of the above mentioned region as the asymptotic region.

Define this subregion to be consisting of all points \((k^+, k_\perp)\) satisfying:

\[\begin{align*}
k_\perp^2 &< \frac{k^+ \Delta}{p^+}, \\
k^+ &< \frac{p^+ \Delta}{m^2}.
\end{align*}\]

This choice of the asymptotic region leads to

\[\Theta_\Delta(k) = \theta \left( \frac{k^+ \Delta}{p^+} - k_\perp^2 \right) \theta \left( \frac{p^+ \Delta}{m^2} - k^+ \right)\]
Asymptotic states

\[ \Omega_{\pm}^A|n: p_i\rangle = \exp \left[ -e \int dp^+ d^2 p_\perp \sum_{\lambda=1,2} [d^3 k][f(k, \lambda : p)a^\dagger(k, \lambda) - f^*(k, \lambda : p)a(k, \lambda)] + e^2 \int dp^+ d^2 p_\perp \sum_{\lambda_1, \lambda_2=1,2} [d^3 k_1][d^3 k_2] \right. \\
\left. [g_1(k_1, k_2, \lambda_1, \lambda_2 : p)a^\dagger(k_2, \lambda_2)a(k_1, \lambda_1) - g_2(k_1, k_2, \lambda_1, \lambda_2 : p)a(k_2, \lambda_2)a^\dagger(k_1, \lambda_1)]\rho(p) \right] |n: p_i\rangle \]

[AM, Phys. Rev. D 50, 4088 (1994)]
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Here

\[ [d^3 k] = \int \frac{d^2 k_\perp}{(2\pi)^{3/2}} \int \frac{dk^+}{\sqrt{2k^+}} \]

\[
 f(k, \lambda; p) = \frac{p_\mu \epsilon^\mu_\lambda(k)}{p \cdot k} \theta\left(\frac{k^+ \Delta}{p^+} - k^2_\perp\right) \theta\left(\frac{p^+ \Delta}{m^2} - k^+\right),
\]

\[
 f(k, \lambda; p) = f^*(k, \lambda; p),
\]
One fermion coherent state

\[ |p, \sigma : f(p)\rangle = \exp \left[ -e \sum_{\lambda=1,2} [d^3 k] [f(k, \lambda : p) a^\dagger(k, \lambda) - f^*(k, \lambda : p) a(k, \lambda)] + e^2 \sum_{\lambda_1, \lambda_2=1,2} [d^3 k_1][d^3 k_2] [g_1(k_1, k_2, \lambda_1, \lambda_2 : p) a^\dagger(k_2, \lambda_2) a(k_1, \lambda_1)] - g_2(k_1, k_2, \lambda_1, \lambda_2 : p) a(k_2, \lambda_2) a^\dagger(k_1, \lambda_1) \right] |p, \sigma\rangle \]
The transition matrix is given by

\[ T = V + V \frac{1}{p^- - H_0} V + \cdots \]

The electron mass shift is obtained by calculating \( T_{pp} = \langle p, s \vert T \vert p, s \rangle \)

\[ \delta m^2 = p^+ \sum_s T_{pp} \]

We expand \( T_{pp} \) in powers of \( e^2 \) as

\[ T_{pp} = T^{(1)} + T^{(2)} + \cdots \]

For example

\[ T^{(1)}_{pp} \equiv T^{(1)}(p, p) = \langle p, s \vert V_1 \frac{1}{p^- - H_0} V_1 \vert p, s \rangle + \langle p, s \vert V_2 \vert p, s \rangle \]
**Introduction**

Coherent state formalism in LFFT

**Mass renormalization up to \( O(e^2) \) in Fock state basis**

Mass renormalization up to \( O(e^2) \) in the coherent state basis

Mass renormalization up to \( O(e^4) \) in Fock basis

Mass renormalization up to \( O(e^4) \) in the coherent state basis

**All Order Cancellation**

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**Summary**

\[ O(e^2) \] self energy correction in Fock basis

\[ O(e^4) \] self energy correction in Fock basis

\[ \langle \text{Diagrams for } O(e^2) \text{ self energy correction in Fock basis corresponding to } T_1 \rangle \]

\[ T_{pp}^{(1)} \equiv T^{(1)}(p, p) = \langle p, s | V_1 \frac{1}{p^- - H_0} V_1 | p, s \rangle + \langle p, s | V_2 | p, s \rangle \]

