Generalised Resummation of Jet Observables

P. F. Monni
Rudolf Peierls Centre for Theoretical Physics
University of Oxford

Based on work in collaboration with:
G. Zanderighi (Oxford University)
A. Banfi, H. McAslan (Sussex University)

and ongoing work

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Outline

• Perturbation theory in the large-coupling (perturbative) regime

• Practical relevance of resummation for QCD jet observables

• State of the art and factorisation theorems

• Devising a general technique without any required factorisation theorem

  • Observable’s properties
  • Amplitudes
  • Relevant phase space regions

• Applications in e+e- -> 2 jets and p p -> V production

• Current work in progress and outlook
Fixed order QCD and resummation

• High energy strong interaction can be very well described by perturbative QCD (PT) through a power series in the (small) coupling constant

• Each extra power of the coupling corresponds to an extra real emission and relative virtual corrections

• Each real emission has a different scale for its strong coupling which is of the order of its transverse momentum

• In a PT expansion the coupling is evaluated at some common scale of the order of the hard scale(s) of the process - this ensures the coupling to be small (high transverse momentum)

• The PT expansion can be taken and the hard event is usually very well described by the first few terms in the power series
Fixed order QCD and resummation

• When the transverse momentum of the QCD radiation is constrained to be small, the strong coupling becomes large and an arbitrary amount of QCD emissions become equally important - Need for a all-orders description of the reaction

• The large coupling manifests itself in terms of large single logarithms in the perturbative series

\[ \alpha_s(k_t^2) \sim \frac{\alpha_s(Q^2)}{1 - \alpha_s(Q^2) \beta_0 \ln \frac{Q^2}{k_t^2}} \]

• One way to account for the large coupling effects is to sum up all these logarithms to all orders in PT series

• Resummation recasts predictive power of the PT series in this regime and provides a reliable description for the reaction
Fixed order QCD and resummation

• Constraining the real radiation’s kinematics leads to additional (double) logarithms

• real emission forced to be soft and/or collinear to the emitter

• virtual corrections are unaffected

\[
\frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)
\]

\[
- \frac{dk_t}{k_t} d\eta \alpha_s(k_t^2)
\]

\[
P(k_t < v) \sim 1 - \frac{\#\alpha_s C_F}{2\pi} \ln^2 v + \ldots
\]
Fixed order QCD and resummation

• In the perturbative regime these logarithms can become as large as (breakdown of the PT below this limit)

\[ L \sim \frac{1}{\alpha_s} \]

• This makes “higher order” corrections as large as leading order ones, i.e. \((\alpha_s L)^n L \sim \alpha_s L^2\)

• The PT series breaks down and the probability of the reaction diverges logarithmically in the large \(L\) limit instead of being suppressed

• The resummation of the large logarithms to all perturbative orders restores the correct physical (Sudakov) suppression and rescues the predictive power of perturbation theory
Logarithmic accuracy

• **Double** logarithms due to constraining the radiation kinematics commonly happen to exponentiate exactly (see later)

• non-exponentiating observables are avoided because of issues with the simulation in event generators, e.g. JADE algorithm

• For such observables we can define a new perturbative order by expressing the cross section as an exponential function

\[
\Sigma(v) = \int_0^v \frac{1}{\sigma_{\text{Born}}} \frac{d\sigma}{dv'} dv' \sim e^{\alpha_s^L L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1} + \ldots}
\]

• In the region where \( L \sim 1/\alpha_s(Q) \), LL are enhanced w.r.t. the Born, NLL are as large as the Born cross section itself, NNLL count as NLO corrections, and so on
Are these regimes of interest?

- The large-logarithms (Sudakov) regime is probed in several collider reactions whenever tight phase space cuts are applied in the definition of the physical observables (e.g. event-shapes, jet rates, definition of fiducial/control regions for signal or background, ...)

- Phenomenology interest:
  - fit of the strong coupling constant
  - precise simulation of background/signal
  - tuning/developing Monte Carlo event generators
  - ...