In the limit \( k_1^+ \to 0, k_1^\perp \to 0, \)

\[ (\delta m_{1a}^2)^{IR} = -\frac{e^2}{(2\pi)^3} \int d^2 k_1^\perp \int \frac{dk_1^+}{k_1^+} \frac{(p \cdot \epsilon(k_1))^3}{(p \cdot k_1)} \]

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**$O(e^2)$ self energy correction in coherent state basis**

![Diagram of Feynman diagrams](image)

- Additional diagrams in coherent state basis for $O(e^2)$ self energy correction corresponding to $T_2$.

$$ T'(p, p) = \langle p, s : f(p) | V_1 | p, s : f(p) \rangle $$

- Calculated using coherent state properties

$$ a(k, \rho) | 1 : p_i \rangle = - \frac{e}{(2\pi)^{3/2}} \frac{f(k, \rho : p_i)}{\sqrt{2k^+}} | 1 : p_i \rangle, $$

$$ a^\dagger(k, \rho) | 1 : p_i \rangle = \frac{e}{(2\pi)^{3/2}} \frac{f^*(k, \rho : p_i)}{\sqrt{2k^+}} | 1 : p_i \rangle \mathrel{\leftrightarrow} | 2 : p_i, k_i \rangle. $$
Extra contribution in coherent state basis

\[ T'(p, p) = \frac{e^2}{(2\pi)^3} \int \frac{d^2 k_{1\perp}}{2p^+} \int \frac{dk_1^+}{2k_1^+} \bar{u}(\bar{p}, s') \epsilon^\lambda(k_1) u(p, s)f(k_1, \lambda : p) \]

where

\[ f(k, \lambda : p) = \frac{p_\mu \epsilon^\mu_\lambda(k)}{p \cdot k} \theta \left( \frac{k^+ \Delta}{p^+} - k_{\perp}^2 \right) \theta \left( \frac{p^+ \Delta}{m^2} - k^+ \right) \]

\[ (\delta m^2)' = \frac{e^2}{(2\pi)^3} \int d^2 k_{1\perp} \int \frac{dk_1^+}{k_1^+} \frac{(p \cdot \epsilon(k_1))^2 \Theta_{\Delta}(k_1)}{p \cdot k_1} \]

Equal and opposite to Fock space expression in asymptotic region
Electron mass correction in Fock basis up to $O(e^4)$ to self energy is given by $T^{(2)} = T_3 + T_4 + T_5 + T_6 + T_7$

where

$T_3 = \langle p, s | V_1 \frac{1}{p - H_0} V_1 \frac{1}{p - H_0} V_1 \frac{1}{p - H_0} V_1 | p, s \rangle$

$T_4 = \langle p, s | V_1 \frac{1}{p - H_0} V_1 \frac{1}{p - H_0} V_2 | p, s \rangle$

$T_5 = \langle p, s | V_1 \frac{1}{p - H_0} V_2 \frac{1}{p - H_0} V_1 | p, s \rangle$

$T_6 = \langle p, s | V_2 \frac{1}{p - H_0} V_1 \frac{1}{p - H_0} V_1 | p, s \rangle$

$T_7 = \langle p, s | V_2 \frac{1}{p - H_0} V_2 | p, s \rangle$


$O(e^4)$ self energy correction in Fock basis corresponding to $T_3$. 

\begin{align*}
\text{(a)} & \quad (p, s) \quad\quad (p, \sigma) \\
\text{(b)} & \quad (p, s) \quad\quad (p, \sigma) \\
\text{(c)} & \quad (p, s) \quad\quad (p, \sigma)
\end{align*}
IR divergences in these diagrams appear when

I \[ p \cdot k_1 \to 0 \text{ i.e. } k_1^+ \to 0, k_{1\perp} \to 0, \text{ but } p \cdot k_2 \neq 0. \]