- Theoretical interest:
  - Properties of the QCD radiation to all-orders
  - Understanding of IRC singular structure (subtraction)
  - Probing the boundary with the non-perturbative regime, and study of non-perturbative dynamics
  - ...
State of the art

- Several NLL resummations exist for a number of observables at lepton-lepton (hadron) and hadron-hadron colliders (extensive literature, ~1 observable per article)

- (~) 4 approaches to Sudakov resummation are available:
  - Branching algorithm
    [Catani, Marchesini, Webber et al.]
  - Soft Collinear Effective Theory
    [Bauer, Fleming, Pirjol, Stewart; Beneke, Chapovski, Diehl, Feldmann]
  - Sterman (- Collins - Soper) approach
    [Sterman, Collins, Soper et al.]
  - CAESAR approach
    [Banfi, Salam, Zanderighi]

- Possible to devise a synergy for a new more powerful approach (this talk)?
State of the art

- NNLL corrections are often sizeable and important for precision physics. Few NNLL results exist for 2 scale observables in e+e- collisions and even fewer for hadronic collisions.

- Most important limitation being the “factorisability” requirement (see later) for the observable in some (smartly defined) conjugate space - resummation often leads to very tedious calculations (~14-16 yrs to go from NLL to the next order).

- **GOAL**: Devise a (numerical) resummation approach that:
  - does not rely on factorisation properties of the observable
  - is NNLL accurate and extendable to higher orders
  - is fully general for a very broad category of observables (~all that can be possibly resummed to NNLL)
  - is flexible and automated (only input: observable’s routine)
Definition of the problem

• The problem consists of describing the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way.

• To achieve that, we need to study the behaviour of both the cross section and the observable in the presence of an arbitrary number of emissions.

• We divide the problem (and its solution) in three parts:
  • Observable’s properties
  • Amplitudes in the logarithmic regime
  • Relevant phase space regions
Definition of the problem

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  • Relevant phase space regions
Observable’s properties to all orders

- We consider an Infrared and Collinear (IRC) safe observable normalised as

\[ V = V(\{\tilde{p}\}, k_1, ..., k_n) \leq 1 \]

- We study the limit \( V \to 0 \)

- In this limit the radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions - QCD amplitudes factorise in these regimes w.r.t. the Born up to regular (giving rise to non-logarithmic corrections) terms

\[
|\mathcal{M}(\{\tilde{p}\}, k_1, ..., k_n)|^2 \approx |M_{\text{Born}}(\{\tilde{p}\})|^2 |M(k_1, ..., k_n)|^2 + \ldots
\]

- “Standard” approach: factorisation is achievable if also the (constrained) phase space factorises (contains observable’s definition). Resummation can be achieved once a factorisation theorem is available (modulo complex calculations)
Observable’s properties to all orders

• The observable does not trivially factorise in a product of terms arising from each kinematic mode contributing to the factorised amplitude - it often requires to transform into a conjugate space where the factorisation is explicit (e.g. Mellin - Laplace, Fourier)

• OK for simple semi-inclusive cases: e.g. thrust in e+e-

\[ 1 - T \simeq \sum_{i=1}^{n} \frac{k_{ti}}{Q} e^{-\eta_i} \quad \rightarrow \quad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau} \prod_{i=1}^{n} e^{-\nu \frac{k_{ti}}{Q} e^{-\eta_i}} \]

• Tedious for involved observables: e.g. jet broadening in e+e- or inclusive vector-boson kt in hadron collisions

• KO for observables which mix various kinematic modes or require iterative optimisations: e.g. 2 jet rate, thrust major

• Factorisation is an unnecessary request for resummation. All one needs is some scaling properties of the observable
Requirements on the observable

• Parametrisation for single emission and collinear splitting

\[ V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i ; \quad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \mu^2 = (\kappa_{ia} + \kappa_{ib})^2 / \kappa_{ii}^2 \]

• The standard requirement of IRC safety implies that

\[
\begin{align*}
\lim_{\zeta_{m+1} \to 0} V(\{\tilde{p}\}, \kappa_1(\bar{\nu}\zeta_1), \ldots, \kappa_m(\bar{\nu}\zeta_m), \kappa_{m+1}(\bar{\nu}\zeta_{m+1})) &= V(\{\tilde{p}\}, \kappa_1(\bar{\nu}\zeta_1), \ldots, \kappa_m(\bar{\nu}\zeta_m)) \\
\lim_{\mu \to 0} V(\{\tilde{p}\}, \kappa_1(\bar{\nu}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{\nu}\zeta_i, \mu), \ldots \kappa_m(\bar{\nu}\zeta_m)) &= V(\{\tilde{p}\}, \kappa_1(\bar{\nu}\zeta_1), \ldots, \kappa_i(\bar{\nu}\zeta_i), \ldots \kappa_m(\bar{\nu}\zeta_m))
\end{align*}
\]

• We limit ourselves to \textit{continuously} global observables*, i.e. the transverse momentum dependence is the same everywhere (it ensures the absence of non-global logarithms)

*Not a real limitation, although currently NNLL structure of non-global logarithms unknown
Requirements on the observable

• Parametrisation for single emission and collinear splitting

\[
V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i; \quad \kappa_i(\zeta) \rightarrow \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \quad \mu^2 = (\kappa_{ia} + \kappa_{ib})^2 / \kappa_{li}^2
\]

• Impose the following conditions, known as recursive IRC (rIRC) safety [Banfi, Salam, Zanderighi]

\[
\lim_{v \rightarrow 0} \frac{1}{v} V(\{\tilde{p}\}, \kappa_1(\tilde{v}\zeta_1), \ldots, \kappa_m(\tilde{v}\zeta_m)) = l i m \bar{v} \rightarrow 0 \bar{v} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \quad (1)
\]

• The above limit must be well defined and non-zero (except possibly in a phase space region of zero measure)

• Condition (1) simply requires the observable to scale in the same fashion for multiple emissions as for a single emission (IRC divergences have an exponential form)

• It is enough to ensure the exponentiation of double logarithms to all orders
Requirements on the observable

• Parametrisation for single emission and collinear splitting

\[ V(\{\vec{p}\}, \kappa_i(\zeta_i)) = \zeta_i; \quad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \mu^2 = (\kappa_{ia} + \kappa_{ib})^2 / \kappa_{ii}^2 \]

• Impose the following conditions, known as recursive IRC (rIRC) [Banfi, Salam, Zanderighi]

\[ \lim_{\vec{v} \to 0} \frac{1}{\vec{v}} V(\{\vec{p}\}, \kappa_1(\vec{v}\zeta_1), \ldots, \kappa_m(\vec{v}\zeta_m)) \quad (1) \]

\[ \lim_{\zeta_{m+1} \to 0} \lim_{\vec{v} \to 0} \frac{1}{\vec{v}} V(\{\vec{p}\}, \kappa_1(\vec{v}\zeta_1), \ldots, \kappa_m(\vec{v}\zeta_m), \kappa_{m+1}(\vec{v}\zeta_{m+1})) \]

\[ = \lim_{\vec{v} \to 0} \frac{1}{\vec{v}} V(\{\vec{p}\}, \kappa_1(\vec{v}\zeta_1), \ldots, \kappa_m(\vec{v}\zeta_m)) \quad (2.a) \]

\[ \lim_{\mu \to 0} \lim_{\vec{v} \to 0} \frac{1}{\vec{v}} V(\{\vec{p}\}, \kappa_1(\vec{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\vec{v}\zeta_i, \mu), \ldots \kappa_m(\vec{v}\zeta_m)) \]

\[ = \lim_{\vec{v} \to 0} \frac{1}{\vec{v}} V(\{\vec{p}\}, \kappa_1(\vec{v}\zeta_1), \ldots, \kappa_i(\vec{v}\zeta_i), \ldots \kappa_m(\vec{v}\zeta_m)) \quad (2.b) \]
Requirements on the observable