II \[ p \cdot k_2 \to 0 \text{ i.e. } k_2^+ \to 0, k_{2\perp} \to 0, \text{ but } p \cdot k_1 \neq 0. \]

III \[ p \cdot k_1 \to 0 \text{ and } p \cdot k_2 \to 0 \text{ i.e. } k_1^+ \to 0, k_{1\perp} \to 0, \]
\[ k_2^+ \to 0, k_{2\perp} \to 0. \]
$O(e^4)$ self energy correction in Fock basis corresponding to $T_4$, $T_5$ and $T_6$ respectively.
Mass renormalization up to $O(e^4)$ in the coherent state basis

- Additional contributions at $O(e^4)$ in coherent state basis
  \[ T^{(2)} + T'_8 + T'_9 + T'_{10} + T'_{11} \]

  where

  \begin{align*}
  T'_8 &= \langle p, s : f(p) | V_1 \frac{1}{p - H_0} V_1 \frac{1}{p - H_0} V_1 | p, s : f(p) \rangle \\
  T'_9 &= \langle p, s : f(p) | V_1 \frac{1}{p - H_0} V_1 | p, s : f(p) \rangle \\
  T'_{10} &= \langle p, s : f(p) | V_1 \frac{1}{p - H_0} V_2 | p, s : f(p) \rangle + \langle p, s : f(p) | V_2 \frac{1}{p - H_0} V_1 | p, s : f(p) \rangle \\
  T'_{11} &= \langle p, s : f(p) | V_2 | p, s : f(p) \rangle.
  \end{align*}
Additional diagrams in coherent state basis for $O(e^4)$ self energy correction corresponding to $T_8$ and $T_9$ respectively.
Additional diagrams in coherent state basis for $O(e^4)$ self energy correction corresponding to $T_{10}$. 
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Additional diagrams in coherent state basis for $O(e^4)$ self energy correction corresponding to $T_{11}$. 

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\[(\delta m^2)_3 + (\delta m^2)_8 + (\delta m^2)_9\] is IR finite.
\[(\delta m^2)_4 + (\delta m^2)_4 + (\delta m^2)_6 + (\delta m^2)_{10} + (\delta m^2)_{11}\] is IR finite.
Cancellation of true IR divergences to all orders

- Use method of induction
  Yennie et al, Annals of Physics 13, 379 (1961): Real virtual cancellation of IR divergences to all orders
- LFQED: divergences cancel to $O(e^4)$
- Assume IR divergences cancel up to $O(e^{2n})$
- Express $O(e^{(2n+2)})$ contribution in terms of IR finite $O(e^{2n})$ matrix elements
- Show the additional divergences also cancel in coherent state basis
• Represent the $O(e^{2n})$ IR finite amplitude by a blob i.e. a blob represents the sum of the Fock and coherent state contributions to the self energy correction that being added up together give IR finite amplitude.

• The blob is of $O(e^{2n})$ and contains $n$ photon lines
• Express the $O(e^{2(n+1)})$ contributions in terms of this blob
• Show the cancellation of IR divergences in $O(e^{2(n+1)})$ using the coherent state basis.
General expression for transition matrix element in $O(e^{2n})$ is a sum of terms of the form:

$$T_j^{(n)} = -\frac{e^{2n}}{2p^+(2\pi)^3} \int \prod_{i=1}^{n} \frac{d^3k_i}{2k_i^+ 2p_{2i-1}^+}$$

$$\times \bar{u}(p_1, s_1)\ell_1(p_1 + m)\ell_2(p_2 + m) \cdots \cdots \cdots (p_i + m)\ell_i u(p_i, s_i)$$

$$\prod_{r=1}^{n} (p^- - p_r^- - \sum_{i=1}^{n} k_i)$$

$$T^{(n)} = \sum_j T_j^{(n)} = \sum_j \frac{\bar{u}(\bar{p}, s').M^{(j)}_n u(p, s)}{D(j)}$$

where $j$ is summed over all possible diagram in $O(e^{2n})$ and will be assumed to be IR divergence free.