• Parametrisation for single emission and collinear splitting

\[ V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i; \quad \kappa_i(\zeta) \rightarrow \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ii}^2 \]

• Impose the following conditions, known as recursive IRC (rIRC) safety

\[
\lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \kappa_m(\bar{\zeta}_m)) = l_i \enspace V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \kappa_i(\bar{\zeta}_i), \ldots, \kappa_m(\bar{\zeta}_m)) \quad (1)
\]

\[
\lim_{\zeta_{m+1} \to 0} \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \kappa_m(\bar{\zeta}_m), \kappa_{m+1}(\bar{\zeta}_{m+1}))
\]

\[
= \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \kappa_m(\bar{\zeta}_m)) \quad (2.a)
\]

\[
\lim_{\mu \to 0} \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{\zeta}_i, \mu), \ldots, \kappa_m(\bar{\zeta}_m))
\]

\[
= \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{\zeta}_1), \ldots, \kappa_i(\bar{\zeta}_i), \ldots, \kappa_m(\bar{\zeta}_m)) \quad (2.b)
\]
Requirements on the observable

- Parametrisation for single emission and collinear splitting

\[ V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i ; \quad \kappa_i(\zeta) \rightarrow \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu), \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ii}^2 \]

- Impose the following conditions, known as recursive IRC (rIRC) safety

[Banfi, Salam, Zanderighi]

- Conditions (2.a) and (2.b), in addition to plain IRC safety, require that for sufficiently small \( \bar{v} \) there exists some \( \epsilon \) that can be chosen independently of \( \bar{v} \) such that we can neglect any emissions at scales \( \sim \epsilon \bar{v} \)

- The order with which one takes the limit is different in fixed-order and resummed calculations, and the final result must not change

\[
\lim_{\mu \to 0} \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m)) = \lim_{\bar{v} \to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \quad (2.b)
\]
Definition of the problem

• The problem consists of describing the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way

• To achieve that, we need to study the behaviour of both the cross section and the observable in the presence of an arbitrary number of emissions

• We divide the problem (and its solution) in three parts
  
  • Observable's properties
  
  • Amplitudes in the logarithmic regime
  
  • Relevant phase space regions
Virtual and (unresolved) real emissions

• RGE evolution for virtual corrections leads to a complete exponentiation
  [Parisi; Magnea, Sterman; later generalised to any process]

• We define a subset of (unresolved) emissions, such that \( V_{\text{sc}}(\{\tilde{p}\}, k_i) < \epsilon v \)

  • the soft-collinear approximation is enough to ensure the cancellation of the IRC poles arising from the virtual corrections (a different scheme can be defined - final result is scheme-invariant)

• By definition of rIRC safety, these emissions do not contribute significantly to the observable

\[
V(\{\tilde{p}\}, k_1, \ldots, k_n, \ldots, k_m) \simeq V(\{\tilde{p}\}, k_1, \ldots, k_n) + \epsilon^p v, \quad p \text{ some positive power}
\]

• Unresolved radiation largely unconstrained - squared amplitudes exponentiate by classical QCD exponentiation theorems
  [Soft non-abelian exp.: Frenkel, Gatheral, Taylor - Collinear evolution: Altarelli, Parisi]

• Using rIRC safety and RGE evolution equations it is possible to show that these unresolved and virtuals sum up to an exponential function
  [Work in progress…]
Virtual and (unresolved) real emissions

- The combination of unresolved real and virtual corrections gives rise to an exponential factor that defines the no-emission probability at scales larger than $\varepsilon v$
  \[ P(\text{no \ emissions}) \sim e^{-R(\varepsilon v)} \]

- Since we’re interested in vetoing emissions above $v$, this can be further expanded as
  \[ P(\text{no \ emissions}) \sim e^{-R(v) - R'(v) \ln \frac{1}{\varepsilon} + \frac{1}{2!} R''(v) \ln^2 \frac{1}{\varepsilon} + \ldots} \]