Here,

$$D^{(j)} = \prod_{r=1}^{n} D^{(j)}$$
**O(e^4) revisited**

- **O(e^2)** correction

\[ T^{(2)} = \sum_j \frac{\bar{u}(\bar{p}, s') M_2^{(j)} u(p, s)}{D(j)} \]

**Figure:** IR finite \(O(e^2)\) blob with an external photon line results into \(O(e^4)\) diagram
In our new notation it is,

\[ T_{3a}^{(2)} = T_{3b}^{(2)} + T_{3c}^{(2)} \]

\[ = \frac{e^2}{(2\pi)^3} \int \frac{d^3 k_1}{2k_1^+} \bar{u}(p, \sigma)\epsilon(k_1)(\not{\rho}_1 + m)M_2^{(j)}(\not{\rho}_1 + m)\epsilon(k_1)u(p, s) \]

\[ \frac{(p \cdot k_1)^2 D(j)}{(p \cdot k_1)^2 D(j)} \]

Blob is IR finite

IR divergences can appear "only" from the vanishing of energy denominators of the kind \( p^- - k_1^- - (p - k_1) \)

Additional contribution in \( O(e^4) \) in coherent state basis
In coherent state basis, we have extra contributions

\[ T_{4a}^{(2)} = -\frac{e^2}{(2\pi)^3} \int \frac{d^3k_1}{2k_1^+} \frac{\bar{u}(p, \sigma) \gamma(k_1)(p_1 + m) M_j^{(j)} u(p, s)(p \cdot k_1)}{(p \cdot k_1)^2 D(j)} \]

which cancel the Fock contribution in the limit \( k^+ \to 0, k_\perp \to 0 \).
Introduction
Coherent state formalism in LFFT

- Mass renormalization up to $O(e^2)$ in Fock state basis
- Mass renormalization up to $O(e^2)$ in the coherent state basis
- Mass renormalization up to $O(e^4)$ in Fock basis
- Mass renormalization up to $O(e^4)$ in the coherent state basis

All Order Cancellation
Improved Method of Asymptotic Dynamics
Summary
• Same holds for other diagrams as well

To construct an \( O(e^{2n+2}) \) diagram in Fock basis, we can add a photon to nth order blob in three different ways.
Introduction
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Mass renormalization up to $O(e^2)$ in the coherent state basis
Mass renormalization up to $O(e^4)$ in Fock basis
Mass renormalization up to $O(e^4)$ in the coherent state basis

All Order Cancellation
Improved Method of Asymptotic Dynamics
Summary
The contributions from Figs. (a), (b) and (c) are given by

\[
T^{(n+1)}_{6a} = \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, \sigma)\epsilon(q)(P + m)M_{n}^{(j)}(P + m)\epsilon(q)u(p, s)}{(p \cdot q)^2D^{(j)}}
\]

\[
T^{(n+1)}_{6b} = -\frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, s')M_{n}^{(j)}(\bar{p}' + m)\epsilon(q)(P + m)\epsilon(q)u(p, s)}{(p \cdot q)(p^- - p'^-)D^{(j)}}
\]

\[
T^{(n+1)}_{6c} = \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, s')M_{n}^{(j)}(P + m)\epsilon(q)u(p, s)}{(p \cdot q)D^{(j)}}
\]

where \(P = p - q\)
Note that in case of overlapping diagram, the structure is different.

However, one can show that for our purpose it is sufficient to consider the limit \( q^+ \to 0, q_\perp \to 0 \), in which case

\[
M^{\ell(j)}_n = \ell(k_1)(P_1 + m)\ell(k_2)(P_2 + m) \cdots \ell(k_\ell)(P_\ell + m)\ell(q)\ell(p_{\ell+1} + m) \cdots
\]

Also, the energy denominators corresponding to the intermediate states will be

\[
D^{(j)} = (p^- - p_1^- - k_1^- - q^-)(p^- - p_2^- - k_1^- - k_2^- - q^-) \cdots
\]

\[
(p^- - p_\ell^- - \sum_i k_i^- - q^-) \cdots \cdots \cdots
\]

The additional contributions in coherent state basis are given by the following diagrams
Introduction
Coherent state formalism in LFFT
Mass renormalization up to $O(e^2)$ in Fock state basis
Mass renormalization up to $O(e^4)$ in the coherent state basis
Mass renormalization up to $O(e^6)$ in Fock basis
Mass renormalization up to $O(e^4)$ in the coherent state basis

All Order Cancellation
Improved Method of Asymptotic Dynamics
Summary

Figure:
Dipartimento di Fisica, Universita di Torino, March 14, 2017
Coherent state formalism in LFFT
Mass renormalization up to $O(e^2)$ in Fock state basis
Mass renormalization up to $O(e^2)$ in the coherent state basis
Mass renormalization up to $O(e^4)$ in Fock basis
Mass renormalization up to $O(e^4)$ in the coherent state basis

All Order Cancellation
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Summary

\[
T_{7a}''(n+1) = -\frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, s')\xi(q)(P + m)M^{(j)}_n u(p, s)(p \cdot \epsilon) \Theta_\Delta(q)}{(p \cdot q)^2 D^{(j)}}
\]

\[
T_{7b}''(n+1) = \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, s')M^{(j)}_n (P' + m)\xi(q)u(p, s)(p \cdot \epsilon) \Theta_\Delta(q)}{(p \cdot q)(p^- - p'^-) D^{(j)}}
\]

\[
T_{7c}''(n+1) = -\frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(\bar{p}, s')M^{(j)}_n u(p, s)(p \cdot \epsilon) \Theta_\Delta(q)}{(p \cdot q) D^{(j)}}
\]

In the limit, $k^+ \to 0, k_\perp \to 0, P\xi(q) \to p \cdot \epsilon$, the coherent state contribution exactly cancels the IR divergences in the original (Fock space) diagrams. Thus by induction, the IR divergences cancel to all orders.
**Improved Method of Asymptotic Dynamics**

- KF method does not work for theories involving 4-point interaction
- Asymptotic states in QCD are bound states
- In QCD a recursive proof of cancellation of IR divergences cannot be obtained using just KF method
- Asymptotic Hamiltonian should contain the confining potential in case asymptotic states are bound states

**An ‘improved’ method of asymptotic dynamics should take into account the separation of particles also**

Anuradha Misra, Few-Body Systems 36, 201-204 (2005).]
Improved method of asymptotic dynamics (Horan, Lavelle & McMullan 2000)

- based on asymptotic properties of matrix elements instead of operators
- takes into account appropriate boundary conditions corresponding to the separation of particles at large distances
- first criteria suggests not only the energy denominators but their partial derivatives also become zero

For theory involving 4-point interactions, KF method does not work, but the improved method leads to cancellation of IR divergences
Criteria in the Method of Asymptotic Dynamics

- In LFQED,
  \[ \nu_i = p^- - k^- - (p - k)^- \]

- Condition to obtain the asymptotic region KF approach \( \implies \nu_i = 0 \)

- Improved Method \( \implies \frac{\partial \nu_i}{\partial p^-} = \frac{\partial \nu_i}{\partial p^+} = \frac{\partial \nu_i}{\partial k^-} = \frac{\partial \nu_i}{\partial k^+} = 0 \)

- For QED, both the criteria lead to same asymptotic region for constructing coherent states

- For LFQCD, one may need to use the criteria of separation of particles?
Future directions

- Develop the improved method of asymptotic dynamics in LFFT for simple model like Yukawa theory, $\phi^4$ theory
- Extend this method to QCD to analyze the nature of IR divergences
- Construct an artificial potential that is needed for bound state calculation in LFQCD
- Combine the coherent state method with the BLFQ methods (J.Vary et al) to deal with the IR problem in LF bound state calculations
To summarize

- The true IR divergences are cancelled to all orders when coherent state basis is used to calculate the matrix elements in lepton self energy correction in light-front QED.

- Apply improved method to LFQCD beyond one loop order to obtain IR finite amplitudes

- Connection between asymptotic dynamics and IR divergences needs to be investigated

- Combine coherent state method with BLFQ methods (J.Vary et al, Phys. Rev. D 91, 105009 (2015)) for practical use of coherent state methods

GRAZIE