- The factor $R(v)$ is called the radiator, and defines the physical region where no radiation is allowed

- Owing to the above definition of unresolved emissions, the radiator is universal for all observables with the same soft-collinear scaling in the presence of a single emission

- The remaining terms in the above expansion cancel against the singularities of the resolved real emissions
Virtual and (unresolved) real emissions

- The resulting cumulative resummed cross section takes the form

\[
\Sigma(v) = \int_{0}^{v} \frac{1}{\sigma_{\text{Born}}(v')} \frac{d\sigma}{dv'} dv' \sim e^{-R(v)} F(v)
\]

- Property (1) of rIRC safety ensures that all double logarithms are contained in the radiator, and that the multiple emissions function is non trivial (at most) at NLL level

- Because of the lower cutoff defined by the unresolved emissions, the remaining (resolved) real emissions are bounded both from above and from below (i.e. each emission “loses” one logarithm power)
Soft (resolved) matrix elements

• It is useful to decompose the matrix element for n soft emissions (w.r.t. the Born) as a sum of terms with an increasing number of color-correlated emissions

\[ \begin{align*}
&\sim \alpha_s^n L^n \\
+ &\sim \alpha_s^n L^n \\
+ &\sim \alpha_s^n L^n \\
\vdots &
\end{align*} \]

• Which diagrams do we need to achieve NNLL (in the above picture neglect terms of order \( \sim \alpha_s^n L^{n-2} \))?
Soft (resolved) matrix elements

- It is useful to decompose the matrix element for \( n \) soft emissions (w.r.t. the Born) as a sum of terms with an increasing number of color-correlated emissions.

\[
\begin{align*}
\sim \alpha_s^n L^n & \sim \alpha_s^n L^{n-1} \\
& \sim \alpha_s^n L^n \\
& \sim \alpha_s^n L^{n-2} \\
\end{align*}
\]

- Which diagrams do we need to achieve NNLL (in the above picture neglect terms of order \( \sim \alpha_s^n L^{n-2} \) )?
Collinear (resolved) matrix elements

• The above counting allows us to keep (at NNLL) only configurations with an arbitrary number of independent soft-collinear emissions, and a single soft gluon branching either in a quarks or gluons pair

• This neglects contributions from the real matrix element which would give rise to subleading contributions

• Analogously, if we repeat the same analysis in the hard collinear limit, we find that only a single hard emission can be emitted collinearly to the Born hard leg at this logarithmic order (accompanied by the usual ensemble of soft-collinear emissions)

• Terms beyond this accuracy can be systematically included for extensions to higher orders (if ever necessary)
Definition of the problem

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• To achieve that, we need to study the behaviour of both the cross section and the observable in the presence of an arbitrary number of emissions.

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Phase space at NLL with \( n = 2 \) legs

- Once we have defined a way to handle the relevant amplitudes, we need to integrate them over the proper phase space in order to achieve a given logarithmic accuracy.

- At NLL the multiple emission function is given by an ensemble of soft and collinear independent (abelian) emissions widely separated in rapidity.

\[
F_{\text{NLL}}(v) = \langle \Theta(1 - \lim_{v \to 0} \frac{V_{\text{sc}}(\tilde{p}, \{k_i\})}{v}) \rangle
\]

- Differences in rapidity bounds of different emissions contribute at NNLL (can be neglected at this order).
Phase space at NNLL with \( n = 2 \) legs

- Extension to NNLL involves additional kinematic configurations:
  - (at most) two soft-collinear emissions get close in rapidity

\[
\delta F_{\text{correl}}(\lambda) = \int_{0}^{\infty} \frac{d\zeta}{\zeta} \int_{0}^{2\pi} \frac{d\phi}{2\pi} \sum_{a=1,2} \left( \frac{2C_{\ell_a} \lambda R_{\ell_a}''(v)}{\beta_0} \alpha_s(Q) \right) \int_{0}^{\infty} \frac{dk}{k} \int_{-\infty}^{\infty} d\eta \int_{0}^{2\pi} \frac{d\phi}{2\pi} \frac{1}{2!} C_{ab}(\kappa, \eta, \phi) \times 
\]

\[
\times \int d\mathcal{Z}\{R'_{\text{NLL,}l_i, k_i}\}[\Theta(v - V_{\text{sc}}(\{\bar{p}\}, k_a, k_b, \{k_i\}))-\Theta(v - V_{\text{sc}}(\{\bar{p}\}, k_a + k_b, \{k_i\}))]
\]

\[
C_{ab}(\kappa, \eta, \phi) = \frac{\tilde{M}^2(k_a, k_b)}{M^2_{\text{sc}}(k_a)M^2_{\text{sc}}(k_b)}
\]

All corrections in terms of four-dimensional integrals
Phase space at NNLL with $n=2$ legs

[Banfi, McAslan, Monni, Zanderighi]

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)

- Corrections affect both matrix element ($hc$) and observable ($rec$)

\[
\delta F_{hc}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b\epsilon)}Q)}{\alpha_s(Q)(a + b\epsilon)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int dZ[\{R_{\text{NLL},\ell_i, k_i}\}] \times \\
\quad \times \int_0^1 \frac{dz}{z} (zp_\ell(z) - 2C_\ell) \left[ \Theta \left( 1 - \lim_{v\to 0} \frac{V_{\text{sc}}(\{\hat{p}\}, k, \{k_i\})}{v} \right) - \Theta \left( 1 - \lim_{v\to 0} \frac{V_{\text{sc}}(\{\hat{p}\}, \{k_i\})}{v} \right) \right] \Theta(1 - \zeta)
\]

\[
\delta F_{rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b\epsilon)}Q)}{\alpha_s(Q)(a + b\epsilon)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int dZ[\{R_{\text{NLL},\ell_i, k_i}\}] \times \\
\quad \times \int_0^1 dz p_\ell(z) \left[ \Theta \left( 1 - \lim_{v\to 0} \frac{V_{hc}^{(k')}(\{\hat{p}\}, k', \{k_i\})}{v} \right) - \Theta \left( 1 - \lim_{v\to 0} \frac{V_{\text{sc}}(\{\hat{p}\}, k, \{k_i\})}{v} \right) \right]
\]
Phase space at NNLL with $n=2$ legs

[Banfi, McAslan, Monni, Zanderighi]

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one soft-collinear emission has the correct rapidity bounds (approximated in the NLL ensemble)

$$
\delta F_{sc}(\lambda) = \frac{\pi}{\alpha_s(Q)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \sum_{\ell=1,2} \left( \delta R'_{NNLL, \ell} + R'_{\ell_i} \ln \frac{d_{\ell g\ell}(\phi)}{\zeta} \right) \int \mathcal{D}[\{R'_{NLL, \ell_i, k_i}\}] \times \left[ \Theta \left(1 - \lim_{v \to 0} \frac{V_{sc}(\{\bar{p}, k, \{k_i\})}{v}\right) - \Theta(1 - \zeta) \Theta \left(1 - \lim_{v \to 0} \frac{V_{sc}(\{\bar{p}, k, \{k_i\})}{v}\right) \right],
$$

- (at most) one soft emission can have very small rapidity (wide angle)

$$
\delta F_{wa}(\lambda) = \frac{2C_F}{a} \frac{\alpha_s(v^{1/a}Q)}{\alpha_s(Q)} \int_0^\infty \frac{d\zeta}{\zeta} \int_{-\infty}^{\infty} d\eta \int_0^{2\pi} \frac{d\phi}{2\pi} \int \mathcal{D}[\{R'_{NLL, \ell_i, k_i}\}] \times \left[ \Theta \left(1 - \lim_{v \to 0} \frac{V_{wa}^{(k)}(\{\bar{p}, k, \{k_i\})}{v}\right) - \Theta \left(1 - \lim_{v \to 0} \frac{V_{sc}(\{\bar{p}, k, \{k_i\})}{v}\right) \right],
$$

**With $n>2$ there are additional (NLL) contributions due to the colour correlation between hard legs**

[Botts, Sterman; Kidonakis, Oderda, Sterman et al.]
Application to processes with $n = 2$ legs

- $pp \rightarrow H \ (Z)$ production with a jet veto at the LHC

- Need to suppress massive background due to $t\bar{t} \rightarrow W^+W^-b\bar{b}$

- Veto all jets with a transverse momentum larger than $p_{t,veto} \approx 25 - 30$ GeV

- Jets defined with a $kt$ class clustering algorithm ($p=0,1,-1$), i.e.

$$d_{ij} = \min(k_{ti}^p, k_{tj}^p) \frac{\sqrt{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}}{R}, \quad d_{iB} = k_{ti}^p$$
Application to processes with $n=2$ legs

• $pp\rightarrow H$ (Z) production with a jet veto at the LHC

  * Massive reduction of the theory uncertainty in the $pp\rightarrow H$ case

*NNLO fixed order from MCFM + h(dy)nnlo

[Banfi, Monni, Salam, Zanderighi]

[Campbell, Ellis, Williams; Grazzini et al.]
Application to processes with $n=2$ legs

- event shapes in $e^+e^- \rightarrow 2$ jets production

- Define observables sensitive to QCD corrections

- Relevant (e.g.) for precise determination of the strong coupling constant - deviation from a dijet proportional to the strong coupling

- Toy model for final-state radiation (conceptually complete)

- Clean theory/exp laboratory to study non-perturbative corrections
Application to processes with $n = 2$ legs

• Reproduce known results and present new ones

• thrust and heavy jet mass (known)

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \quad \rho_H = \max_{i=1,2} \frac{M_i^2}{Q^2}, \quad M_i^2 = \left( \sum_{j \in \mathcal{H}^{(i)}} p_j \right)^2$$

• total and wide jet broadening (known)

$$B_L = \sum_{i \in \mathcal{H}^{(1)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q}, \quad B_R = \sum_{i \in \mathcal{H}^{(2)}} \frac{|\vec{p}_i \times \vec{n}_T|}{2Q} \quad B_W = \max\{B_L, B_R\} \quad B_T = B_L + B_R$$

• C parameter (new)

$$C \equiv 3 \left( 1 - \frac{1}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)} \right)$$

• thrust major and oblateness (new)

$$T_M = \max_{\vec{n} \cdot \vec{n}_T = 0} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \quad T_m = \frac{\sum_i |p_{i,x}|}{Q} \quad O = T_M - T_m$$
Application to processes with $n = 2$ legs

- event shapes in $e^+e^- \rightarrow 2$ jets production

*NNLO fixed order from EERAD3

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich]

[Banfi, McAslan, Monni, Zanderighi]
Application to processes with $n=2$ legs

- Observables with very different logarithmic structures can be modelled with the same method (fully general)

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Conclusions

• Novel general method for the resummation of any rIRC safe, global two-scales observable

  • weak applicability conditions

  • Cancellation of virtual poles against unresolved radiation is performed analytically using dimensional regularisation

  • Contribution of resolved real radiation formulated in terms of four dimensional integrals (suitable for efficient Monte Carlo implementation)

  • New results already available

• Modulo technical work for $n>2$ the NNLL resummation for any two scales global observables is a theoretically solved problem
Future developments and work in progress

- Application to resummation for non-trivial jet resolution parameters (no analytic form already at NLL, non factorisable beyond NLL)
  
  [Work in progress…]

- The method offers a solid theoretical framework to study the logarithmic behaviour of a Parton Shower
  
  [Work in progress…]

  - Determine the accuracy of existing generators

  - Can we learn how to design a more accurate one?

- Extend the method to processes with $n>2$ hard Born legs (theoretically solved - mainly technical work)

- Study of all-order structure of 3 scale problems (e.g. nested resummations, pp $\rightarrow$ H+1 jet,…)

- Structure of non-global logarithms still unknown at this order (theoretically attractive - some recent progress in this direction)